

# Stabilization of Angular-Shaped Riprap under Overtopping Flows

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**Abstract**—Riprap is mostly used to prevent erosion by flows down the steep slopes in river engineering. A total of 53 stability tests performed on angular riprap with a median stone size ranging from 15 to 278 mm and slope ranging from 1 to 40% are used in this study. The existing equations for the prediction of medium size of angular stones are checked for their accuracy using the available data. Predictions of median size using these equations are not satisfactory and results show deviation by more than  $\pm 20\%$  from the observed values. A multivariable power regression analysis is performed to propose a new equation relating the median size with unit discharge, bed slope, riprap thickness and coefficient of uniformity. The proposed relationship satisfactorily predicts the median angular stone size with  $\pm 20\%$  error. Further, the required size of the rounded stone is more than the angular stone for the same unit discharge and the ratio increases with unit discharge and also with embankment slope of the riprap.

**Keywords**—Angularity, Gradation, Riprap, Stabilization

## I. INTRODUCTION

ROCK protection currently referred to as riprap is used in protecting bridge abutments, piers, channels, guide banks, spillways, embankment dams, and other structures vulnerable to erosion. Gradation and shape has a significant effect on the stability of riprap. Well-graded rocks enhance stability in situations of overtopping than uniformly graded rocks due to better interlocking of rocks. Further, the interstitial velocity of water is much higher in a uniformly graded rock layer compared to well-graded which results in instability of the riprap. With regard to shape, “angularity” is often used as a qualitative descriptor of shape, in as much as it affects the angle of repose. “Angular” particles are defined as having sharply defined edges and corners, whereas “rounded” particles are more potato-shaped, having been worn and abraded by physical contact, typically during fluvial transport. Intermediate between these two extremes are particles that are “sub-angular” or “sub-rounded.”

Current design procedures prefer stones tending toward sub-angular to angular due to the higher degree of interlocking, unity of rock mass and, hence greater stability, compared to rounded particles of the same weight and size. However angular shaped stones are not easily available at all construction locations as compared to round shaped stones. However, if a rock quarry or a crushing operation site is

located at lesser haul distance, then apart from increasing the stability; it will also reduce the project cost and hence will led to efficient and economical solution.

## II. REVIEW OF TECHNICAL LITERATURE

Stability is an important aspect in the design of a riprap. Several investigators have studied riprap stability on steep slopes when subjected to flow [5], [7]-[8], [14]-[15], [19]-[20], [22]. Isbash [10] studied the construction of rock-fill dams by dumping stones into flowing rivers and formulated a dimensionally homogenous relationship which relates minimum velocity necessary to move stones of known size and specific gravity. Flow hydraulics on steep embankment slopes cannot be analyzed with standard flow and sediment transport equations. Uniform flow and tractive shear equations do not apply to shallow flow over large roughness elements, highly aerated flow, or chute and pool flow—all of which can occur during overtopping. Riprap design criteria for overtopping protection of embankment dams should prevent stone movement and ensure the riprap layer does not fail. An empirically derived design criterion offers a better approach for design for riprap. Riprap design to resist overtopping flow is dependent upon the material properties (median size, shape, gradation, porosity, and unit weight), the hydraulic gradient or embankment slope, and the unit discharge.

Oliver [13] developed a rock stability theory on the basis of flume study pertaining to flow through and over rock-fill dams. He empirically derived a relationship for overtopping flow linking the design parameters of unit flow, slope, and median rock size for crushed or rough stones to threshold flow.

Hartung and Scheuerlein [9] presented a set of stability equations for angular stones on steep slopes that incorporate the effects of flow aeration. Their theory incorporated the results of extensive aerated flow observations over fixed rock beds coupled with the equation for the stability of rock dumped underwater developed by Isbash [10]. They determined that the stone stability could be related to the maximum unit discharge, roughness height, specific weight of water, specific weight of the stone, and the angle of the embankment slope.

Knauss [11] developed a rock stability function based on unit discharge, slope, rock packing, and air concentration for sizing riprap, and determined that aeration of flow increases the critical velocity for which riprap on a steep slope remains stable.

Abt et al. [2]-[3] and Abt and Johnson [1] conducted experimental study on a riprap with 25.4–152 mm median stone diameters and having angular and round shape on steep beds with slope range of 1–20%. On the basis of their results, the

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following relationship was developed for sizing angular stones to resist movement for a specific design unit discharge.

$$D_{50} = 0.5074 S^{0.43} q_d^{0.56} \quad (1)$$

$$q_d = 1.35q_f \quad (2)$$

where  $D_{50}$  = size of graded rock representing 50% finer;  $S$  = embankment slope; and  $q_d$  = design discharge and  $q_f$  is the critical unit discharge. On comparing the test results of both the round and angular-shaped stones, they found that the round-shaped stones failed at a unit discharge of 32 and 45% lower than the angular stone for 52 and 104 mm stone sizes, respectively.

Robinson et al. [17]-[18] and Rice et al. [16] comprehensively investigated the design aspect of rock chutes. Riprap, with median stone diameters of 15–278 mm, were placed on embankment slopes of 2–40% and tested in overtopping conditions. The riprap was angular in shape with coefficients of uniformity  $C_u = D_{60}/D_{10}$  varying from 1.47 to 1.73. They developed a two stage prediction equation from their stability tests as follows;

$$D_{50} = \left[ \frac{q_f S^{1.5}}{9.76 \times 10^{-7}} \right]^{1/1.89} \quad S < 0.10 \quad (3)$$

$$D_{50} = \left[ \frac{q_f S^{0.58}}{8.07 \times 10^{-6}} \right]^{1/1.89} \quad 0.10 \leq S \leq 0.40 \quad (4)$$

where  $D_{50}$  in mm and  $q$  in  $m^2/s$ . Equations (3) and (4) apply to rock chute constructed with angular riprap with a rock layer thickness of  $2D_{50}$ .

Mishra [12] carried out angular riprap tests on a near-prototype size embankment built at 50% slope, under a cooperative agreement between the U.S. Bureau of Reclamation (USBR) and Colorado State University (CSU). Failure was defined as removal of the riprap by erosion and movement of rock until bedding material was exposed. From the experimental results, it was found that the interstitial velocity of water is strongly influenced by the void sizes inside the rock layer. The void sizes are determined by the gradation of the rock. For this application the coefficient of uniformity  $C_u (D_{60}/D_{10})$  provides a good representation of the rock gradation and should be a factor in the predictive equation for the interstitial velocity of water. The USBR/CSU studies, which took into account the data obtained from previous studies, did show that the predictive (1) by Abt and Johnson [1] under-predicts the interstitial velocity for large riprap [12]. The equation for interstitial velocity  $V_i$  developed in the USBR/CSU study is

$$\frac{V_i}{\sqrt{gD_{50}}} = 2.48C_u^{-2.22} S^{0.58} \quad (5)$$

A universal formula for designing the riprap was derived which satisfactorily predict the size of the riprap to be used for a specified unit discharge and a given embankment slope;

$$D_{50} C_u^{0.25} = K_u (q_f)^{0.52} S^{-0.75} \times \left( \frac{\sin \theta}{(G \cos \theta - 1)(\cos \theta \tan \phi - \sin \theta)} \right)^{1.11} \quad (6)$$

Where  $K_u = 0.55$  in SI units;  $G$  = Specific gravity of rock;  $\theta$  = Slope of the embankment;  $\phi$  = angle of repose of the riprap material.

Ullmann [21] conducted a series of experiments to investigate round-shaped stones placed on embankments with slope varying from 0.2 to 0.3. The characteristic size  $D_{50}$  varied from 24 to 99 mm and coefficients of uniformity  $C_u$  from 1.21 to 1.33. Stone roundness ( $R$ ) ranged from 76 to 95%. He found that the determination of a stable round-shaped stone size could be expressed as a function of unit discharge, embankment slope, coefficient of uniformity, and the percent of stone roundness. It was observed that round-shaped stones should be approximately 47% larger than angular rocks to remain stable at the same slope and discharge respectively. As per the experimental results, the design equation was developed for round shaped stone in overtopping flow and was expressed as;

$$C_u^{0.25} \frac{D_{50}}{(0.0112R + 0.39)} = 6.48S^{0.43} q_f^{0.56} \quad (7)$$

Peirson and Cameron [14] made a comparison of the large scale flow test data (Table 1) for crushed angular materials with available theoretical design approaches for rock protection against erosion by flows down steep slopes.

TABLE 1  
DETAILS OF DATA USED BY PEIRSON AND CAMERON [14]

Investigator	Rock size $D_{50}$ (mm)	Slope Range	No. of data points available
Oliver [13]	13-59	0.08-0.2	32
Hartung and Scheuerlein [9]	45-180	0.1-0.667	0
Abt et al. [2]	51,102	0.1	8
Abt et al. [3]	26,56	0.01-0.1	4
Abt and Johnson [1]	26-158	0.01-0.2	21
Robinson et al. [18]	15-192	0.02-0.4	32

As per their study, the Hartung and Scheuerlein [9] design method is not conservative and a criterion can be developed for bed instability based on the applied bed stress being equal to the critical stress to induce motion. After analyzing the database, they developed a relationship for stability of the form;

$$q_f = 0.0781 \sqrt{g} (\sin \theta)^{-7/6} D_{50}^{1.5} [\cos \theta (\tan \phi - \tan \theta) (\rho_s - \rho) / \rho]^{5/3} \quad (8)$$

where  $\rho$  = density of water;  $g$  = acceleration due to gravity; and  $\rho_s$  = density of armor.

Abt et al. [4] analyzed the existing database for round shaped stones with median size ranging from 32.3 to 99.1 mm and within the slope range of 10 to 45%. As per their study, requisite round shaped stones range from 5 to 42% larger than angular stones to stabilize the riprap layer for similar flow conditions with unit flows of 0.002 m<sup>2</sup>/s and slopes of 40%. They developed a simple relationship for round shaped stones as;

$$D_{50} = 0.9782 C_u^{0.70} S^{0.70} q_f^{0.68} \quad (9)$$

Eli and Gray [6] investigated the hydraulic performance of a steep single layer riprap ( $t/D_{50} = 1$ ) drainage channel. They placed a uniform size angular riprap stone on the channel bottom at slope 0.5. On the basis of their experimental results, they developed a relationship which relates the stable discharge with mean depth ( $d$ ) and channel slope ( $\theta$ ) at critical flow conditions;

$$q_f = d^{1.5} \sqrt{g \cos \theta} \quad (10)$$

Equation (10) is valid within the tested range of  $0.25 < d/D_{50} < 0.5$ . They also observed that at very steep slopes of the order of 50%, the weight component of water in the downstream flow direction is considerable.

Now as per literature review, there is sufficient information for sizing the angular stable riprap for placement on embankment slopes ranging from 1 to 40% as a function of the unit discharge, mean layer thickness, coefficient of uniformity and embankment slope. Robinson et al. [18] had place riprap in two layers with thickness to median stone size ratio i.e.  $t/D_{50}$  equal to 2, whereas in case of Abt and Johnson [1], this ratio varies from 1.9 to 3. The objective of this study is to check the accuracy of the existing equations for designing the size of the riprap boulders and to propose a new relationship using the available data in the literature. This will provide a design criterion for the potential use of angular shaped stone as erosion protection.

### III. COLLECTED DATA

Data related to stability tests of angular stones riprap has been collected from the literature. A total 53 data sets from Robinson et al. [18] and Abt and Johnson [1] were collected and listed in Table 2 (Appendix). Experimental set up and procedure adopted for studying the stability of riprap by the investigators are briefly described herein.

Robinson et al. [18] performed a total of 32 rock chute stability tests in three separate rectangular flumes with widths of 0.76, 1.07 and 1.83m, respectively on slopes ranging from 2 to 40%. Angular crushed limestone with a  $D_{50}$  of 15 to 278 mm was placed in layers with thickness to  $D_{50}$  ratio i.e. ( $t/D_{50}$ ) equal to 2 for all the stability tests. The specific gravity of stones varied from 2.54 to 2.82 whereas coefficient of uniformity varied from 1.25 to 1.73. The tests were performed by introducing a base flow in the rock chute, then increasing the flow incrementally. Failure was defined as the flow

condition that exposed the underlying geo-fabric or bedding material. Initial stone movement was generally observed at flow rates below the design discharge. As the flow rate reached, the highest stable discharge, larger stones were observed to vibrate, move on the slope, and/ or tilt up into the flow. At higher discharges channelization and scour holes were observed in the rocky layer and the chute was considered to have failed.

Abt and Johnson [1] performed a total of 21 stability tests on angular stones in two rectangular flumes having width of 2.4 m each, one outdoor flume for simulating steep slopes ( $\geq 10\%$ ) and other an indoor flume for simulating flatter slopes ( $< 10\%$ ). Each flume was modified to enable prototype testing of stone-covered embankments in order to evaluate flow conditions and stone movement. Angular limestone with  $D_{50} = 50.8$  to 157.5 mm was placed in layers with thickness to  $D_{50}$  ratio i.e. ( $t/D_{50}$ ) varying from 1.9 to 3. The specific gravity of stones varied from 2.65 to 2.72 whereas coefficient of uniformity varied from 1.69 to 2.30. The failure criterion of the riprap layer was when the filter blanket or geo-fabric was exposed.

TABLE II  
SUMMARY OF DATA OF ANGULAR SHAPED STONES

Test No.	$S$	$D_{50}$ (mm)	$C_u$	$\rho_s g$ kN/m <sup>3</sup>	$t$ (mm)	$q_f$ (m <sup>2</sup> /s)
(a) Robinson et al. [18]						
1	0.1	15	1.65	27	30	0.0057
2	0.125	15	1.65	27	30	0.0052
3	0.167	15	1.65	27	30	0.0037
4	0.222	15	1.65	27	30	0.0031
5	0.1	33	1.65	27	66	0.0248
6	0.125	33	1.65	27	66	0.0235
7	0.167	33	1.65	27	66	0.0186
8	0.222	33	1.65	27	66	0.0147
9	0.4	46	1.25	27	92	0.0381
10	0.1	52	1.72	28.2	104	0.0762
11	0.125	52	1.72	28.2	104	0.0624
12	0.167	52	1.72	28.2	104	0.0578
13	0.222	52	1.72	28.2	104	0.0483
14	0.4	52	1.72	28.2	104	0.0349
15	0.1	89	1.58	25.4	178	0.1738
16	0.125	89	1.58	25.4	178	0.1514
17	0.167	89	1.58	25.4	178	0.1596
18	0.222	89	1.58	25.4	178	0.1105
19	0.125	89	1.58	25.4	178	0.1663
20	0.222	89	1.58	25.4	178	0.1003
21	0.4	89	1.58	25.4	178	0.0865
22	0.125	145	1.54	25.5	290	0.3307
23	0.222	145	1.54	25.5	290	0.2239
24	0.4	145	1.54	25.5	290	0.1951

25	0.222	188	1.73	25.8	376	0.5416
26	0.4	188	1.73	25.8	376	0.3279
27	0.08	188	1.73	25.8	376	0.7525
28	0.06	52	1.72	28.2	104	0.1858
29	0.06	33	1.65	27	66	0.0892
30	0.04	33	1.65	26.5	66	0.183
31	0.02	15	1.65	27	30	0.0427
32	0.06	192	1.58	25.6	384	1.6258

(b) *Abt and Johnson [1]*

33	0.1	25.9	1.75	26.7	76	0.033
34	0.1	25.9	1.75	26.7	76	0.031
35	0.1	25.9	1.75	26.7	76	0.028
36	0.1	25.9	1.75	26.7	76	0.039
37	0.1	50.8	2.4	26.7	102	0.078
38	0.1	50.8	2.4	26.7	152	0.092
39	0.1	50.8	2.4	26.7	152	0.102
40	0.1	55.9	2.09	26.7	152	0.103
41	0.1	55.9	2.09	26.7	152	0.115
42	0.1	55.9	2.09	26.7	152	0.115
43	0.2	55.9	2.09	26.7	152	0.046
44	0.1	101.6	2.3	26	203	0.322
45	0.1	101.6	2.3	26	305	0.34
46	0.1	101.6	2.3	26	305	0.378
47	0.2	104.1	2.15	26	305	0.166
48	0.2	129.5	1.62	26	305	0.327
49	0.2	157.5	1.69	26	305	0.407
50	0.08	55.9	2.09	26.7	152	0.166
51	0.02	55.9	2.09	26.7	152	0.416
52	0.02	25.9	1.75	26.7	76	0.102
53	0.01	25.9	1.75	26.7	76	0.138

## IV. TESTS OF THE EXISTING EQUATIONS

A database of 53 angular-shaped stone stability tests conducted by Robinson et al. [18] and Abt and Johnson [1] with embankment slopes ranging from 1 to 40 % was used to test the accuracy of the existing equations for the prediction of  $D_{50}$  of stone. The unit discharge at rock failure varied from 0.003 - 0.752  $\text{m}^2/\text{s}$  for rock sizes of 15 to 278 mm, respectively.

The results from the 53 stability tests for angular shaped stones were evaluated using (9). The predicted  $D_{50}$  is compared to the observed one as shown in Fig. 1. It is evident that (9) prominently over-predicts the desired stone size to stabilize the riprap layer. Also this (9) doesn't take into account the effect of riprap layer thickness. It is to be noted that (9) was proposed by Abt et al. [4] for round shaped stones. Therefore, over-prediction indicates that the size of round stone required for stable riprap is more than the angular stone for the same flow parameters.

The other equations like (6) and (8) are too tedious to use i.e. contain too many parameters. Equation (10) is valid for only single layer and also at slope of 50%. The collected data was also used to check the accuracy of the (3) and (4) of Robinson et al. [18] and (1) of Abt and Johnson [1]. Comparison of predicted values of  $D_{50}$  of angular stones using (3), (4) and (1) with observed ones are also shown in Fig. 1. As evident from the Fig. 1, the predictions of  $D_{50}$  using these equations is not satisfactory and results show deviation by more than  $\pm 20\%$  from the perfect agreement line. Thus there is a need to devise a relationship which will fairly predict the desired angular stone size for the design discharge.

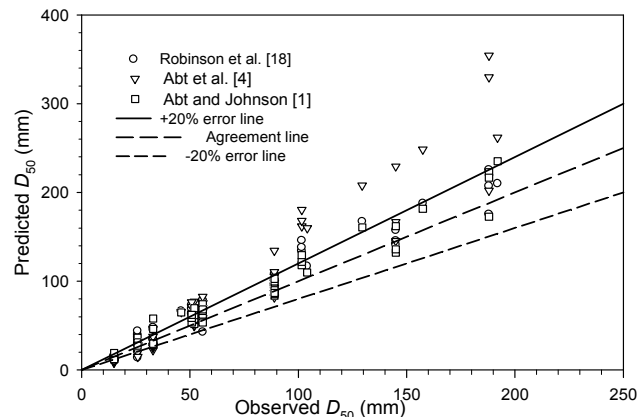


Fig. 1 Comparison of predicted  $D_{50}$  of stone using existing equation with observed one

V. PROPOSED EQUATION FOR  $D_{50}$ 

A multivariable power regression analysis was performed using the 53 angular shaped stone data points from Table 2 to include  $D_{50}$ ,  $C_u$ ,  $S$ ,  $t$  and  $q_f$ . The proposed relationship is

$$D_{50} = 0.66t^{0.58}S^{0.22}C_u^{-0.45}q_f^{0.22} \quad (\text{SI units}) \quad R^2=0.98 \quad (11)$$

The coefficient of uniformity  $C_u$  is an integral parameter that collapsed the database into a well defined relationship allowing the prediction of angular-shaped rock at failure in overtopping- flow conditions. The thickness parameter  $t$  which is taken as  $n$  times  $D_{50}$ , where  $n$  is the number of rock layers in which riprap has been placed on the sloping bed, was also taken into consideration to increase the validity of this relationship to multi-layered riprap.

Fig. 2 presents a comparison between the predicted  $D_{50}$  and the observed one within the tested range. Evident from Fig. 2, the proposed equation satisfactorily predicts the desired angular stone size within error of  $\pm 20\%$  of the observed one.

The required  $D_{50}$  for rounded and angular stones were predicted using (9) and (11), respectively for  $S = 10\text{-}40\%$ ;  $q_f = 0$  to 0.5  $\text{m}^2/\text{s}$ ;  $C_u = 1.65$  and for thickness of riprap equal to two times of the  $D_{50}$ . The variation of ratio of  $D_{50}$  for rounded and angular stones with unit discharge is shown in Fig. 3 for the different slopes. It is apparent that the required size of the rounded stone is more than the angular stone for the same unit

discharge. The ratio increases with unit discharge and also with embankment slope of the riprap.

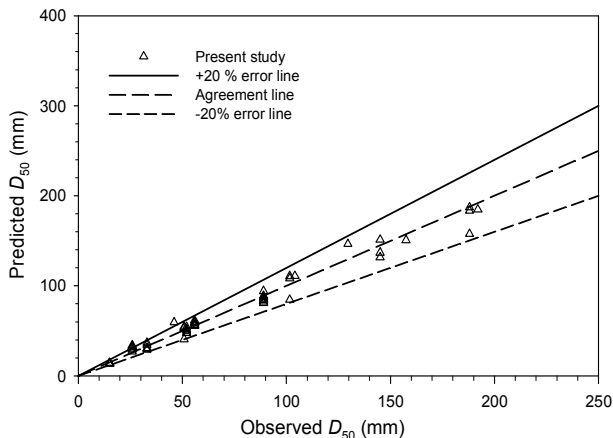


Fig. 2 Comparison of predicted  $D_{50}$  of stone using proposed equation with observed one

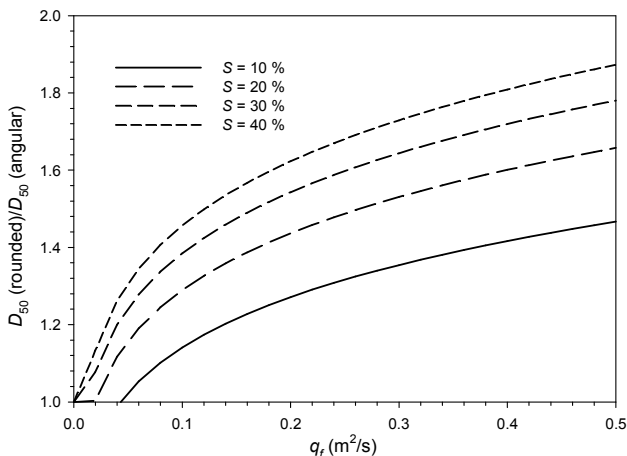


Fig. 3 Variation of ratio of  $D_{50}$  for rounded and angular stones with unit discharge

## VI. CONCLUSION

It can be concluded from this study that the predictions of  $D_{50}$  using the available equations are not satisfactory and results show deviation by more than  $\pm 20\%$  from the observed values. The developed equation using available 53 satiability tests of riprap, satisfactorily predicts the desired angular stone size within error of  $\pm 20\%$  of the observed one. The required size of the rounded stone is more than the angular stone for the same unit discharge. The ratio increases with unit discharge and also with embankment slope of the riprap.

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