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Texture Characterization Based on a Chandrasekhar Fast Adaptive filter

Mounir Sayadi and Farhat Fnaiech

Abstract—In the framework of adaptive parametric modelling of images, we propose in this paper a new technique based on the Chandrasekhar fast adaptive filter for texture characterization. An Auto-Regressive (AR) linear model of texture is obtained by scanning the image row by row and modelling this data with an adaptive Chandrasekhar linear filter. The characterization efficiency of the obtained model is compared with the model adapted with the Least Mean Square (LMS) 2-D adaptive algorithm and with the co-occurrence method features. The comparison criteria is based on the computation of a characterization degree using the ratio of "between-class" variances with respect to "within-class" variances of the estimated coefficients. Extensive experiments show that the coefficients estimated by the use of Chandrasekhar adaptive filter give better results in texture discrimination than those estimated by other algorithms, even in a noisy context.

Keywords—Texture analysis, Statistical features, Adaptive filters, Chandrasekhar algorithm.

I. INTRODUCTION

TEXTURE analysis plays an important role in several image processing and pattern recognition applications such as remote sensing, cartography, robot vision, military surveillance and medical imaging. It has long been the topic of intense research [1][6][13]. Texture can be found in the background of natural scenes as well, as filling elements of surface images, thus the textural features are an important pattern elements in image interpretation. Various methods for texture features extracting for texture characterisation have been proposed during the last two decades. One such method characterizes texture by discrete Wavelet representation [1][8][15]. Fractal based features have been also used as features for texture characterization [6]. These features depend mostly on textural characteristics than on intensity information. Several authors have made a comparison of the performance of various features for texture characterisation purpose. In addition, the co-occurrence matrix is a popular statistical technique for extracting textural features [15]. In [10], Ojala *et al* have compared four texture features: gray level differences, Laws texture features, center-symetric covariance features, and local binary patterns. One of the most promising methods for texture features extraction is the parametric modelling [7][12][13], where the coefficients of the AR parametric model are used for texture characterisation and synthesis.

M. Sayadi (Corresponding author) is with the Unit SICISI, High school of sciences and techniques of Tunis (ESSTT), 5 Av. Taha Hussein, 1008, Tunis, Tunisia. mounir.sayadi@esstt.rnu.tn

F. Fnaiech is with the director of the Unit SICISI, High school of sciences and techniques of Tunis (ESSTT), 5 Av. Taha Hussein, 1008, Tunis, Tunisia. farhat.fnaiech@esstt.rnu.tn

Various adaptive parametric filters have been proposed for texture characterization. In [12], the AR coefficients estimated with the 2-D Fast Lattice RLS filter [7] are used for texture classification in a noiseless context. In [13], an adaptive approach in texture characterization with the transversal 2-D AR coefficients obtained from the 2-D OLRIV (Overdetermined Lattice Recursive Instrumental Variable) algorithm is presented. This algorithm uses higher order statistics. It is compared in [12] to the 2-D Fast Lattice RLS algorithm, based on second order statistic. It has been shown that although the effect of the additive gaussian noise is more important on the 2-D Fast Lattice RLS filter than the 2-D OLRIV filter, the 2-D FLRLS provides a good characterization of the texture model. It presents a large classification sensitivity.

On the other hand, one of the approaches for decreasing the computation cost of the adaptive filtering algorithms is the use of the Chandrasekhar factorisation techniques [9][11]. The strength of this approach derives from the fact that it avoids the resolution of the standard Riccati equation of the Kalman filter. The derivation of fast adaptive algorithms based on Chandrasekhar fast equations using a state space model was presented in [3] and [4] for MA and ARMA linear filtering, respectively. It has been extended to the multichannel and non-linear filtering in [11]. The main contribution of the present work is the use of the coefficients issued from the fast Chandrasekhar adaptive filter as features for texture characterization. We will show how much these coefficients can improve the texture characterization in comparison with those estimated by other adaptive filters.

II. THE IMAGE PARAMETRIC ADAPTIVE LINEAR MODEL

A texture image of size $(L \times L)$ can be represented by a 2-D AR parametric linear model with quarter-plane support of order (p,q) i.e. $R = \{i, j : (0 \le i \le p-1, 0 \le j \le p-1)\}$ with $((i,j) \neq (0,0))$ as shown in Fig. 1. The value of a pixel at position (n,r) is represented by the following relationship:

$$y(n,r) = \sum_{l=0}^{p-1} \sum_{m=0}^{q-1} {}_{(l,m)\neq(0,0)} a_{lm} y(n-l, r-m) . \qquad (1)$$

n and r are in the interval $0..L$-1. In the stationary case, the 2-D AR filter coefficients a_{lm} don't depend on the position of the pixel (n,r).

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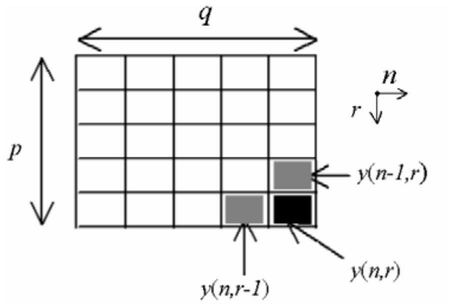

Fig. 1 The quarter-plane support 2-D scheme

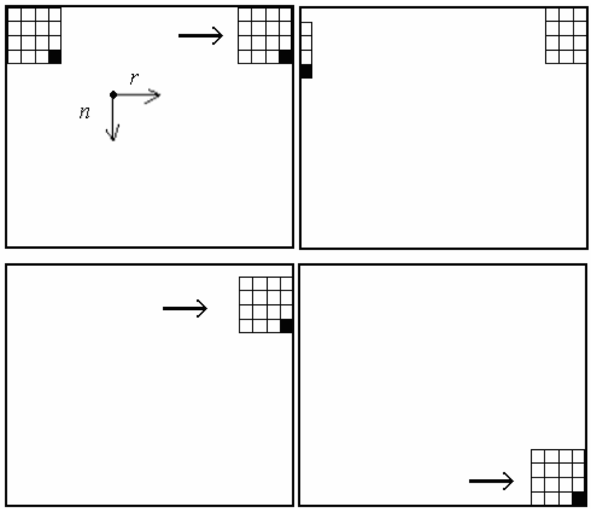

Fig. 2 The adaptive scanning mask from left to right and top to bottom

To produce an approximate value for a given pixel, all the values of the pixel image which are covered by the sliding mask *R*, weighted by the 2-D AR filter coefficients and summed. The mask is then moved to the following location (Fig. 2).

In the following, we give an overview on the 2-D Least Mean Square algorithm [5], based on the minimization of the mean square error between the filter output and the desired output. This algorithm is the most widely used adaptive filter due to its implementation simplicity.

The steps of the 2-D LMS algorithm, when it is used for modelling a texture *d* of size ($L\times L$), are [5]:

Step 1: Using a sliding window from left to right and top to bottom, calculate the adaptive filter output:

$$y(n,r)=\sum_{l=0}^{p-1}\sum_{m=0}^{q-1}{}_{(l,m)\neq(0,0)}\overset{k}{a}_{lm}.y(n-l,r-m) \qquad (2)$$

where $\overset{k}{a}_{lm}$ are the AR coefficients at index time *k,* being expressed according to coordinates of the pixel (n,r) by $k=nL+r$. The filter coefficients $\overset{k}{a}_{lm}$ are initialised to zero in the first iteration.

Step 2: Adaptation of the 2-D AR coefficients:

For l from 0 to $p-1$, For m from 0 to q-1 :

$$\overset{k+1}{a}_{lm}=\overset{k}{a}_{lm}+\mu\left(d(n,r)-y(n,r)\right)y(n-l,r-m)\,. \qquad (3)$$

In texture modelling, the desired output $d(n,r)$ is the gray level value of the pixel (n,r) of the texture image to be modelled. μ is the step size of the algorithm.

III. The proposed Chandrasekhar adaptive filter

To use the Chandrasekhar adaptive filter, we propose to scan the quarter plane model window row by row as shown in Fig. 3.

Equation (1) can be then written in the following form:

$$x(k)=\sum_{i=0}^{pq-1}b_i.x(k-i) \qquad (4)$$

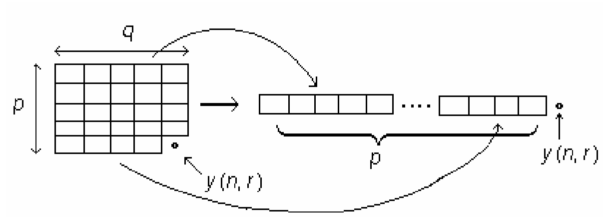

Fig. 3 The transformation of the 2-D model

In this model, two linear scanning indexes *i* and *k* are respectively $i=m+lq$ and $k=nL+r$.

The sample $x(k)$ corresponds to $y(n,r)$ and $x(k\text{-}1)$ to $y(n,r\text{-}1)$. The coefficients b_i corresponding to a_{lm} are set in a vector $\underline{b}$ of size $N=pq$-1. According to the works presented in [3][4] and [11], the Chandrasekhar fast adaptive algorithm can be applied to estimate the coefficients of the model (4).

We summarize in the following the steps of this algorithm applied in texture modelling: [11]

$$\underline{V}(k)=S(k)^t\begin{bmatrix}\underline{U}(k-1)\\ x(k)\end{bmatrix}\ , \qquad \text{where}$$

$$\underline{U}(k-1)=\left[x(k-N).........x(k-1)\right]^{\,t} \qquad (5)$$

$$\underline{W}(k)=M(k)\underline{V}(k) \qquad (6)$$

$$M(k+1)=M(k)+\underline{W}(k)(R(k))^{-1}\underline{W}^t(k) \qquad (7)$$

$$R(k+1)=R(k)+\underline{V}^t(k)\underline{W}(k) \qquad (8)$$

$$\underline{K}(k+1)=\begin{bmatrix}0\\ \underline{C}(k+1)\end{bmatrix}=\begin{bmatrix}\underline{C}(k)\\ 0\end{bmatrix}+S(k)\underline{W}(k) \qquad (9)$$

$$S(k+1)=S(k)-\underline{K}(k+1)(R(k+1))^{-1}\underline{V}^t(k) \qquad (10)$$

Prediction error: $e(k)=d(k)-\underline{U}(k-1)^t\ \underline{b}(k)$ (11)

Estimated coefficient vectors:

$$\underline{b}(k+1)=\underline{b}(k)+\underline{C}(k)(R(k))^{-1}e(k)\,. \qquad (12)$$

$d(k)$ represents the corresponding pixel of index (n,r) in the desired texture image. The algorithm is initialised with: $\underline{K}(1) = 0_{N+1}$ and $R(1) = \sigma_d^2$. By defining a pinning vector of length p: $\underline{\rho}_i^p = \left[0_{i-1}^t \, 1 \, 0_{p-i}^t \right]^t$, the initialisation of $M(1)$ and $S(1)$ are: $M(1) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $S(1) = \left[\sigma_d \underline{\rho}_1^{N+1} \middle| \, \sigma_d \underline{\rho}_{N+1}^{N+1} \middle| \, 0_{N+1} \right]$. The matrix 0_k is a k vectors whose elements are zeros. The initial values of the coefficients b_i are set to zero. For more details about the Chandrasekhar algorithm, the reader is referred to [11].

IV. Characterization Degree of the Filters Coefficients

Consider a set of 25 different gray-scale textures of (256×256) pixels extracted from the Brodatz album [2] (Fig. 5). From each texture, 100 sample images of (32×32) pixels are randomly chosen. Both Chandrasekhar filter and 2-D LMS AR coefficients are then evaluated for each one of the resulting 2500 sample images. A quarter-plane support with a sliding (3×3) order window. In order to check whether the use of the Chandrasekhar adaptive filter improves the texture characterization in comparison with the 2-D LMS adaptive filter, we propose to evaluate the characterization efficiency of the coefficients estimated by each adaptive filter. Thus we define a "characterization degree" J as the ratio between "inter-class" and "intra-class" deviations of each texture class [14]. For a given estimated coefficient, we define the "inter-class" (between-class) deviation as the standard deviation of this coefficient with respect to the texture class variation. The "intra-class" (within-class) deviation is defined as the standard deviation of this coefficient in the same texture class with respect to various realizations. Total inter-class and intra-class deviations are calculated by averaging out all the coefficient standard deviations obtained from all independent realizations. Large inter-class deviations along with a small intra-class one yield a high characterization degree. The greater the characterization degree is, the more robust the classification process is. The comparison of the capability of the filter coefficients will be presented through the next two experiments.

Experiment 1: Comparison of the characterization degree in a noisy context

For each coefficient family class i.e. the Chandrasekhar adaptive coefficients and the 2-D LMS filter ones, we compute the "characterization degree" J as follows [14]:

Note $\underline{x}_{k,n}$ the n^{th} estimated vector of coefficients for the k^{th} texture class $(1 \le k \le 25, \; 1 \le n \le 100)$. The mean of the k^{th} texture class vectors of coefficient is noted $\underline{\mu}_k = \frac{1}{100}\sum_{n=1}^{100} \underline{x}_{k,n}$ and $\underline{\mu}_c = \frac{1}{25}\sum_{k=1}^{25} \underline{\mu}_k$ notes the mean of all the coefficient vectors. The mean of the within-class (intra-class) dispersion matrices is given by the matrix:

$$S_{\text{intra}} = \frac{1}{2500}\sum_{k=1}^{25}\sum_{n=1}^{100}\left(\underline{x}_{k,n} - \underline{\mu}_k\right)\left(\underline{x}_{k,n} - \underline{\mu}_k\right)^t \tag{13}$$

which is the maximum likelihood estimation of the covariance matrix of the class. Complementary to this is the mean of the between-class (inter-class) dispersion matrices which describes the scattering of the class sample means and is calculated by the matrix:

$$S_{\text{inter}} = \frac{1}{25}\sum_{k=1}^{25}\left(\underline{\mu}_k - \underline{\mu}_c\right)\left(\underline{\mu}_k - \underline{\mu}_c\right)^t . \tag{14}$$

The "characterization degree" J proposed and detailed in [14] is given by:

$$J = trace\left(S_{\text{intra}}^{-1} S_{\text{inter}}\right). \tag{15}$$

The greater the characterization degree is, the more robust the classification process is. The comparison of the ability of the studied 2-D coefficients in presence of additive noise will be presented through the next simulation results. In Fig. 4, we plot the characterization degree J for both Chandrasekhar adaptive filter and 2-D LMS coefficients.

For reason of comparison, the method based on the co-occurrence matrices features [15] is also applied. The elements of the co-occurrence matrix $C_{i,j}$, represents how often pairs of pixels with values i and j separated by a specific distance occurs. The horizontal direction is used to obtain the co-occurrence matrices. A set of seven standard features are extracted from these matrices for each studied image for each direction. These features are: the contrast, the energy, the entropy, the homogeneity, the maximum probability, the cluster shade and the cluster prominence of the co-occurrence matrix [15].

Four cases are considered: the noiseless case, and two Signal to Noise Ratio (SNR) values: 5 dB and 0 dB. An additive Gaussian noise and a 2-D order of 3×3 quarter-plane support have been used providing 8 AR coefficients. Clearly, the characterization degree of the Chandrasekhar filter is greater than that of the 2-D LMS filter and the co-occurrence method. In all cases, the increase of additive noise variance causes attenuation in the characterization degree. The additive noise perturbs the classification process. Therefore, the Chandrasekhar filter based coefficients are better texture discriminators than those of the other filters.

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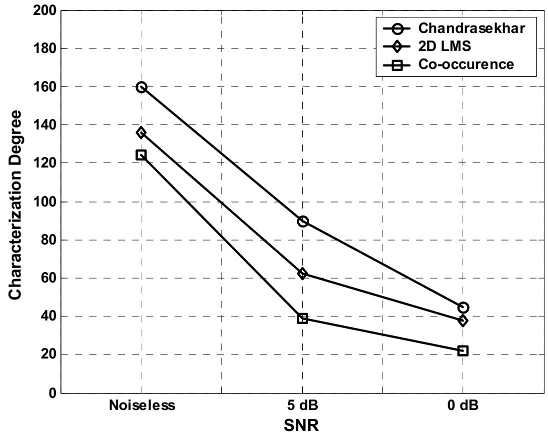

Fig. 4 Characterization degree of the Chandrasekhar filter coefficients, the 2-D LMS filter ones and the co-occurrence method features for various SNR values
(Filter order of 3×3).

Experiment 2: Influence of the filter order

In this experiment, the characterization degree is calculated for different filter orders ranging from 3×3 to 6×6 without any additive noise. Fig. 6 depicts the characterization degree with respect to the 2-D filter orders for both Chandrasekhar adaptive filter and 2-D LMS coefficients. It should be noted that for any order, the characterization degree provided by the Chandrasekhar adaptive filter coefficients is greater than the one provided by the 2-D LMS filter ones. Furthermore, the coefficients seem to give better characterization degree for a high filter order.

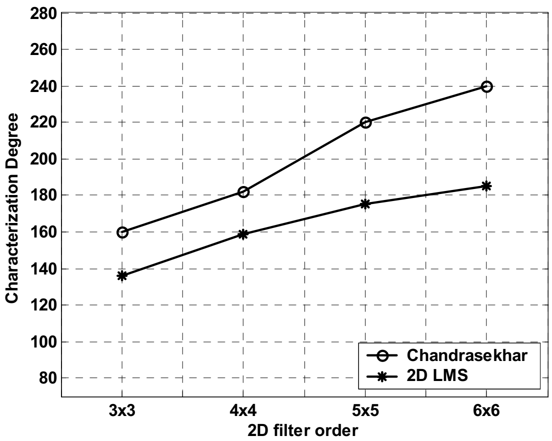

Fig. 6 Plot of the characterization degree with respect to the filter orders for the Chandrasekhar filter coefficients (*) and the 2-D LMS filter ones (o)

V. Conclusion

In this paper, we have proposed for the first time the adaptive Chandrasekhar filter coefficients as new parametric features for texture characterisation. The texture image is scanned row by row and filtered with the adaptive Chandrasekhar mono dimensional filter. We have shown how much the use of the Chandrasekhar adaptive filter in texture modelling can improve the texture characterization in comparison with the classical 2-D LMS adaptive filter. The estimated coefficients by the Chandrasekhar adaptive filter give better results in texture discrimination than those estimated by other classical algorithms, even in a noisy context. Further work taking into account the effect of the texture orientation and the scaling of the texture images on the estimated coefficients still remain to be done.

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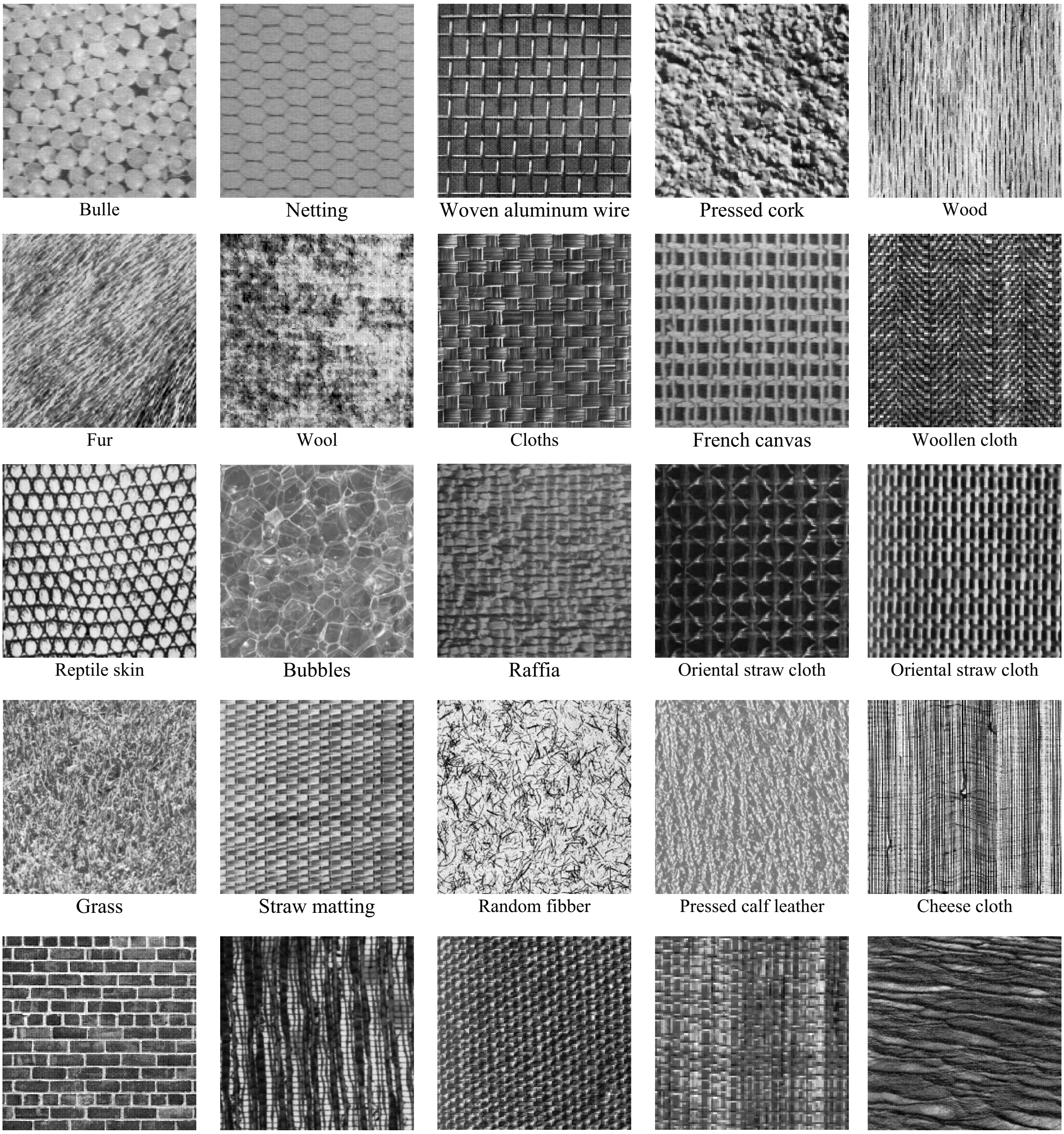

Fig. 5 The 25 Brodatz textures used in the study