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Positive solutions of second-order singular differential equations in Banach space

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Abstract—In this paper, by constructing a special set and utilizing fixed point index theory, we study the existence of solution for the boundary value problem of second-order singular differential equations in Banach space, which improved and generalize the result of related paper.

Keywords—Banach space, cone, fixed point index, singular equation.

I. INTRODUCTION

THE singular differential equation arises in a variety of applied mathematics and physics, the theory of singular differential equation is emerging as an important area of investigation since it is much richer than the corresponding theory of concerning equation without singular. In recent years, some new results concerning the Dirichlet boundary value problem of singular differential equation have been obtained by a variety of method ([1-5]). In thesis [6-7] the author investigate the singular equation

$$\begin{cases} x''(t) + f(t, x(t)) = \theta \quad t \in (0, 1) \\ x(0) = \theta, x(1) = \theta \end{cases}$$

the nonlinear term $f(t, x)$ may be singular at $t = 0, 1$ and $x = 0$, θ is zero element of real Banach space E. Motivated by the work of thesis [6-7], the present paper investigates the existence of positive solution for the general boundary value problem of the differential equation (we call it BVP(1)).

$$\begin{cases} x''(t) + f(t, x(t)) = \theta \quad t \in (0, 1) \\ ax(0) - bx'(0) = \theta \\ cx(1) + dx'(1) = \theta \end{cases} \tag{1}$$

where $a > 0, c > 0, b \geq 0, d \geq 0$. Our approaches are method of fixed point index theory and a new constructed cone. The organization of this paper is as follows, we shall introduce some definitions and lemmas in the rest of this section. The main result will be stated and proved in section 2. Finally, we give sone examples to demonstrate our main result.

Let E be a real Banach space,$J = [0, 1]$, P is a a regular cone in E, let the regular constant $c = 1$, we consider BVP (1) in $C[J, E]$, for $\forall x \in C[J, E]$, let $\|x\|_c = \max_{t \in J} \|x(t)\|$, then($C[J, E], \|.\|_c$)is a real Banach space.

Definition 1.1 A map $x \in C[J, E] \bigcap C^2[(0, 1), E]$ is said to be a solution of (1) if it satisfies equation (1). If $x(t) > \theta$, then x is called positive solution. Suppose $x(t) : (0, 1) \to E$ is continuous, if $\lim_{\varepsilon \to 0^+} \int_\varepsilon^1 x(t)dt$ exist, we call $\int_0^1 x(t)dt$ is convergence in abstract space.

Denotes α the $Kuratowski$ noncompactness measure in E, $\alpha(.)$ and $\alpha_c(.)$ the $Kuratowski$ noncompactness measure in E and $C[J, E]$ respectively.

lemma 1.1 Suppose $S \subset C[J, E]$ is a bounded and equicontinuous set on J, then $\alpha_c(S) = \sup_{t \in J} \alpha(S(t))$, where $S(t) = \{x(t) : x \in S\}$.

lemma 1.2 Suppose P is a regular and solid cone in Banach space, $P_r = \{x \in P : \|x\| < r\}$, $F : P_r \to P$ is strictly set contraction, if for $\forall x \in \partial P_r$ and $\lambda \geq 0$, $u_0 \in P \backslash \{\theta\}$, we have $x - Fx \neq \lambda u_0$, then $i(F, P_r, P) = 0$.

lemma 1.3 Suppose P is a cone in Banach space, $P_R = \{x \in P : \|x\| < R\}$, $F : \overline{P}_R \to P$ is a strictly set contraction, $Fx = \lambda x; x \in \partial P_R \Rightarrow \lambda < 1$, then $i(F, P_R, P) = 1$.

lemma 1.4 Suppose $V = \{x_n\} \subset L[J, E]$, there exist $g \in L[J, R_+]$, for all $x_n \in V, \|x_n\| \leq g(t), a.e. t \in J$, then $\alpha\Big(\{\int_a^t x_n(s)ds : n \in N\}\Big) \leq 2 \int_a^t \alpha(V(s))ds$.

II. CONCLUSION

For convenience, we list the following assumptions:

$(\mathbf{H_1})$ Let $\varphi(t) = c + d - ct, \psi(t) = b + at$, $t \in [0, 1]$. Suppose $f \in C[(0, 1) \times P, P]$ with $\|f(t, x)\| \leq k(t)\|q(x)\|$, where $k : (0, 1) \to (0, +\infty)$ satisfying $\int_0^1 \varphi(s)\psi(s)k(s)ds < +\infty$, and $q \in C[P, P]$.

$(\mathbf{H_2})$ Let $\rho = ac+ad+bc$, $G(s, s) = \dfrac{\varphi(s)\psi(s)}{\rho}$, $q[r_1, R_1] = \sup_{x \in P_{R_1} \backslash P_{r_1}} \|q(x)\| < +\infty$. For any $R_1 > r_1 > 0$, suppose $\int_0^1 G(s, s)k(s)q[\dfrac{\varphi(s)\psi(s)}{\rho + bd} r_1, R_1]ds < +\infty$, and there exists $R > 0$ such that $\int_0^1 G(s, s)k(s)q[\dfrac{\varphi(s)\psi(s)}{\rho + bd} R, R]ds < R$.

$(\mathbf{H_3})$ $f(t, x)$ is continuous uniformly on $[\delta, 1-\delta] \times \overline{P_{R_1}} \backslash P_{r_1}$, where $\delta \in (0, \frac{1}{2})$.

$(\mathbf{H_4})$ for $\forall t \in (0, 1)$ and bounded set $D \subset \overline{P_{R_1}} \setminus P_{r_1}$ there exists an L with $0 \leq L < \dfrac{2ac}{\rho}$ such that $\alpha(f(t, D)) \leq L\alpha(D)$.

$(\mathbf{H_5})$ there exists $h^* \in P^*, \|h^*\| = 1, h \in L[0, 1]$ such that $\lim_{x \to 0} \inf_{\forall x \in P} h^*(f(t, x)) \geq h(t)$ holds uniformly with respect to $t \in (0, 1)$, and $0 < \int_0^1 G(s, s)h(s)ds < +\infty$, where P^* is a dual cone of P.

$(\mathbf{H_6})$ There exists $k^* \in P^*, \|k^*\| = 1$ and $[s_1, s_2] \subset [0, 1]$ such that $\liminf_{x \to \infty, \forall x \in P} \dfrac{k^*(f(t, x))}{\|x\|} = \infty$ holds uniformly with

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respect to $t \in [s_1, s_2]$.

The following theorem is our main result.

Theorem 2.1 suppose that conditions $(H_1)-(H_6)$ hold, then there exists $r \in (0, R)$ such that BVP (1) has positive solution $x \in K_R \setminus \overline{K_r}$ and $y \in K \setminus \overline{K_R}$ respectively.

Before giving the proof of Theorem 2.1, we list some preliminaries and prove some lemmas. We first consider the following equivalence problem of BVP (1).

$$Ax(t) = \int_0^1 G(t,s)f(s,x(s))ds \tag{2}$$

where

$$G(t,s) = \begin{cases} \dfrac{\varphi(t)\psi(s)}{\rho}, 0 \le s \le t \le 1 \\ \dfrac{\varphi(s)\psi(t)}{\rho}, 0 \le t \le s \le 1 \end{cases} \tag{3}$$

$$\varphi(t) = c + d - ct, \psi(t) = b + at, \rho = ac + ad + bc.$$

It is clear that $x \in C[J,E] \cap C^2[(0,1),E]$ is a solution of BVP (1) if and only if A has a fixed point $x \in C[J,E]$, so we only need to show A has a nontrivial fixed point $x \in C[J,E]$. In order to overcome the difficulty caused by singular, we construct a special cone:

$$K = \{x \in C[J,P] : x(t) \ge \frac{\varphi(t)\psi(t)}{\rho + bd}x(s)\}. \tag{4}$$

where $t, s \in J$, obviously K is a cone in $C[J,E]$. Now we show $AK \subset K$, i.e.A is a self-mappiny in K.

Note $\quad G(t,s) \le \dfrac{\varphi(s)\psi(s)}{\rho} = G(s,s), 0 \le t,s \le 1,$

and $\quad G(t,\tau) \ge \dfrac{\varphi(t)\psi(t)}{\rho + bd}G(s,\tau)$, so if $x \in K$, then

$$\begin{aligned} Ax(t) &= \int_0^1 G(t,\tau)f(\tau,x(\tau))d\tau \\ &\ge \frac{\varphi(t)\psi(t)}{\rho + bd}\int_0^1 G(s,\tau)f(\tau,x(\tau))d\tau, \end{aligned}$$

so $Ax(t) \ge \dfrac{\varphi(t)\psi(t)}{\rho + bd}Ax(s)$, consequently $AK \subset K$.

In order to obtain the existence of fixed point of A, we first prove the following two lemmas:

Lemma (2.1) Suppose $(H_1)-(H_2)$ hold, then for $\forall r > 0, A$ is a continuous and bounded operator from K_r to K. where $K_r = \{x \in K : \|x\|_c < r\}$.

Proof By (4), we know $A : \overline{K_r} \to K$, first we show the continuity. Let $x_m, x \in \overline{K_r}, \|x_m - x\|_c \to 0, m \to +\infty$, on account of H_1, for $\forall m, \forall s \in J$ we have

$$\|f(s,x_m(s))\| \le k(s)q[0,r], \tag{5}$$

so by $(H_1)-(H_2)$ we know $\lim\limits_{m\to+\infty}(Ax_m)(t) = (Ax)(t)$, meanwhile by (5) we can see $\{(Ax_m)(t)\}_{m\ge1}$ is equicontinuous family on J, so we should get

$$\lim_{m\to+\infty}\|Ax_m - Ax\| = 0.$$

In fact, if this is false, then there exist $\varepsilon_0 > 0$ and $\{x_{m_i}\} \subset \{x_m\}$ such that$\|Ax_{m_i} - Ax\|_c \ge \varepsilon_0 (i = 1,2,...)$, since $\{Ax_m\}$ is relatively compact in $C[J,E]$, the relative compactness of $\{Ax_m\}$ implies that $\{Ax_{m_i}\}$ contains a subsequence which converges to some $y \in K$, no loss of generality we may assume that $\lim\limits_{m\to+\infty} Ax_{m_i} = y$, i.e. $\lim\limits_{m\to+\infty}\|Ax_{m_i} - y\|_c = 0$, obviously this is in contradiction with $y = Ax$, so A is continuous.

On the other hand, by virtue of $(H_1)-(H_2)$ and the inequality (5), we know A is bounded from K_r to K.

Lemma 2.2 suppose $(H_1)-(H_4)$ hold, then for $\forall r > 0$, A is strictly set contraction from K_r to K.

Proof for $\forall r > 0$, suppose $S \subset K_r$, by virtue of lemma 2.1 we know AS is bounded set and equicontinuous on J, by lemma 1.1 we know

$$\alpha_c(A(S)) = \sup_{t\in J}\alpha(AS(t)) \tag{6}$$

where $AS(t) = \{Ax(t) : x \in S, t \in J\}$, let $D_\delta = \{\int_\delta^{1-\delta} G(t,s)f(s,x(s))ds : x \in S, \delta \in (0, \frac{1}{2})\}$, then by virtue of $(H_1)-(H_2)$, for $x \in S, t \in J$ we have

$$\begin{aligned} &\|\int_\delta^{1-\delta} G(t,s)f(s,x(s))ds - \int_0^1 G(t,s)f(s,x(s))ds\| \\ &\le C_1\Big(\int_0^\delta G(s,s)k(s)ds + \int_{1-\delta}^1 G(s,s)k(s)ds\Big) \end{aligned} \tag{7}$$

where $C_1 = q[0,r]$. by virtue of (H_1) and (6)-(7), we know the Hausdorff distance of D_δ and $\{A(S)\}$

$$d_H(D_\delta, AS) \to 0, \quad \delta \to 0+.$$

so

$$\alpha(AS) = \lim_{\delta\to0^+}\alpha(D_\delta). \tag{8}$$

Next we estimate $\alpha(D_\delta)$. because $\int_\delta^{1-\delta} G(t,s)f(s,x(s))ds \in (1-2\delta)\overline{co}\{G(t,s)f(s,x(s)) : s \in [\delta, 1-\delta]\}$, so by $(H_3)-(H_4)$ and (9.4.11) in therein [8] we obtain

$$\begin{aligned} \alpha(D_\delta) &= \alpha\big(\{\int_\delta^{1-\delta} G(t,s)f(s,x(s))ds : x \in S\}\big) \\ &\le (1-2\delta)\alpha\big(\overline{co}\{G(t,s)f(s,x(s)) : s \in [\delta,1-\delta],\}\big) \\ &\le \alpha\big(\{G(t,s)f(s,x(s)) : s \in [\delta,1-\delta], x \in S\}\big) \\ &\le \frac{\rho}{4ac}\alpha(\{f(s,x(s)) : s \in [\delta,1-\delta], x \in S\}) \\ &\le \frac{L\rho}{4ac}\max_{t\in[\delta,1-\delta]}\alpha(S(I_\delta)) \\ &\le \frac{1}{2}\alpha_c(S), I_\delta = [\delta, 1-\delta]. \end{aligned} \tag{9}$$

Note (8), let $\delta \to 0$ when $\alpha_c(S) \ne 0$, we have $\alpha(A(S)) \le \frac{1}{2}\alpha_c(S) < \alpha_c(S)$, so A is strict set contraction. On the other hand by lemma 2.1, for $\forall r > 0, A$ is strictly set contraction from K_r to K.

Now we give the proof of theorem 2.1.

Proof: By virtue of (H_5), for $\forall t \in (0,1)$ we have $0 < \int_0^1 G(t,s)h(s)ds \leq \int_0^1 G(s,s)h(s)ds$, otherwise $h(s) = 0, a.e. s \in J$. Chose $\varepsilon^{'} > 0$ sufficiently small such that

$$r^{'} = \int_0^1 \int_0^1 G(t,s)(h(s) - \varepsilon^{'})dsdt > 0, \qquad (10)$$

by (H_5), there exists $r^{''} \in (0,R)$ when $\|x\|_c \leq r^{''}$, for $\forall t \in (0,1)$, we have

$$h^*(f(t,x(t))) \geq h(s) - \varepsilon^{'}. \qquad (11)$$

take $0 < r < l = \min\{r^{'}, r^{''}\}$, for $\forall x \in \partial K_r, \lambda \geq 0$, we have $x - A(x) \neq \lambda e$, where $e \in P, e \neq \theta$. In fact, if it is false, then there exists $\lambda \geq 0, x \in \partial K_r$ such that $x - A(x) = \lambda e$, i.e. $x(t) = Ax(t) + \lambda e \geq \int_0^1 G(t,s)f(s,x(s))ds$, consequently by (11) we can get

$$\begin{aligned} h^*x(t) &\geq \int_0^1 G(t,s)h^*f(s,x(s))ds \\ &\geq \int_0^1 G(t,s)(h(s) - \varepsilon^{'})ds. \qquad (12) \end{aligned}$$

In addition, by virtue of (10) and (12), we have

$$\int_0^1 h^*x(t)dt \geq \int_0^1 \int_0^1 G(t,s)(h(s) - \varepsilon^{'})dsdt = r^{'} > r. \qquad (13)$$

But for $\forall t \in J$, since $\mid h^*x(t) \mid \leq \|x(t)\| \leq \|x\|_c = r$, this is in contradiction with (13). According to lemma 1.2, we have

$$i(A, K_r, K) = 0. \qquad (14)$$

Next by (H_2) we will show $i(A, K_R, K) = 1$, by the homotopy invariance of fixed point index, we only need to show: for $\forall x \in \partial K_R$ and $\forall \lambda \geq 1$, $Ax \neq \lambda x$.
In fact, if it is false, then there exist $x \in \partial K_R$ and some $\lambda \geq 1$ such that $Ax = \lambda x$, then $x = \frac{1}{\lambda}Ax$, therefore by (2) we know, for $\forall t \in (0,1)$, we have

$$\begin{aligned} x(t) &= \frac{1}{\lambda\rho}\Big\{[c(1-t)+d]\int_0^t (as+b)f(s,x(s))ds\Big\} \\ &+ \frac{1}{\lambda\rho}\Big\{(at+b)\int_t^1 [c(1-s)+d]f(s,x(s))ds\Big\}, \end{aligned}$$

therefore

$$\begin{aligned} x^{'}(t) &= \frac{1}{\lambda\rho}\int_0^t (-c)(as+b)f(s,x(s))ds \\ &+ \frac{a}{\rho\lambda}\int_t^1 [c(1-s)+d]f(s,x(s))ds \\ &\leq \frac{a}{\rho\lambda}\int_t^1 [c(1-s)+d]f(s,x(s))ds, \end{aligned}$$

(where $\leq$ is partial order induced by cone.) So for $\forall t \in J$, we have

$$\begin{aligned} 0 \leq x(t) &\leq \frac{a}{\rho\lambda}\int_0^1 \int_t^1 [c(1-s)+d]f(s,x(s))dsdt \\ &+ \frac{b}{\rho\lambda}\int_0^1 [c(1-s)+d]f(s,x(s))ds \\ &= \frac{1}{(\rho\lambda)}\int_0^1 (as+b)[c(1-s)+d]f(s,x(s))ds \\ &= \frac{1}{\lambda}\int_0^1 G(s,s)f(s,x(s))ds, \end{aligned}$$

so

$$\begin{aligned} R &= \|x\|_c = \max_{t\in J}\|x(t)\| \\ &\leq \frac{1}{\lambda}\int_0^1 G(s,s)k(s)\|q[\frac{\varphi(s)\psi(s)}{\rho+bd}R, R]ds \\ &\leq \int_0^1 G(s,s)k(s)q[\frac{\varphi(s)\psi(s)}{\rho+bd}R, R]ds \\ &< R. \end{aligned}$$

This is in contradiction with (H_2), so we have

$$i(A, K_R, K) = 1. \qquad (15)$$

Select $R^{'} > \max_{t\in J}\Big[\frac{(b+as_1)(c+d-cs_2)}{\rho+bd}\int_{s_1}^{s_2} G(t,s)ds\Big]^{-1}$, by (H_6), for $x > N$, there exists $N > R$ such that $k^*(f(t,x)) \geq R^{'}\|x\|$, let $\overline{R} = R+1$ then for $\forall x \in \partial K_{\overline{R}}, \lambda \geq 0$, we have $x - A(x) \neq \lambda e$. In fact, if there exist $\lambda \geq 0, x \in \partial K_{\overline{R}}$ such that $x - A(x) = \lambda e$ then

$$\begin{aligned} \overline{R} &\geq k^*(x(t)) \geq k^*(Ax(t)) \\ &\geq \int_0^1 G(t,s)k^*(f(s,x(s)))ds \\ &\geq \int_{s_1}^{s_2} G(t,s)k^*(f(s,x(s)))ds \\ &\geq R^{'}\int_{s_1}^{s_2} G(t,s)\|x(s)\|ds \\ &\geq R^{'}\int_{s_1}^{s_2} G(t,s)\frac{(b+as_1)(c+d-cs_2)}{\rho+bd}\|x\|ds \\ &> \overline{R}. \end{aligned}$$

This is a contradiction, so by lemma1.2,

$$i(A, K_{\overline{R}}, K) = 0.$$

Moreover, by (14)(15), we can see

$$i(A, K_R \setminus \overline{K_r}, K) = i(A, K_R, K) - i(A, K_r, K) = 1,$$

$$i(A, K_{\overline{R}} \setminus \overline{K_R}, K) = i(A, K_{\overline{R}}, K) - i(A, K_R, K) = -1.$$

So A has fixed point $x \in K_R \setminus \overline{K_r}$ and $y \in K \setminus \overline{K_R}$ respectively.
Finally we show $x \neq y$, we only need to show A has not fixed point in ∂K_R. Otherwise, assume $z \in \partial K_R$ is a fixed point, so when $t \in J$, $z(t) = \int_0^1 G(t,s)(f(s,z(s)))ds$ and

$R = \|z(t)\| \geq \frac{(b+at)(c+d-ct)}{\rho+bd} R.$
By $(H_1)-(H_2)$ we have

$$\begin{aligned} R &= \max_{t \in J} \|z(t)\| \\ &\leq \int_0^1 G(t,s)k(s)q[\frac{\varphi(s)\psi(s)}{\rho+bd}R, R]ds \\ &\leq \int_0^1 G(s,s)k(s)q[\frac{\varphi(s)\psi(s)}{\rho+bd}R, R]ds \\ &< R. \end{aligned}$$

This is a contradiction, and our conclusion follows.□

Corollary Suppose conditions $(H_1)-(H_5)$ hold, or conditions $(H_1)-(H_4)$ and (H_6) hold, BVP (1.1) has at least one positive solution.

Example: Suppose $E = l^\infty = \{x = (x_1, x_2, ..., x_n, ...) : \sup\limits_n \mid x_n \mid < +\infty\}$, for $x \in E$, let $\|x\| = \sup\limits_n \mid x_n \mid$, then $(E, \|.\|)$ is a Banach space, and $P = \{x \in E : x_n \geq 0, n = 1, 2, ...\}$ is a regular cone in E, let the regular constant $c = 1$, we consider the following equations in E

$$\begin{cases} -x_n^{''}(t) = \frac{\cos t}{\sqrt{t(t-1)}}(1 + \frac{1}{n}(tx_{2n} + \ln(1+x_n))), \\ x_n(0) = x_n(1) = 0, n = 1, 2, ... \end{cases} \tag{16}$$

problem (16) can be think as the type of BVP(1), it is equivalent to $x(t) = (x_1(t), x_2(t), ...), f(t) = (f_1, f_2, ...)$; $f_n(t,x) = \frac{\cos t}{\sqrt{t(1-t)}}(1+\frac{1}{n}(tx_{2n}+\ln(1+x_n)))$, we can see $f(t,x)$ is singular at $t = 0, 1$. Now we check $H_1 - H_5$ hold. Chose $k(t) = \frac{1}{\sqrt{t(1-t)}}, q(x) = (q_1(x), q_2(x), ...), q_n(x) = 1 + \frac{1}{n}(tx_{2n} + \ln(1+x_n))$for $\forall R_1 > r_1 > 0$, it is easy to get $q[r_1, R_1] = \sup\limits_{x \in \overline{P}_{R_1} \setminus P_{r_1}} \|q(x)\| \leq 1 + R_1 + \ln(1+R_1)$. since

$$\int_0^1 \sqrt{s(1-s)}ds = \frac{\pi}{8}, \int_0^1 \frac{ds}{\sqrt{s(1-s)}} = \pi,$$

therefore $\int_0^1 s(1-s)k(s)q[s(1-s)r_1, R_1]ds \leq \frac{\pi}{8}(1 + R_1 + \ln(1+R_1)) < +\infty$. It is easy to see, if R is sufficiently large then $\int_0^1 s(1-s)k(s)q[s(1-s)R, R]ds \leq R$ hold, so H_1, H_2 hold, obviously H_3 hold, for $\forall t \in (0,1)$, we give a bounded sequence $\{x^n\} \subset \overline{P}_{R_1} \setminus P_{r_1}$ using diagonal rule, we can choose a convergent subsequence from $\{f(t, x^{(n)})\}$ (where $R_1 > r_1 > 0$ is arbitrary), so H_4 hold, and it is equivalent to the case L=0. Choose $h^* \in P^*$ such that $h^*(x) = x_1$, so H_5 hold. To sum up, $H_1 - H_5$ hold, by the Corollary of theorem 2.1, we know problem (16) has at least one positive solution.

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