

Numerical Method based on Initial Value-Finite Differences for Free Vibration of Stepped Thickness Plates

Ahmed M. Farag, Wael F. Mohamed, Atef A. Ata, and Burhamy M. Burhamy

Abstract—The main objective of the present paper is to derive an easy numerical technique for the analysis of the free vibration through the stepped regions of plates. Based on the utilities of the step by step integration initial values IV and Finite differences FD methods, the present improved Initial Value Finite Differences (IVFD) technique is achieved. The first initial conditions are formulated in convenient forms for the step by step integrations while the upper and lower edge conditions are expressed in finite difference modes. Also compatibility conditions are created due to the sudden variation of plate thickness. The present method (IVFD) is applied to solve the fourth order partial differential equation of motion for stepped plate across two different panels under the sudden step compatibility in addition to different types of end conditions. The obtained results are examined and the validity of the present method is proved showing excellent efficiency and rapid convergence.

Keywords—Vibrations, Step by Step Integration, Stepped plate, Boundary.

I. INTRODUCTION

PLATES with varying thickness are extensively used in modern structures due to their unique functions. For example, stepped plates possess a number of attractive features, such as material saving, weight reduction, stiffness enhancing, designated strengthening and fundamental vibration frequency increasing. With the availability of inexpensive and high performance computers, theoretical analysis is employed to optimize stepped plates in practical engineering designs [1]-[2]-[3]. In particular, buckling and free vibration analysis of stepped plates has attracted much attention in the past few decades [2]. A few numbers of theoretical approaches have been formulated for this class of problems. Although, Chopra [4] has attempted an exact solution for a simply supported stepped thickness plate with two panels, Warburton [4] pointed out that the continuity conditions used by Chopra were incorrect. He offered a modified analytical technique for two paneled stepped plate with different properties of orthotropy. However, dealing with

A. M. Farag is with Department of Engineering Mathematics and Physics, Faculty of Engineering, Alexandria University, Alexandria, Egypt (e-mail: afarag59@yahoo.com).

W. F. Mohamed is with Department of Engineering Mathematics and Physics, Faculty of Engineering, Alexandria University, Alexandria, Egypt (e-mail: wfmobarak@gmail.com).

A. A. Ata is with Department of Engineering Mathematics and Physics, Faculty of Engineering, Alexandria University, Alexandria, Egypt (e-mail: atefata@gmail.com).

B. M. Burhamy is with Department of Engineering Mathematics and Physics, Faculty of Engineering, Alexandria University, Alexandria, Egypt.

the proposed plates by analytical method (Levy-type) was successful [2], but it was restricted to limited edges boundary conditions. On the other hand there was [1] a flexibility of applying variety of boundary condition with power matrix exponential method. Other developed numerical methods like Finite Strip Method (FSM) [5], or a developed one like finite strip transition matrix method (FSTM) [4] are used to get dynamical solutions. Since analytical solutions of the partial differential equation of motion for some plate problems are so complicated, numerical and approximate methods can be used [6]-[7]-[8]. In the engineering applications of the theory of plates, Finite Differences (FD) method is a classical field [9]-[10]-[11] while among the numerical techniques, finite differences method is one of the most general. Recently, despite the existence of a broad variety of numerical method for the structural analysis, finite differences method is still regarded as a numerical method for straightforward analysis of specialized problems [12]. On the other hand, the initial value method is one of the fast approximate methods, which proves a good convergence with the exact solution [13]-[14]. In the present paper, an improved technique (IVFD) combining the initial value method (IV) and finite differences method (FD) is achieved to analyze the flexural vibration of stepped plate. A selected mesh grid is applied for the partial differential equation of plate motion where the initial value method is utilized in one direction. In the other direction, the partial derivatives are replaced with quantities of the finite differences method. The developed IVFD method is successfully applied for free vibration analysis of stepped thickness plates with different combinations of boundary conditions. The natural frequency parameters are calculated and the efficiency of the method is validated by comparing the deduced results with those published in the literature.

II. INITIAL VALUE FINITE DIFFERENCES MATHEMATICAL FORMULATION

The governing partial differential equation of small transverse deflection of a thin isotropic elastic plate, of thickness h , for free vibration [15] has the form:

$$w_{,xxxx} + 2w_{,xxyy} + w_{,yyyy} + \frac{\rho h}{D} w_{,tt} = 0 \quad (1)$$

where $D = \frac{Eh^3}{12(1-\nu^2)}$ is the cylindrical stiffness of the plate,

w is the plate deflection measured along the z axis, E is the modulus of elasticity and ν is Poisson's ratio of the plate material. Also, $w_{,xxxx}$ and $w_{,yyyy}$ are the fourth-order partial derivatives of w with respect to x and y respectively. By analogy, the other partial derivatives of w are written according to the independent variables. To investigate the vibration modes, the displacement is defined as time harmonic function as:

$$w(x, y, t) = \hat{w}(x, y)e^{i\omega t} \quad (2)$$

where ω is the circular natural frequency. Substitution of (2) in (1) yields the equation governing the dynamic stationary free vibration behavior of Kirchhoff plates [16]-[17]-[18]:

$$\hat{w}_{,xxxx} + 2\hat{w}_{,xyxy} + \hat{w}_{,yyyy} - \lambda^2 \hat{w} = 0 \quad (3)$$

where $\lambda^2 = \frac{\rho h \omega^2}{D}$ is the plate natural frequency parameter.

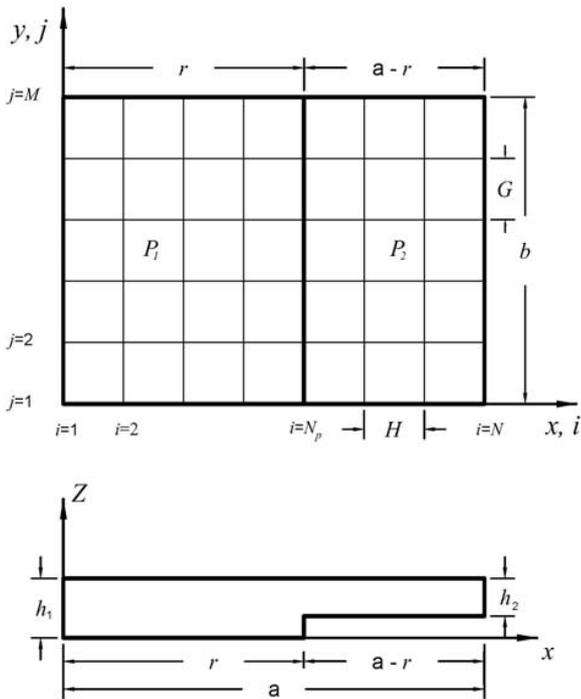


Fig. 1 Model of stepped thickness plate

The investigated model is thin elastic isotropic plate of two panels p_1 and p_2 with two different thicknesses h_1 and h_2 respectively as shown in Fig. 1. The dimensions of the plate are a and b in x and y directions respectively, and the plate aspect ratio is $\beta = \frac{a}{b}$. Also, $\gamma = \frac{r}{(a-r)}$ is called the panel width ratio. A mesh of $M \times N$ numbers of divisions is constructed with equal divisions in each direction, $H = a/N$ in x -direction

and $G = b/M$ in y -direction, as shown in Fig. 1.

In the present paper a step by step integration technique is used by initial value method in the x -direction. The partial derivatives $\hat{w}_{,xyxy}, \hat{w}_{,xxxx}$ at any point (i, j) , in (3), are represented in y -direction by the lower displacement derivatives $\hat{w}, \hat{w}_{,xx}$ of four neighborhood points [19]-[20], while the derivatives $\frac{\partial^n \hat{w}}{\partial x^n}; n = 1, 2, 3, 4$ in x -direction are retained. Using the finite differences rules [21], one can obtain:

$$(\hat{w}_{,xyxy})_{(i,j)} = \frac{1}{G^2} [(\hat{w}_{,xx})_{(i,j+1)} - 2(\hat{w}_{,xx})_{(i,j)} + (\hat{w}_{,xx})_{(i,j-1)}] \quad (4)$$

$$(\hat{w}_{,yyyy})_{(i,j)} = \frac{1}{G^4} [(\hat{w}_{(i,j-2)} - 4\hat{w}_{(i,j-1)} + 6\hat{w}_{(i,j)} - 4\hat{w}_{(i,j+1)} + \hat{w}_{(i,j+2)})] \quad (5)$$

Then by substitution from (4) and (5) into (3), the nodal equation of motion of isotropic plates is transformed to:

$$(\hat{w}_{,xxxx})_{(i,j)} = \lambda^2 \hat{w} - \frac{2}{G^2} [(\hat{w}_{,xx})_{(i,j+1)} - 2(\hat{w}_{,xx})_{(i,j)} + (\hat{w}_{,xx})_{(i,j-1)}] - \frac{1}{G^4} [\hat{w}_{(i,j-2)} - 4\hat{w}_{(i,j-1)} + 6\hat{w}_{(i,j)} - 4\hat{w}_{(i,j+1)} + \hat{w}_{(i,j+2)}] \quad (6)$$

Trapezoidal rule is used as an integration method to solve the differential equation in x -direction, by which the derivatives applied at a point (i, j) are represented as:

$$\hat{w}_{(i+1,j)}^{(n)} = \hat{w}_{(i,j)}^{(n)} + \frac{H}{2} [\hat{w}_{(i,j)}^{(n+1)} + \hat{w}_{(i+1,j)}^{(n+1)}] \quad ; n = 0, 1, 2, 3 \quad (7)$$

where, $\hat{w}_{(i,j)}^{(n)}$ is the n -order derivative of the displacement \hat{w} at a point (i, j) with respect to x .

The procedure of the step by step integration is illustrated by a flow chart diagram, see Fig. 2. This procedure can be explained in the following steps:

Step1: Initial Values at Starting Line $i = 1$

According to the proposed initial conditions at the edge $i = 1$, two quantities of $(\frac{\partial^n \hat{w}}{\partial x^n})_{(1,j)}$; $n = 0, 1, 2, 3$, $j = 1, 2, 3, \dots, M$ are

known. The other two values have to be assumed. Here are $2M$ -initial values corresponding to $2M$ -homogenous solutions. Elementary, for one homogenous solution, only one non-trivial value of $2M$ -initial values is assumed while the other values are considered zeros. Applying (6) at any point $(1, j)$; $j = 1, 2, 3, \dots, M$, the magnitude of the fourth derivative $(\hat{w}_{,xxxx})_{(1,j)}$ are determined at $i = 1$.

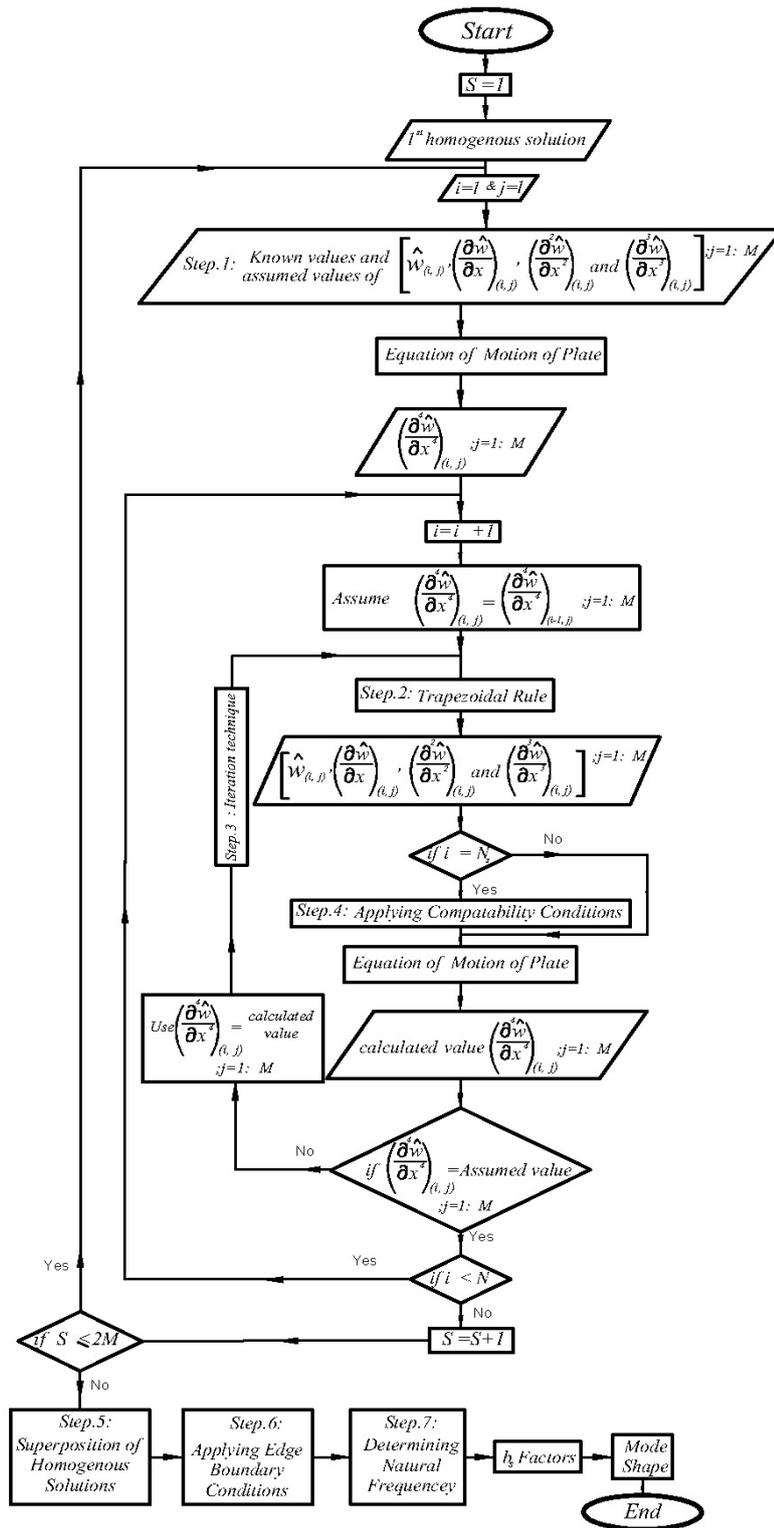


Fig. 2 Flow chart diagram of IVFD technique

Step2: Trapezoidal Rules for Step by Step Integration Technique

Beginning with the assumption $(\hat{w}_{,xxxx})_{(i+1,j)} = (\hat{w}_{,xxxx})_{(i,j)}$
 $=$
 $j = 1,2,3,\dots, M$ and using the trapezoidal rules of (7), one can calculate the first trial values of $(\frac{\partial^n \hat{w}}{\partial x^n})_{(i+1,j)}$; $n = 0,1,2,3$,
 $j = 1,2,3,\dots, M$ from the corresponding obtained values $(\frac{\partial^n \hat{w}}{\partial x^n})_{(i,j)}$
 $; n = 0,1,2,3, j = 1,2,3,\dots, M$. Consequently by substituting the obtained quantities from trapezoidal rule into (6), the values of the fourth derivative $(\hat{w}_{,xxxx})_{(i,j)}$ for all j_s can be determined at the current line.

Step3: Iteration Technique

For the determined value of the fourth derivative $(\hat{w}_{,xxxx})_{(i,j)}$, from (6), which is not coincident with the assumed one, the new determined value is taken as the assumed value. The procedure of step 2 has to be repeated for all $j = 1,2,3,\dots, M$ until the assumed value agrees as closely as with the deduced one for all j_s .

Step4: Compatibility Conditions

Steps 2 and 3 are applied to the next lines until $i = N_p$ is reached, where N_p is the line number at the sudden of plate.

Consequently, compatibility conditions [1]-[2]-[3] have to be applied at line $i = N_s$. These conditions can be represented in the form:

$$\begin{Bmatrix} \hat{w}_{(i,j)} \\ (\hat{w}_{,x})_{(i,j)} \\ M_{x(i,j)} \\ V_{x(i,j)} \end{Bmatrix}_{p_1} = \begin{Bmatrix} \hat{w}_{(i,j)} \\ (\hat{w}_{,x})_{(i,j)} \\ M_{x(i,j)} \\ V_{x(i,j)} \end{Bmatrix}_{p_2} \quad (8)$$

The suffixes p_1 and p_2 refer to the values before and values after the sudden step respectively. Expressing the moment and shear in finite differences forms, one gets [15]:

$$\begin{Bmatrix} \hat{w}_{(i,j)} \\ (\hat{w}_{,x})_{(i,j)} \\ (\hat{w}_{,xx})_{(i,j)} \\ (\hat{w}_{,xxx})_{(i,j)} \end{Bmatrix}_{p_2} = \begin{Bmatrix} \hat{w}_{(i,j)} \\ (\hat{w}_{,x})_{(i,j)} \\ k(\hat{w}_{,xx})_{(i,j)} + c_1 D_1 \\ k(\hat{w}_{,xxx})_{(i,j)} + c_2 D_2 \end{Bmatrix}_{p_1} \quad (9)$$

where $c_1 = \frac{(\alpha^3 - 1)\nu}{G^2}$, $c_2 = \frac{(\alpha^3 - 1)(2 - \nu)}{G^2}$, $k = \alpha^3 = \left(\frac{h_1}{h_2}\right)^3$

The magnitude $\alpha = \frac{h_1}{h_2}$ is the stepped plate thickness ratio

and

$$D_1 = [\hat{w}_{(i,j-1)} + 2\hat{w}_{(i,j)} - \hat{w}_{(i,j+1)}],$$

$$D_2 = [(\hat{w}_{,x})_{(i,j+1)} - 2(\hat{w}_{,x})_{(i,j)} + (\hat{w}_{,x})_{(i,j-1)}]$$

The previous steps 2 and 3 are applied to the next lines i ; $i = N_p : N$ until the deflection \hat{w} and their partial derivatives $\frac{\partial^n \hat{w}}{\partial x^n}$; $n = 1,2,3$ are calculated at the points of the terminal edge $i = N$.

Step5: Superposition of Homogenous Solutions

Because of existing $2M$ -assumed initial values at the beginning edge $i = 1$, the terminal boundary conditions are not satisfied for each homogenous solution. At the terminal edge $i = N$, there are $2M$ -homogeneous solutions (each homogeneous solution corresponds each assumed initial value at point $(1, j)$).

There are $2M$ -sets of displacement quantities at the points (i, j) of the mesh (each corresponds to one of the $2M$ -assumed initial values). The superposition method is used to derive each individual solution. The true superimposed solution $\hat{w}^{(n)}$ of plate is the sum of the $2M$ -homogeneous solutions such as:

$$\hat{w}^{(n)} = \sum_{s=1}^{2M} b_s \hat{w}_s^{(n)} ; s = 1,2,3,\dots,2M, n = 0,1,2,3,4 \quad (10)$$

where:

$\hat{w}_s^{(n)}$ is the n -derivative of the displacement of the homogenous solutions and b_s ; $s = 1,2,3,\dots,2M$ are unknowns coefficients to be determined from the boundary conditions.

Step6: Boundary Condition at the end Edge $i = N$

The individual solutions of the end edge $i = N$ will not satisfy boundary condition, so it is necessary to force the superimposed solutions to coincides with the boundary conditions. For example, the boundary conditions for simply supported plate are:

$$\hat{w}_{(N,j)}^{(0)} = \hat{w}_{(N,j)}^{(2)} = 0 ; j = 1,2,3,\dots, M \quad (11)$$

The unknown b_s ; $s = 1,2,3,\dots,2M$ factors can be determined by satisfying the boundary conditions of the terminal edge $i = N$. Boundary conditions will be expressed in a matrix form such as:

$$[\mathfrak{R}_s][b_s] = [0]; s = 1,2,3,\dots,2M \quad (12)$$

where \mathfrak{R}_s is $2M \times 2M$ known matrix and b_s is $2M$ -

unknown vector.

Step7: The Circular Natural Frequency

For a non-trivial solution of (12), the determinate of $[R_s]$ must be zero [22]-[23]-[24]. All values that satisfy zero determinate of $[R_s]$ are the natural frequencies ω of plate. The corresponding displacement of obtained natural frequency is the mode shape [25]-[26]-[27] such as:

$$(\hat{w})_{(i,j)} = \sum_{s=1}^{2M} b_s (\hat{w}_s)_{(i,j)} \quad ; \quad s = 1, 2, 3, \dots, 2M \quad (13)$$

III. FORMULATION OF THE BOUNDARY CONDITIONS

Boundary conditions, at the edges of the rectangular plate, are formulated in convenient forms to deal with the IVFD method. The assumed initial values at the initial edges are depending on the boundary conditions along this edge.

The assigned initial values at the first edge for all j_s for different boundary conditions [25] are:

i. *Simply supported plate edge, S.*

a- At $(i = 1, N)$:

$$\hat{w}_{(i,j)} = (\hat{w}_{,xx})_{(i,j)} = 0$$

The other two magnitudes $\hat{w}_{(i,j)}$, $(\hat{w}_{,xxx})_{(i,j)}$ have to be assumed non-trivial values.

b- At $(j = 1, M)$:

$$\hat{w}_{(i,j-1)} = -\hat{w}_{(i,j+1)} \text{ and } \hat{w}_{(i,j)} = 0$$

ii. *Clamped supported plate edge, C.*

a- At $(i = 1, N)$:

$$\hat{w}_{(i,j)} = (\hat{w}_{,x})_{(i,j)} = 0$$

The other two magnitudes $(\hat{w}_{,xx})_{(i,j)}$, $(\hat{w}_{,xxx})_{(i,j)}$ have to be assumed non-trivial values.

b- At $(j = 1, M)$:

$$\hat{w}_{(i,j-1)} = \hat{w}_{(i,j+1)} \text{ and } \hat{w}_{(i,j)} = 0$$

iii. *Free plate edge, F.*

For F-edges at $(i = 1, N)$, the values of $V_{x(i,j)}$ and $M_{x(i,j)}$ must be vanish. The magnitudes $(\hat{w})_{(1,j)}$ and $(\hat{w}_{,x})_{(1,j)}$ at the initial edges, have to be assumed.

IV. NUMERICAL VERIFICATION AND DISCUSSION

Several natural frequency parameters of different modes are calculated for different cases of plate composed of two panels with unequal thickness and panel widths. Different aspect ratio β , panel width ratio γ and various magnitude of thickness

ratio α are investigated. The natural frequency parameter is pointed out for every case where Poisson's ratio is taken as $\nu = 0.3$. Tables I and II show the values of normalized dimensionless natural frequency $\Gamma_{mn} = \left(\frac{\lambda_{mn}^2}{\pi^2}\right)$ parameter for two

cases of boundary conditions of square plates SSSS and CSCS where S, C refers to simply supported and clamped edges respectively. The plate panel width ratio is taken as $\gamma = 0.5$ with different values of thickness ratio α . The obtained results are compared with the exact values showing good agreements.

TABLE I
NORMALIZED DIMENSIONLESS NATURAL FREQUENCY PARAMETER Γ_{mn}

Boundary conditions	α	Γ_{mn}	Method		
			IVFD	Farag [1]	Xiang[2]
SSSS	0.5	Γ_{11}	2.8168	2.9015	2.9015
		Γ_{12}	7.0926	7.1156	7.1156
		Γ_{13}	13.5798	13.7848	13.7850
CSCS	0.5	Γ_{11}	4.0352	4.1711	4.1711
		Γ_{12}	9.9327	9.9047	9.9047
		Γ_{13}	17.8633	18.0453	18.0450

TABLE II
NORMALIZED DIMENSIONLESS NATURAL FREQUENCY PARAMETER Γ_{mn}

Boundary conditions	α	Γ_{mn}	Method		
			IVFD	Farag [1]	Xiang[2]
SSSS	2/3	Γ_{11}	2.4124	2.4470	2.4471
		Γ_{12}	6.2622	6.2229	6.2229
		Γ_{13}	11.8552	11.9124	11.9480
CSCS	2/3	Γ_{11}	3.5094	3.5609	3.5610
		Γ_{12}	8.7183	8.7199	8.7199
		Γ_{13}	15.5305	15.6215	15.6210

The first fundamental natural frequency parameter for the case of full clamped plate is calculated when the thickness ratios α vary from 0.1 to 1.0, and the panel width ratio is taken as $\gamma = 0.75$, as shown in Table III. The results show that the natural frequency parameters increase by decreasing the thickness ratio α .

TABLE III
DIMENSIONLESS NATURAL FREQUENCY PARAMETER λ_{mn}^2

IVFD	α	0.1	0.2	0.3	0.4	0.5
	λ_{11}^2		50.54	49.78	48.76	46.91
α	0.6	0.7	0.8	0.9	1.0	
λ_{11}^2		41.8	40.2	38.15	36.59	35.536

Another case is studied for CCCC square plate of Panel width ratio $\gamma = 0.5$ and thickness ratio $\alpha = 2.0$ for different aspect ratios β as shown in Table IV. As seen from the results, the

natural frequency parameter decreases whenever the aspect ratio increases.

TABLE IV
DIMENSIONLESS NATURAL FREQUENCY PARAMETER λ_{nm}^2

IVFD					
$\alpha = 2.0$					
β	1.2	1.4	1.6	1.8	2.0
λ_{11}^2	22.5549	20.1156	18.649	17.6919	17.0598
λ_{12}^2	49.8966	48.0585	47.4841	46.5267	46.0672
λ_{21}^2	41.9315	34.1196	29.1031	25.7716	23.4357
λ_{22}^2	66.1331	59.0487	54.7599	51.122	51.1603
λ_{31}^2	70.8815	55.1428	44.4972	38.1021	33.3154
λ_{32}^2	86.5435	77.1999	68.1626	61.842	56.6745

The plate mode shapes of a full clamped CCCC plate with thickness ratios $\alpha = 2.0$ and $\alpha = 1.0$, panel width ratio $\gamma = 0.5$ and aspect ratio $\beta = 1.6$, are plotted as shown in Figs. 3-a,b and 4-a,b in the appendix.

V. CONCLUSION

The developed method is successfully applied to free vibration analysis of stepped thickness plates with different combinations of boundary conditions. It has been shown that the natural frequencies calculated using the IVFD method agrees closely with the results in the published literature. It has also been found that the solutions converge rapidly with the increase in the number of grid lines in the direction of the step-by-step integration technique. However, it is noticed that the number of the grid lines in the direction of the initial value method have to be chosen for the other direction of finite difference method. This is due to the efficiency of the two different techniques used in the two directions. The method can be developed to extend in the future work to deal with the dynamic problems of different shapes of plates as well as stepped plates with hollow and circular stepped plate.

APPENDIX

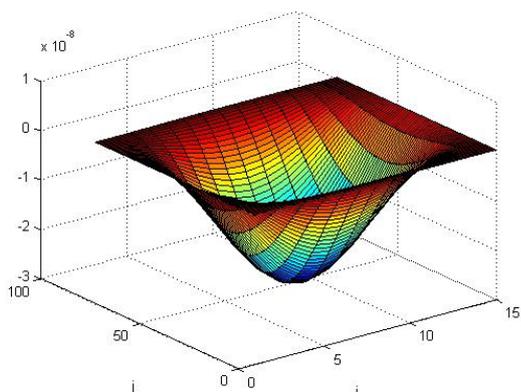


Fig. 3(a) 1stMode shape of $\lambda^2 = 26.1928$ and $\alpha = 1.0$

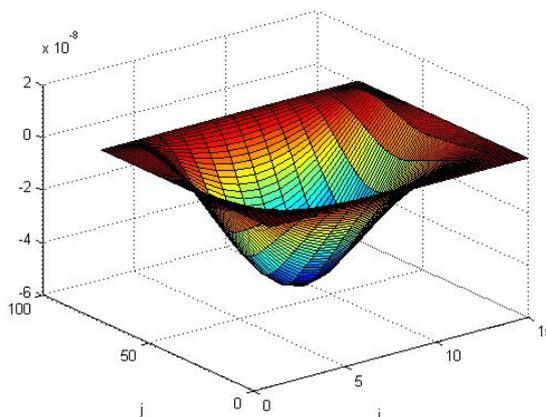


Fig. 3(b) 1stMode shape of $\lambda^2 = 18.649$ and $\alpha = 2.0$

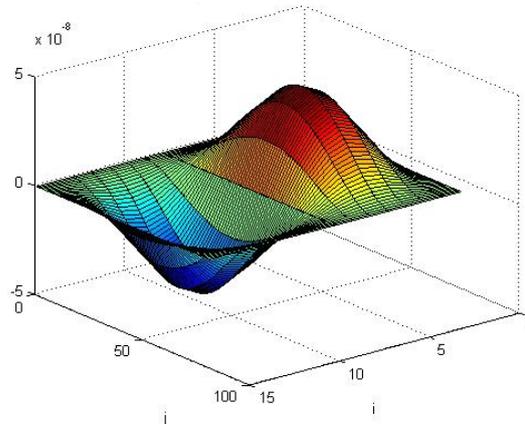


Fig. 4(a) 2ndMode shape of $\lambda^2 = 37.9107$ and $\alpha = 1.0$

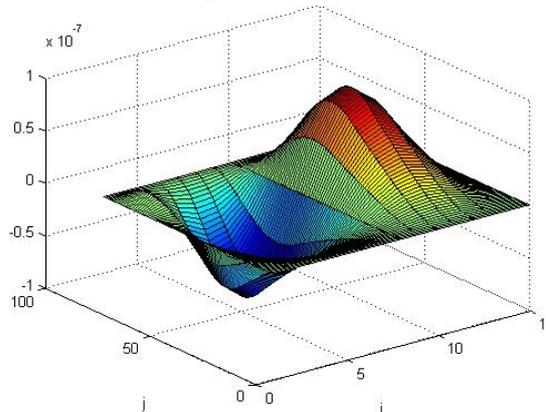


Fig. 4(b) 2ndMode shape of $\lambda^2 = 29.1031$ and $\alpha = 2.0$

REFERENCES

- [1] A. M. Farag "Closed form solution for vibrating surfaces of partially restrained and clamped double-panel plates" European Journal of scientific Research. Vol 29 N0.3 pp.320-333, 2009.
- [2] Y. Xiang, G.W. Wei "Exact solutions for buckling and vibration of stepped rectangular mindlin plates" International journal of solids and structures, vol. 41, pp 279-294, 2004.
- [3] S. Kukla ,M. Szweczyk " free vibration of annular plates of stepped thickness resting on winkler elastic foundation" scientific research of institute of mathematics and computer science, Poland, 2008.

- [4] M. El-sayad, A. M. Farag "numerical solution of vibrating double and triple-panel stepped thickness plates" Applied & Computational Mathematics.vol 1, issue 3, 2012
- [5] S.J. Guo, A. J. Keane, M. Moshrefi "vibration analysis of stepped thickness plates" journal of sound and vibration, vol. 204(4), pp 645-657, 1997.
- [6] G. D. Hatzigeorgiou, D. E. Beskos, "Static and dynamic analysis of inelastic solids and structures by the BEM", Journal of the Serbian Society for Computational Mechanics , Vol. 2 , No. 1, 2008 / pp. 1-27.
- [7] A. M. Farag, "Mathematical analysis of free and forced vibration of rectangular plate", Ph.D Thesis, Faculty of engineering, Alexandria university, 1994.
- [8] A. A. Kuleshov, "Finite difference method for the model of small transverse vibrations in thin elastic plates" Proceeding of the 4th WSEAS international conference of finite differences, pp, 19-22, 2010.
- [9] A. Ergun, N. Kunbasar, "A new approach of improved finite difference scheme on plate bending analysis", Scientific research and essays vol.6(1),pp, 6-17, 2011.
- [10] R. J. LeVeque, "Finite difference methods for ordinary and partial differential equations: steady-state and time-dependent problems", the Society for Industrial and Applied Mathematics, 2007.
- [11] W. Yu, R. Mitra, T. Su, Y. Liu, X. Yang, "Parallel finite-difference time-domain method." ARTECH HOUSE, 2006.
- [12] D. J. Duffy, "Finite Difference Methods in Financial Engineering A Partial Differential Equation Approach", cs-books@wiley.co.uk, 2006.
- [13] H. Al-Khaiat, H. H. West., "Analysis of plates by the initial value method". Computer & structure vol.24 No.3, pp, 475-483, 1986.
- [14] H. Al-Khaiat., "Free vibration analysis of orthotropic plates by the initial value method". Computer & structure vol.33 No.6, pp, 1431-1435, 1989.
- [15] S. Timoshenko, S. Woiowsky-krieger, "Theory of plates and shells", McGRAW-HILL, 1959.
- [16] Y. F. Xing, B.Liu, "New exact solutions for free vibrations of thin orthotropic rectangular plates" , Elsevier, Composite Structure, 89, pp, 567-574, 2009.
- [17] A. K. Gupta, N.Agarwal, H.Kaur, "Free vibration analysis of non-homogeneous orthotropic visco-elastic elliptic plate of non-uniform thickness", Int. J. of Appl. Math and Mech. 7(6): pp, 1-18, 2011.
- [18] A. M. Farag, and A.S. Ashour "Free vibration of orthotropic skew plates", Journal of vibration and acoustics, ASMF, vol. 122, pp, 313-317, 2000.
- [19] I. Chern "Finite difference methods for solving differential equations", Department of Mathematics, National Taiwan University, 2009.
- [20] J. Awrejcewicz , "Numerical Analysis – Theory and Application", Published by InTech, JanezaTrdine 9, 51000 Rijeka, Croatia, 2011.
- [21] C. B. Dolicanin, V.B. Nikolic, D. C. Dolicanin, "Application of finite difference method to study of the phenomenon in the theory of thin plates", Appl. Math. Inform. And Mech. Vol. 2, 1, pp, 29-43, 2010.
- [22] N. Baddour , "Recent Advances in Vibrations Analysis", Published by InTech, JanezaTrdine 9, 51000 Rijeka, Croatia, 2011.
- [23] M. A. El-Sayad and S. A. Ghazy "Rayleigh-Ritz Method for Free Vibration of Mindlin Trapezoidal Plates" International Journal of Emerging Technology and Advanced Engineering, Volume 2, Issue 5, May 2012.
- [24] H. Yunshan "Dsc-Ritz method for the free vibration analysis of mindlin plate" Msc department of computer science, national university of Singapore, 2003.
- [25] H. Khov, W. L. Li b, R. F. Gibson An accurate solution method for the static and dynamic deflections of orthotropic plates with general boundary conditions Composite Structures 90, 474-481, (2009).
- [26] H. Thai, S Kim "Levy-type solution for free vibration analysis of orthotropic plates based on refined plate theory" Elsevier, Applied mathematical modeling, vol. 36, pp. 3870-3882, 2012.
- [27] G. M. Oosterhout, P. J. Van Derhoogt and R. M. Spiering. "Accurate calculation methods for natural frequencies of plates with special attention of the higher modes" Journal of Sound and Vibration" vol. 183(1), pp. 33-47, 1995.



Ahmed M. Farag was born in El- Bohaira, Egypt in 1959. He received his BS.c. Degree with Honor in 1982 in Civil Engineering and his MS.c and Ph.D degrees in Engineering Mathematics from faculty of Engineering, Alexandria University in 1987 and 1994, respectively. His Ph.D thesis was comprised of a study of free and forced vibration of rectangular plates. His research interests include computer graphics, geometric modeling, mathematics and vibration of structures.

He is a professor and the head of Engineering Mathematics department in the Faculty of Engineering - Alexandria University. He also is the executive manager of the Specialized Scientific Programs SSP in the Faculty of Engineering - Alexandria University. Author can be reached by: E-mail: afarag59@yahoo.com Tel: +201222410680 or through postal Address: Department of Engineering Mathematics and Physics, Faculty of Engineering, Alexandria University, Alexandria, 21544-Egypt.

Wael F. Mohamed was born in Egypt in 1979. He received his BS.c Degree in Civil Engineering from Alexandria University in Egypt in 2000. After his graduation he joined the same university as a Demonstrator where he obtained his MS.c. Degree in Engineering Mathematics in 2006.

He works as ASSISTANT LECTURE at Department of Engineering Mathematics & Physics, Faculty of Engineering, Alexandria University, Egypt. Research interests; Geometry and Vibration.

Author can be reached by: E-mail: wfmobarak@gmail.com Tel: +201005337317 or through postal Address: Department of Engineering Mathematics and Physics, Faculty of Engineering, Alexandria University, Alexandria, 21544-Egypt.

Atef A. Ata was born in Alexandria, Egypt in 1962 and received his BS.c. Degree with Honor in Mechanical Engineering from Alexandria University in Egypt in 1985. After his graduation he joined the same university as a Lecturer where he obtained his MS.c. Degree in Engineering Mathematics (Hydrodynamics) in 1990. In 1996 he obtained his Ph. D in Engineering Mathematics (Robotics) as a Joint-Venture between University of Miami, Florida, USA and Alexandria University in Egypt.

He joined Alexandria University again as Assistant Professor till 2001. Then he joined the Mechatronics Engineering Department, International Islamic University Malaysia as an Assistant Professor , Associate Professor (2004) and as a Head of Department (November 2005-June 2007). Currently he is a Professor of engineering mechanics at the Faculty of Engineering, Alexandria University, Egypt. Dr. Atef is a senior member of IACSIT as well as Egyptian Engineering Syndicate. He was also one of the Editorial Consultant Board for 2006-2008 for the International Journal of Advanced Robotic Systems, Austria. Currently he is one of the Editorial board of Mechanical Engineering Research, Canada and an Associate Editor of Alexandria Engineering Journal, Egypt hosted by Elsevier. His research interest includes Dynamic and Control of Flexible Manipulators, Trajectory Planning, Genetic Algorithms and Modeling and Simulation of Robotic Systems.

Author can be reached by: E-mail: atefata@gmail.com or through postal Address: Department of Engineering Mathematics and Physics, Faculty of Engineering, Alexandria University, Alexandria, 21544-Egypt.