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# A Servo Control System Using the Loop Shaping Design Procedure

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***Abstract*****—** This paper describes an expanded system for a servo system design by using the Loop Shaping Design Procedure (LSDP). LSDP is one of the $H_\infty$ design procedure. By conducting Loop Shaping with a compensator and robust stabilization to satisfy the index function, we get the feedback controller that makes the control system stable. In this paper, we propose an expanded system for a servo system design and apply to the DC motor. The proposed method performs well in the DC motor positioning control. It has no steady-state error in the disturbance response and it has robust stability.

***Keywords*****—**Loop Shaping Design Procedure (LSDP), servo system, DC motor.

## I. Introduction

The design of a controlled system needs robust stability. In 1990, McFarlane and Glover proposed the loop shaping design procedure (LSDP) [1]. It does not require the accurate model of controlled plant but it is a valid robust control method. It is used for many kinds of control system [2]-[4]. In this study, we conduct the DC motor positioning control. The motor is one of the devices widely used in the industrial world such as a car, a transport machine, robots, power plants, home appliances, and so on. Especially, the motor control with the hybrid car and the electric vehicle becomes one of the key technologies in recent years. High-precision motor control attracts attention by the expansion of the use range of motor, and its study is done actively [5]-[8].

In this paper, we have proposed an expanded system for a servo system design and applied to a DC motor. The proposed method performs well in the DC motor positioning control. It has no steady-state error in the disturbance response and it has robust stability. The organization of this paper is as follows: In Section 2, Loop shaping design procedure is described. In section 3, we propose an expanded system for servo system design. In section 4, simulation and experimental results are given. Finally some conclusions are remarked in Section 5.

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## II. Loop Shaping Design Procedure

The design of a controlled system needs robust stability. Loop Shaping Design Procedure (LSDP) is one of the robust design methods based on the $H_\infty$ control theory. It is a design method proposed by McFarlane and Glover. It can divide into three systematic procedures as follows:

1) Loop Shaping :
Using a pre-compensator $W_1$ and/or a post-compensator $W_2$, the singular values of the nominal plant are shaped to give a desired open-loop shape. The nominal plant $G$ and shaping functions $W_1$, $W_2$ are combined to form the shaped plant, $Gs = W_2GW_1$, as shown in Fig.1.

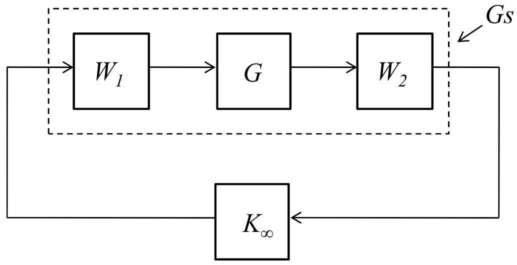


Fig. 1 Loop Shaping

2) Robust stabilization:
A controller $K_\infty$ satisfies

$$\left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (1 - G_s K_\infty)^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_\infty < \gamma. \tag{1}$$

The lowest achievable value $\gamma_{min}$ of $\gamma$ and the corresponding maximum stability margin $\varepsilon$ are derived by Glover and McFarlane [9] as

$$\begin{aligned} \gamma_{\min} = \varepsilon_{\max}^{-1} &= \left\{ 1 - \left\| \begin{bmatrix} N_s & M_s \end{bmatrix} \right\|^2 \right\}^{-\frac{1}{2}} \\ &= \left( 1 + \lambda_{\max}(\boldsymbol{XZ}) \right)^{\frac{1}{2}}, \end{aligned} \tag{2}$$

where $\lambda_{max}$ denotes the maximum eigenvalue. $\boldsymbol{Z}$ and $\boldsymbol{X}$ are positive definite solution of the following Riccati equations:

$$\begin{aligned} &(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{D}^T\boldsymbol{R}^{-1}\boldsymbol{C})\boldsymbol{Z} + \boldsymbol{Z}(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{D}^T\boldsymbol{R}^{-1}\boldsymbol{C})^T \\ &\qquad - \boldsymbol{Z}\boldsymbol{C}^T\boldsymbol{R}^{-1}\boldsymbol{C}\boldsymbol{Z} + \boldsymbol{B}\boldsymbol{S}^{-1}\boldsymbol{B}^T = 0, \end{aligned} \tag{3}$$

$$\begin{aligned} &(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{S}^{-1}\boldsymbol{D}^T\boldsymbol{C})^T\boldsymbol{X} + \boldsymbol{X}(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{S}^{-1}\boldsymbol{D}^T\boldsymbol{C}) \\ &\qquad - \boldsymbol{X}\boldsymbol{B}\boldsymbol{S}^{-1}\boldsymbol{B}^T\boldsymbol{X} + \boldsymbol{C}^T\boldsymbol{R}^{-1}\boldsymbol{C} = 0, \end{aligned} \tag{4}$$

where

$$\boldsymbol{R} = \boldsymbol{I} + \boldsymbol{D}\boldsymbol{D}^T, \quad \boldsymbol{S} = \boldsymbol{I} + \boldsymbol{D}^T\boldsymbol{D}. \tag{5}$$

Then we can get a sub-optimal controller $K_\infty$ as follows:

$$\begin{aligned} \boldsymbol{A_k} &= \boldsymbol{A} + \boldsymbol{BF} + \gamma^2 \boldsymbol{W}_1^{-T} \boldsymbol{Z}\boldsymbol{C}^T(\boldsymbol{C} + \boldsymbol{DF}), \\ \boldsymbol{B_k} &= \gamma^2 \boldsymbol{W_1}^{-T} \boldsymbol{Z}\boldsymbol{C}^T, \\ \boldsymbol{C_k} &= \boldsymbol{B}^T \boldsymbol{X}, \\ \boldsymbol{D}_k &= -\boldsymbol{D}^T, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \boldsymbol{F} &= -\boldsymbol{S}^{-1}(\boldsymbol{B}^T\boldsymbol{X} + \boldsymbol{D}^T\boldsymbol{C}), \\ \boldsymbol{W}_1 &= \boldsymbol{I} + (\boldsymbol{XZ} - \gamma^2 \boldsymbol{I}). \end{aligned} \tag{7}$$

3) The final feedback controller $K$ is then constructed by combining the $H_\infty$ controller $K_\infty$ with the shaping functions $W_1$ and $W_2$ such that

$$K = W_2 K_\infty W_1. \tag{8}$$

It's shown as Fig.2.

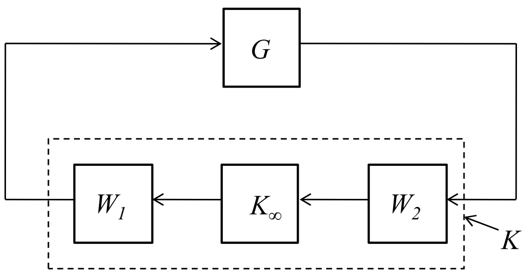


Fig. 2 The final feedback controller K

A typical design works as follows: The open-loop singular values of the nominal plant, and shapes by pre- and/or post-compensation until nominal performance specifications are met. A feedback controller $K_\infty$ with associated stability margin for the shaped pant $\varepsilon \le \varepsilon_{max}$, is then synthesized. If $\varepsilon_{max}$ is small, then the specified loop shape is incompatible with robust stability requirements. And it should be adjusted adequately, so that $K_\infty$ is reevaluated.

## III. An Expanded System for Servo System Design

In the proposed method, we design the controlled system that matches the plant output to the reference value for a servo system. The transfer function of a servo system is given as follows:

$$G(s) = \frac{b}{s(s+a)}. \tag{9}$$

Then the state space equation is expressed as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u + \boldsymbol{e}w, \tag{10}$$

$$y = \boldsymbol{c}\boldsymbol{x} + du, \tag{11}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 0 \\ b \end{bmatrix}, \boldsymbol{c} = \begin{bmatrix} 1 & 0 \end{bmatrix}, d = 0, \boldsymbol{e} : unknown, \tag{12}$$

where $w$ denotes the disturbance.

In the design of the servo system, the plant input becomes a constant. So that the quadratic index function becomes infinity. To avoid it, we use the following signal $v(t)$ instead of $u(t)$.

$$v(t) = \frac{d}{dt} u(t), \tag{13}$$

$$\frac{d}{dt}\begin{bmatrix} \boldsymbol{x} \\ u \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{b} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{x} \\ u \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v + \begin{bmatrix} \boldsymbol{e} \\ 0 \end{bmatrix} w, \tag{14}$$

$$y = \begin{bmatrix} \boldsymbol{c} & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{x} \\ u \end{bmatrix}. \tag{15}$$

If the new state variable is defined as

$$\dot{\boldsymbol{x}}^\# = \begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{u} \end{bmatrix}, \tag{16}$$

Then the state equation and the output equation are obtained as follows:

$$\dot{\boldsymbol{x}}^\# = \boldsymbol{A}^\# \boldsymbol{x}^\# + \boldsymbol{b}^\# v + \boldsymbol{e}^\# w, \tag{17}$$

$$y = \boldsymbol{c}^\# \boldsymbol{x}^\#, \tag{18}$$

where

$$\boldsymbol{A}^\# = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{b} \\ 0 & 0 \end{bmatrix}, \boldsymbol{b}^\# = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \boldsymbol{c}^\# = \begin{bmatrix} \boldsymbol{c} & 0 \end{bmatrix}, \boldsymbol{e}^\# = \begin{bmatrix} \boldsymbol{e} \\ 0 \end{bmatrix}. \tag{19}$$

For this expanded system, the controller is designed based on LSDP. Fig.3 shows the block diagram of the proposed method.

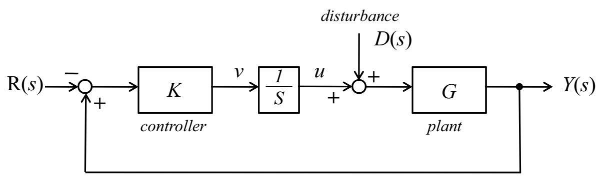


Fig. 3 Block diagram of proposed method

From (13), the plant input is as follows:

$$u(t) = \int_0^t v(t)dt. \tag{20}$$

Therefore, in Fig.3 we use the integrated controller output as the plant input.

LSDP does not consider the noise of the controlled system. However, we consider the noise that comes from power supply. Then the proposed pre-compensator is a low pass filter to reduce the observation noise. We consider the power supply noise to be 50[Hz] (=300[rad/sec]). We define the pre-compensator $W_1$ as follows:

$$W(s) = \frac{30}{s+30}. \tag{21}$$

The cut-off frequency is 30 [rad/sec]. The post-compensator $Q$ is related to the response speed. $G$, $W(s)$ and $Q$ are combined to form the shaped plant. (Fig.4)

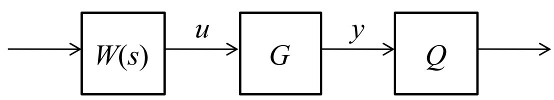


Fig. 4 The shaped plant

## IV. Simulation and Experimental Results

In this paper, the controlled plant is a DC motor. The transfer function is given as follows:

$$G(s) = \frac{b}{s(s+a)}, \tag{22}$$

$$a = \frac{K_m^2}{J_{eq} R_m}, b = \frac{K_m}{J_{eq} R_m}, \tag{23}$$

where $J_{eq}$ is the moment of a inertia, $R_m$ is the armature resistance and $K_m$ is the torque of the motor.

### A. Example 1

This example shows a simulation result. Consider the following parameters $a$ and $b$;

$$a = 10.8, b = 228, \tag{24}$$

Then the state space equation is expressed as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u, \tag{25}$$

$$y = \boldsymbol{c}x, \tag{26}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & -10.8 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 0 \\ 228 \end{bmatrix}, \boldsymbol{c} = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{27}$$

We construct the expanded system and design the controller based on LSDP. The final feedback controller $K$ is expressed as follows:

$$\dot{\boldsymbol{x}}_{\boldsymbol{k}} = \boldsymbol{A}_{\boldsymbol{k}} \boldsymbol{x}_{\boldsymbol{k}} + \boldsymbol{b}_{\boldsymbol{k}} y, \tag{28}$$

$$v = \boldsymbol{c}_{\boldsymbol{k}} \boldsymbol{x}_{\boldsymbol{k}} + d_k y, \tag{29}$$

where

$$\boldsymbol{A}_{\boldsymbol{k}} = \begin{bmatrix} -30 & 0.8 & 0.0717 & 6.2039 & 5.6684 \\ 0 & -48.6122 & 1 & 0 & 0 \\ 0 & -191.9501 & -10.8 & 228 & 0 \\ 0 & -10.4998 & 0 & 0 & 30 \\ 0 & -1.2021 & -0.0717 & -6.2039 & -35.6684 \end{bmatrix}, \tag{30}$$

$$\boldsymbol{b}_{\boldsymbol{k}} = \begin{bmatrix} 0 \\ -48.6122 \\ -191.9501 \\ -10.4998 \\ -0.4021 \end{bmatrix}, \tag{31}$$

$$\boldsymbol{c}_{\boldsymbol{k}} = \begin{bmatrix} 30 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{32}$$

$$d_k = 0. \tag{33}$$

The simulation result is shown in Fig.5. A step set-point is 2 $\pi$=6.28[rad] and introduced at $t$ = 1[s]. A load disturbance $D$(s) = 0.2/s is introduced at $t$ = 15[s]. The post-compensator $Q$ is 0.8. The proposed method has no steady-state error in the disturbance response.

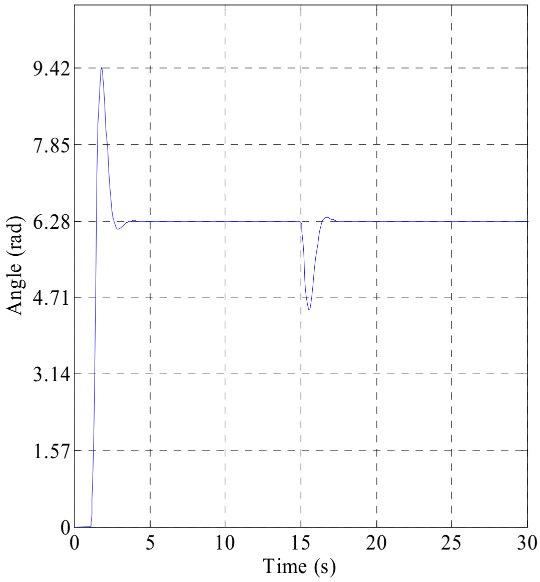


Fig.5 Simulation result of output response in *Example 1*.

### B. Example 2

This example shows a simulation result when there is the controlled plant identification error. The transfer function is given as follows:

$$G(s) = \frac{b}{s(s+a)}, \tag{34}$$

where

$$a = 8.64, b = 274. \tag{35}$$

We assume that there is 20% error in the control parameters $a$ and $b$, respectively. Using the controller $K$ designed by *Example 1*. The post-compensator $Q$ is the same as *Example 1*. The simulation result is shown in Fig.6. It has robust stability. Even if the controlled plant identification has the error, it is easy to construct a stable control system.

### C. Example 3

This example shows an experimental result for the real plant. The controlled plant and the final feedback controller $K$ are the same as *Example 1*. Fig.7 shows the experimental result when proposed method is practically performed. A step set-point is 2 $\pi$=6.28[rad] and introduced at $t$ = 1[s]. A load disturbance $D$(s) = 0.2/s is introduced at $t$ = 15[s]. The post-compensator $Q$ is 0.8. The system is discretized by Zero-order hold method. The sampling time is 0.01[s]. We insert the set-point filter to reduce the overshoot.

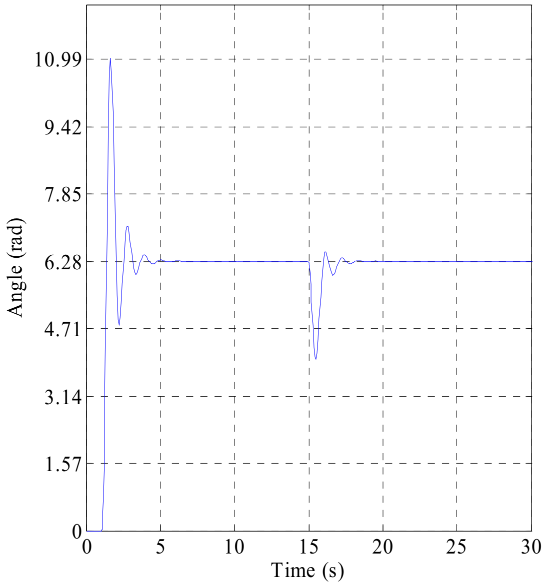


Fig.6 Simulation result of output response in *Example 2*.

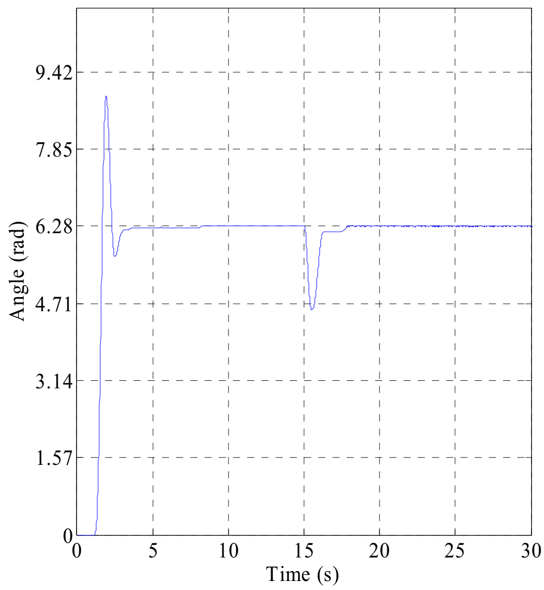


Fig.7 The DC motor positioning control.

## V. Conclusion

In this paper, we have proposed an expanded system for a servo system design by using the LSDP and applied to the DC motor. The experimental result suggests that the proposed method performed well in the DC motor positioning control. The proposed method has no steady-state error in the disturbance response. It has robust stability. Even if the controlled plant identification has the error, it is easy to construct a stable control system.

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