Modeling of Plasticity of Clays Submitted to Compression Test

Otávio J.U. Flores, Fernando A. Andrade, Dachamir Hotza, and Hazim A. Al-Qureshi

***Abstract*—**In the forming of ceramic materials the plasticity concept is commonly used. This term is related to a particular mechanical behavior when clay is mixed with water. A plastic ceramic material shows a permanent strain without rupture when a compressive load produces a shear stress that exceeds the material's yield strength. For a plastic ceramic body it observes a measurable elastic behavior before the yield strength and when the applied load is removed. In this work, a mathematical model was developed from applied concepts of the plasticity theory by using the stress/strain diagram under compression.

***Keywords*—**Plasticity, clay, modeling, coefficient of friction.

I. INTRODUCTION

THE plasticity in the processing of ceramic materials is a fundamental property since it defines the necessary technical parameters to convert a particulate ceramic body to a component with a given shape by application of pressure.

The plasticity, in this case, and particularly in clay mineral systems, is defined as a property that shows shape changes without rupture when a clay body with added water is submitted to an external force. Furthermore, when the force is removed or reduced bellow to a value corresponding to the yield stress the shape is maintained [1], [2].

The plasticity in clays is developed due to morphology of plate-like clay particles that slide over the others when water is added, which acts as lubricant. In any event, it is known that as the water content of clay is increased, its plasticity increases up to a maximum, depending on the nature of the clay. While clay workers are accustomed to speak of a "fat" or highly plastic clay such as a ball clay or a "lean", relatively non-plastic clay such as a residual kaolin, it is very difficult to express these terms in measurable quantities. In industry, plasticity is also referred to as "extrudability", "ductility", "workability" or "consistency" [3].

The main factors that affect the clay plasticity, according to Barba et al. [4] and Haendle [3] are related to physical characteristics of the solid, particularly the particle size distribution and its specific surface area, the water characteristics (viscosity, surface tension, etc.), the solid mineralogical composition (clay mineral type, proportion of non-plastic minerals, etc.), the dispersion state of the particles that depends on the ionic change capacity and nature and proportion of additives as well as on the ceramic body temperature. Process-related factors are application of pressure, body temperature and characteristics of water and additives used [5].

However, the plasticity determination is not always an easy task since it cannot be immediately interpreted and applied. In fact, there are several methods for measurement and characterization of the plasticity of a clay body, although its experimental determination, in some cases, is operator dependent. This hinders a comparison confront between different methods [6]. Among the methods, Atterberg's plasticity index, Pfefferkorn's plasticity index, stress/strain curves, indentation, and rheological measurements are the most applied.

The evaluation of clay plasticity can be divided in two groups: indirect or direct measurements. In the first case, the plasticity is not determined itself, but rather the properties related to it. These methods have been criticized but they are widely used due to the low cost of the equipment employed. In the second group, the methods evaluate the effect of moisture content and other variables on the relationship between an applied force and the resulting deformation. These methods are more common for advanced materials and are more cost intensive due to the equipment used. Nevertheless, they can supply important parameters to correctly evaluate the significance of process and raw material variables on the measured property.

In traditional ceramic materials, due to the addition of clay materials in their composition, the measure and control of the plasticity are basic to characterize a system and to optimize the conditions of their processing [7].

As well as for other types of materials, a compression test can be used to evaluate the plasticity of clays. In the typical test curve, some information is generally obtained, such as modulus of elasticity, yield strength, maximum deformation and rupture strength. According to Ribeiro et al. [7], some parameters obtained in the compression test are strongly influenced by the chemical composition and humidity of the clay.

In this context, a mathematical model for evaluation of the

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plasticity of clay bodies was developed from applied concepts of the plasticity theory by using the stress/strain diagram under compression.

II. THEORETICAL CONSIDERATIONS

Because of several factors related to the particles such as fracture, fragmentation and arrangements it is difficult to formulate the mechanism and the plastic behavior of porous materials. In this case, the mathematical knowledge applied to metallic porous materials was used as a basic tool for plasticity modeling of clays [8], taking into account few experimental parameters.

To define the processing parameters, it was hypothesized that the clay compact, which has a cylindrical shape, will deform axially and symmetrically. When the compressive force is applied, the height of the cylindrical compact decreases and its instantaneous radius increases, since the sample is not confined in a die.

Considering an infinitesimal volume in cylindrical coordinates (r, θ and z) and the equilibrium of forces in the radial direction r [9], it results in

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta r}}{\partial\theta}+\frac{\partial\sigma_{zr}}{\partial z}+\frac{1}{r}\left(\sigma_{rr}-\sigma_{\theta\theta}\right)+F_r=0 \tag{1}$$

In this model the compressive stress is considered constant. For the shape of the infinitesimal volume studied (1) is simplified to:

$$\frac{d\sigma_r}{dr}+\frac{(\sigma_r-\sigma_\theta)}{r}=\frac{-2\mu\sigma_z}{h} \tag{2}$$

where σ_r is the radial stress; σ_θ, the normal stress; σ_z, the axial stress; and μ, the coefficient of friction between plates surfaces of the compression test machine and the clay compact. The coefficient of friction is also considered constant and will be discussed later.

As $\sigma_r = f(\sigma_\theta)$ and using the Levy-Mises equations for the plastic zone, it results that $d\varepsilon_r = d\varepsilon_\theta$ and, consequently, $\sigma_r = \sigma_\theta$. Substituting it in (2), the following equation is obtained:

$$\frac{d\sigma_r}{dr}=-\frac{2\mu\sigma_z}{h} \tag{3}$$

By using the Von Mises criteria related to the effective stress in compression $\overline{\sigma}$ of the clay material, the following equation is obtained:

$$\sigma_r=\sigma_z+\overline{\sigma} \tag{4}$$

Deriving (4) and substituting it in (3):

$$\frac{d\sigma_z}{dr}=-\frac{2\mu\sigma_z}{h} \tag{5}$$

To solve the differential (5), the following boundary conditions are considered:

$$r=r_f \tag{6}$$

and

$$\sigma_z=-\overline{\sigma} \tag{7}$$

where r_f is the sample final radius after compaction.

The resulting equation for the instantaneous axial stress as a function of the compaction processing parameters is given by:

$$\sigma_z=-\overline{\sigma}\exp\left[\frac{2\mu}{h}\left(r_f-r\right)\right] \tag{8}$$

Knowing that the axial force in any compression stage is function of the axial stress and the instantaneous area, it can be directly calculated by the following equation:

$$F=\int_0^{r_f}2\pi\sigma_z dr \tag{9}$$

Finally from (8) and (9) it is possible to relate the applied pressure to the compact instantaneous radius as well as to different variables that affect the plasticity of a given ceramic body. Thus, a more accurate approach to obtaining ceramic bodies with optimized plasticity for a given application is expected.

III. EXPERIMENTAL PROCEDURE

The materials used for the experiments were two clays (AC12 and AC39 supplied by Colorminas, Criciúma, SC, Brazil) and one kaolin (supplied by Caulina, Tenorio, PB, Brazil). Chemical and phase compositions were determined by X-ray fluorescence spectroscopy, XRF (Philips PW 2400) and X-ray diffraction, XRD (Philips X'Pert), and are shown in Table I and Table II, respectively. The quantification of phases was done by rational analysis [10] from the FRX and DRX data.

Kaolin is mainly comprised by kaolinite. The clay minerals present in AC12 and AC39 correspond to about 50% and 83%, respectively, being kaolinite the major clay mineral phase in both samples. Quartz was detected by XRD in agreement with the chemical analysis determined by FRX, which shows a high SiO_2 content. Such results allow supposing that the AC12 clay could not show a high plasticity [11].

TABLE I
CHEMICAL COMPOSITION (WT.%) OF CLAYS OBTAINED BY XRF

Oxide	AC12	AC39	Kaolin
SiO_2	69.41	51.61	45.52
Al_2O_3	18.51	32.57	38.50
Fe_2O_3	2.20	1.04	0.40
MgO	0.82	1.59	–
CaO	0.05	1.48	–
Na_2O	0.08	0.89	0.10
K_2O	2.91	1.57	0.25
TiO_2	0.73	0.08	–
MnO	0.01	0.03	–
P_2O_5	0.14	0.12	–
LoI*	5.15	9.02	13.57

*LoI = Lost on Ignition at 1000°C.

TABLE II
PHASE COMPOSITION (WT.%) OF CLAYS OBTAINED BY XRD AND RATIONAL ANALYSIS

Phase	AC12	AC39	Kaolin
Kaolinite	40.23	72.62	97.48
Mica muscovite	9.91	–	–
Montmorillonite	–	10.31	–
Quartz	47.46	6.62	1.18
Others	2.40	1.10	1.34

The clay, as supplied, was dry desegregated in a ball mill and then sieved (20 mesh, 840 μm) so that 1 kg of clay powder was obtained after processing. Aiming to determine the moisture content in the clay, samples with about 50 g of material were introduced in porcelain crucibles which were then heated at 120±5 °C for 24 h in a laboratory muffle oven. Ceramic bodies with moisture content of about 52%, 56% and 60% were prepared and homogenized by means of a mechanical mixer. Subsequently, the ceramic bodies were maintained in a hermetically closed plastic container during 24 h for moisture homogenization.

In a latter step, cylindrical samples were prepared by manual forming in PVC moulds (diameter = 47.70 mm; height = 49.80 mm). To evaluate in a quantitative way, the stress states in the longitudinal section some samples were sectioned, reticulated and submitted to different compressive loads. In this case, loads were applied by using a Pfefferkorn equipment.

Four samples were then submitted to compressive forces (at 3 mm/min) in a universal test machine (EMIC DL 2000). Each minute the applied force was interrupted and kept stable. At this moment the diameters of the cross sections in the central and extremities regions of the cylindrical samples were measured. From the sample diameter measurements the equivalent radius in the central (r_c) and extremities (r_e) regions were determined. For the radius measurements a metric reference scale was used, which was located beside the sample during the tests.

From digital photographs, taken at each time interval, the diameters were measured with the aid of a caliper. This procedure was adopted since direct measurements cause sample distortions.

IV. RESULTS AND DISCUSSION

Fig. 1 shows a sample longitudinally sectioned after application of a compressive force. As it can be observed the friction occurred between the plates of the test machine and the clay compact significantly affects the internal distribution of stresses.

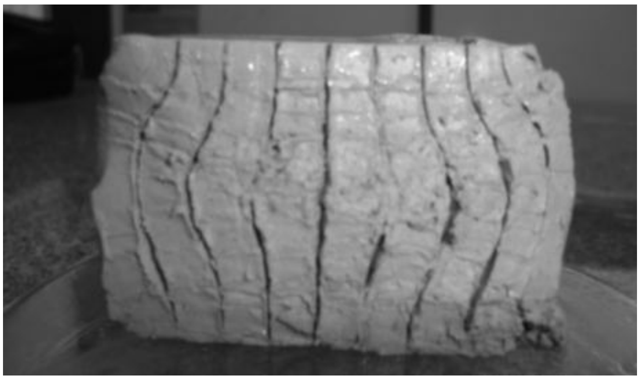
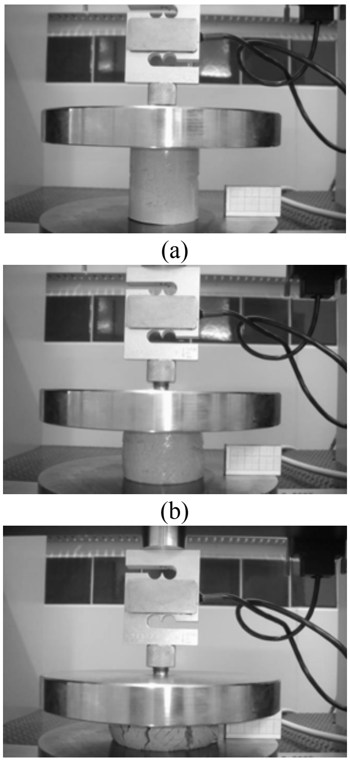


Fig. 1 Sectioned sample showing strain lines representing the stress distribution

When the friction is present the expansion movement of the material attached to the plates is restricted. The friction force is contrary to the material movement meaning that a higher stress must be generated in the center of the contact zone in order to allow the material to expand in this region. At the same time, as the clay compact is not confined in a die, a symmetric distribution of stresses in the sample longitudinal section is observed. The stresses are maximal in the central line and minimal in the compact laterals. This phenomenon is known as "friction hill" and it is related to the sample barreling. Because of this, the coefficient of friction is used as one of the mathematical parameters to analyze the actuating forces in cylindrical clay compacts.

Although the coefficient of friction might be a dynamic variable as function of the actuating stress, processing parameters and porosity, an acceptable procedure, due the difficult of its formulation, is to estimate it as a constant.

Even when barreling occurs, as in this case, it was excluded since it does not significantly affect the proposed analysis. To minimize the influence of this simplification, the instantaneous radius in the contact surface of the compression plates and the compact and in the sample medium height were measured. Fig. 2 shows the applied load evolution in a clay sample.



(a)

(b)

(c)

Fig. 2 Clay sample submitted to increasing compressive loads: (a) low, (b) intermediate, and (c) high load

According to Ribeiro et al. [6], when a clay sample is submitted to a continuous compressive load, a small elastic strain occurs initially followed by a relatively high plastic strain which culminate with the material's rupture.

In fact, according to Fig. 3, that shows the relationship between the applied compressive load and the radial variation (r_f-r_0) in a cylindrical sample, for calculated and experimentally determined values, the same behavior can be observed and a good agreement between the calculated and experimentally obtained data.

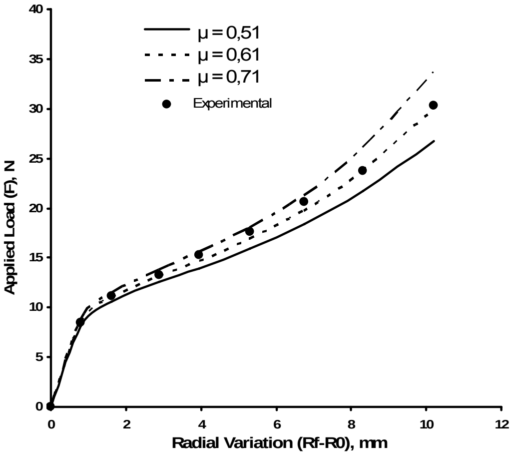


Fig. 3 Applied force as a function of the radial changes for different coefficients of friction

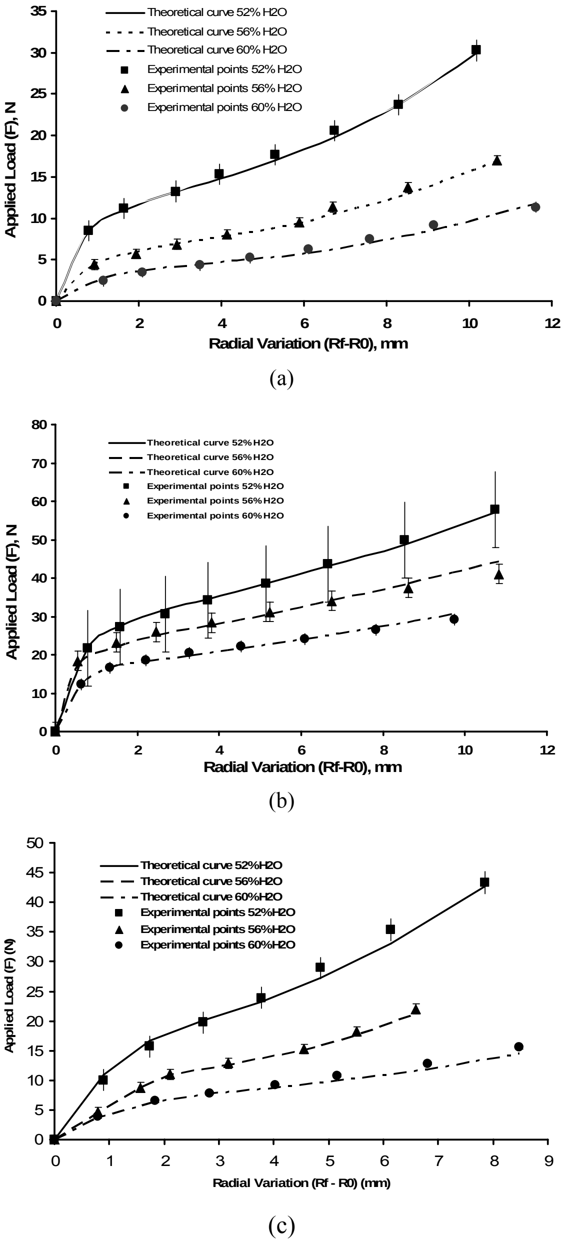
Moreover, as the elastic limit of these materials is relatively small and has not interest in the present analysis, the elastic strain was excluded. Consequently, it was possible to determine, by Eqs. (8) and (9), the average coefficient of friction and the effective compressive stress in the clay compact. The average values were determined by iterative processes which allowed the optimized adjustment of the theoretical curve. Fig. 3 shows that the value of coefficient of friction which best fit the experimental points can be chosen. This is because the values of friction are not readily available in the literature. In this case, for the studied clays, the average coefficient of friction and the effective compressive stress are presented in Table III.

TABLE III
AVERAGE COEFFICIENT OF FRICTION AND EFFECTIVE COMPRESSIVE STRESS

Clay	Moisture (%)	Coefficient of friction	Compressive stress (kPa)
AC12	52	0.61	4.40
	56	0.65	2.18
	60	0.69	1.28
AC39	52	0.05	14.00
	56	0.02	11.80
	60	0.01	8.80
Kaolin	52	0.93	5.64
	56	0.87	3.49
	60	0.60	2.60

The water acts not only as an inert medium to separate particles of clay minerals and to vary the forces of attraction and repulsion between them, but also has a very active role in the property of plasticity, guiding the lamellar particles in the flow direction. As the oriented water molecules are attached on the surface of clay minerals by hydrogen bridges, they also serve to bind the particles together in the wet clay, giving rise to various forms of mechanical strength of the green clay. Fig. 4 shows the influence of water content in the compression process for the same raw material.

Fig. 5 compares the curves of different materials for the same moisture content. The three graphs show a similar behavior of clays. The higher curve refers to the clay AC39, indicating that its plasticity is the highest. The lower curve is related to kaolin, followed indicating that it had the lowest plasticity among the raw materials studied.



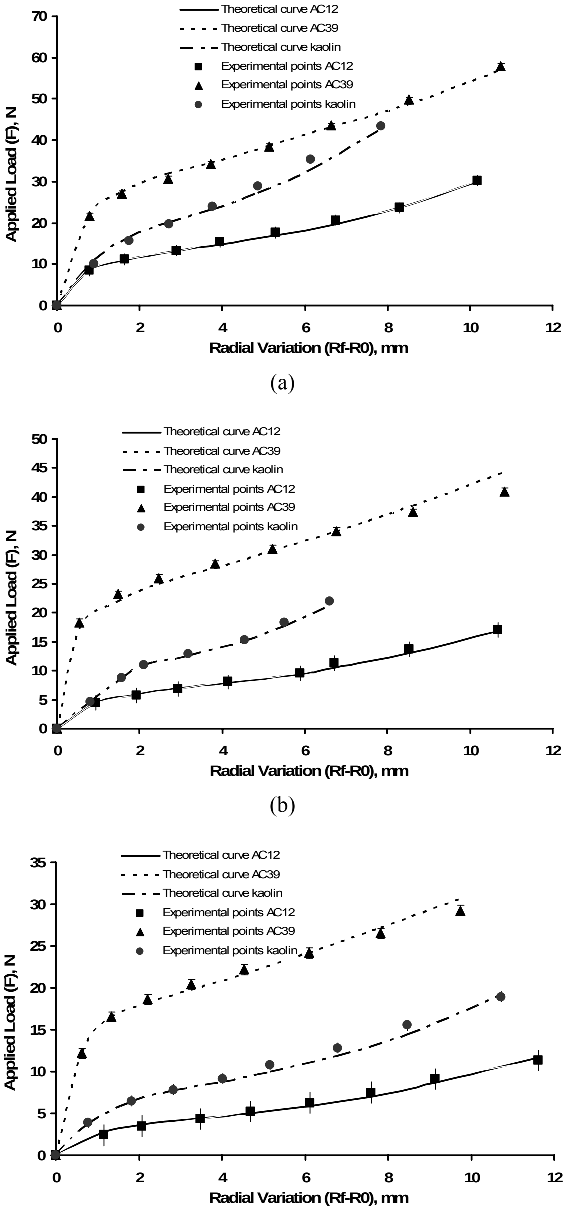
(a)

(b)

(c)

Fig. 4 Applied force as a function of the radial changes for calculated and experimentally determined values for: (a) AC19 clay, (b) AC 32 clay, and (c) kaolin

Considering other variables inherent to the processing of clay materials, which are intimately related to the plasticity, in a latter step, the mechanisms that characterize this important property may be determined. Consequently, a mathematical equation may be obtained to predict the plasticity in optimized conditions for a given application.



(a)

(b)

(c)

Fig. 5 Applied force as a function of the radial changes for calculated and experimentally determined values for different moisture contents: (a) 52%, (b) 56%, and (c) 60%

V. CONCLUSION

The compression mathematical model applied to the evaluation of the clay plasticity presented in this work showed a compatible adjustment with experimental data.

It is possible from the proposed model to determine important parameters such as the coefficient of friction and the effective compressive stress. As demonstrated, the studied materials showed a very low elastic limit which was not considered. These parameters are useful to compare the behavior of clays in different conditions and may serve to minimize the various tests in laboratory scale for the formulation of ideal bodies in the ceramic industry.

Thus, the developed mathematical model is a potential and useful tool for the evaluation of clay materials with optimized plasticity for a given application.

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