Integral tracking control for a piezoelectric actuator system

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Abstract—We propose an integral tracking control method for a piezoelectric actuator system. The proposed method achieves the output tracking without requiring any hysteresis observer or schemes to compensate the hysteresis effect. With the proposed control law, the system is converted into the standard singularly perturbed model. Using Tikhonov's theorem, we guarantee that the tracking error can be reduced to arbitrarily small bound. A numerical example is given to illustrate the effectiveness of our proposed method.

Keywords—Piezoelectric actuator, Tracking control, Hysteresis effect.

I. INTRODUCTION

Piezoelectric actuators have been used in various precise manipulation applications including sperm injection, precision machining, and active vibration control [1]-[3]. Because the actuators have some advantages such as physically infinite displacement resolution, high speed, large bandwidth, high output force, zero stick-slip effect, and little heat generation. These applications have inspired many researchers to study the control methods for the piezoelectric actuators. However, it is often not easy because a highly nonlinear relationship named the hysteresis effect exists between the applied input voltage and the output displacement. This nonlinear effect may cause inaccuracy in the output response and eventually lead to the instability of the closed loop system. To deal with the hysteresis effect, studies have been developed to model and compensate for the effect. The examples of modeling techniques include a nonlinear dynamic model with hysteresis [4], a voltage input electromechanical model [5], a charge steering model [6], a model of physical hysteresis [7], and a neural network hysteresis model [8]. Moreover, there are other approaches to the modeling of piezoelectric actuators, which are based on the established mathematical formulations to approximate the input-output behavior of hysteresis. The examples are presented as the Maxwell slip model, Duhem model, Prandtl-Ishlinskii model, Bouc-Wen model and Preisach model [5], [9]-[12].

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By using these approaches, various control methods to compensate or estimate the hysteresis were presented. In [13], an adaptive variable structure controller design method was presented to control a class of nonlinear systems with unknown Prandtl-Ishlinskii hysteresis. By designing a hysteresis observer to compensate the nonlinearity of the hysteresis, feedforward-feedback control strategy was proposed with PI feedback controller [14]. Authors of [15] proposed an adaptive control for discrete time dynamical system with hysteresis described by Prandtl-Ishlinskii model without constructing the inverse hysteresis. Backstepping design scheme with a robust adaptive dynamic surface control method was used for a class of uncertain perturbed strict-feedback nonlinear system having Prandtl-Ishilinskii hysteresis [16]. In [17], a modified Prandtl-Ishlinskii hysteresis model and its inverse are used to identify and compensate the hysteresis effect. However, there is a main drawback of Prandtl-Ishlinskii model. It is that the model cannot exhibit neither asymmetric hysteresis loops nor saturated hysteresis output. Moreover, design of hysteresis observer can cause complexity of the controller and its wrong design may hinder the performance of the controller.

In this paper, we propose an integral tracking control for piezoelectric actuator system. The proposed method does not require any hysteresis observer or scheme which can compensate the hysteresis effect. Therefore, the method can provide a easier way to implement or analysis when controlling the piezo system. With the proposed control law, the piezoelectric system is converted into the standard singularly perturbed model. Using Tikhonov's theorem, the method guarantees that the tracking error can be reduced to arbitrarily small bound by choosing a design constant as sufficiently small value. Numerical simulation is given to illustrate the effectiveness of our proposed method.

II. PIEZOELECTRIC ACTUATOR SYSTEM AND PROBLEM STATEMENT

The positioning mechanism of piezo actuator system has been described as a second order dynamics. Based on Bouc-Wen model, the dynamical equation can be represented as follows:

$$m\ddot{x} + b\dot{x} + kx = F_h = k(d_e r - h) \qquad (1)$$

where m is the mass of the piezo actuator, b is the damping coefficient, k is the spring coefficient, and x is the displacement, F_h is the net force including hysteresis term, r is the input voltage, d_e is the ratio of the displacement to the input

voltage, and h is the hysteresis nonlinear term. This hysteresis is modelled by

$$\dot{h} = \alpha d_e \dot{u} - \beta |\dot{u}| h - \gamma \dot{u} |h| \qquad (2)$$

where α, β, and γ are the parameters adjusting the shape and magnitude of the hysteresis loop.

It is well-known that the hysteresis is typically in the order of 10%-15% of the commanded motion, and it is a major drawback in utilizing the piezoelectric actuators for motion actuation. Therefore, our objective is to design a controller which make the output of the piezo actuator track the output of a desired model with suppressing the hysteresis effect.

Then, by removing the hysteresis term h of (1), the desired model is represented

$$\dot{x}_d = A x_d + B(f(x_d, r))$$
$$y_d = C x_d \qquad (3)$$

where $x_d \in \mathbb{R}^2$ is the state vector, $f(x_d, r) = -\frac{k}{m} x_{d1} - \frac{b}{m} x_{d2} + \frac{k}{m} d_e r$, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

The actual system has the hysteresis effect and is described as follows:

$$\dot{x} = A x + B [g(x, r) + u]$$
$$y = C x \qquad (4)$$

where $x \in \mathbb{R}^2$ is the state vector, $g(x, r) = -\frac{k}{m} x_1 - \frac{b}{m} x_2 + \frac{k}{m}(d_e r - h)$, the hysteresis term h is governed by (2), and u is the control input which forces to achieve the output tracking between (3) and (4), that is, $(y_d - y) \to 0$ as the time goes to infinity.

III. INTEGRAL TRACKING CONTROL FOR PIEZOELECTRIC ACTUATOR

In this section, we present a high gain integral control method for the piezoelectric actuator with the hysteresis. To obtain the scheme, let us define the following error dynamics between (3) and (4)

$$\dot{e} = A e + B [f(x_d, r) - g(x, r) - u] \qquad (5)$$

where $e = x_d - x \in \mathbb{R}^2$.

The following theorem summarizes the main result of this paper.

Theorem 1: Consider the desired piezoelectric actuator system (3) and actual system (4). Then, there exists $\varepsilon^* > 0$ such that for $\varepsilon \in (0, \varepsilon^*)$ the error e converges to zero as $t \to \infty$, if the control law is designed as

$$u = \frac{\alpha_1}{\varepsilon} e_2 + u_f + \mu \qquad (6)$$

$$\dot{\mu} = \frac{\alpha_2}{\varepsilon^2} e_2 + \frac{1}{\varepsilon} u_f \qquad (7)$$

$$u_f = K e \qquad (8)$$

where ε is a positive small constant, α_1 and α_2 are constants which satisfy that the polynomial $s^2 + \alpha_1 s + \alpha_2$ is Hurwitz, and K is a control gain matrix such that the transfer function $H(s) = (sI - (A - BK))^{-1}$ is Hurwitz.

Proof: By applying the control law (6), the error dynamics (5) can be written as

$$\dot{e} = A e + B \left[f(x_d, r) - g(x, r) - \frac{\alpha_1}{\varepsilon} e_2 - u_f \right]. \qquad (9)$$

To convert the equation into singular perturbed form, let us define the following vector

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \frac{e_2}{\varepsilon} \\ f(x_d, r) - g(x, r) - \mu \end{bmatrix}. \qquad (10)$$

Differentiating (10) and multiplying both sides by ε, then using (9) leads to

$$\varepsilon \dot{\eta}_1 = \dot{e}_2 = -\frac{\alpha_1}{\varepsilon} e_2 - u_f + f(x_d, r) - g(x, r) - \mu$$
$$\varepsilon \dot{\eta}_2 = \varepsilon \left[\dot{\phi} - \dot{\mu} \right] = -\alpha_2 \eta_1 - u_f + \varepsilon \left[\dot{f}(x_d, r) - \dot{g}(x, r) \right]. \qquad (11)$$

In the standard singularly perturbed model, the error system (9) is the slow subsystem and (11) is the fast subsystem.

The fast subsystem can be rewritten in state-space form

$$\varepsilon \dot{\eta} = \bar{A} \eta + \varepsilon \left\{ B \left[\dot{f}(x_d, r) - \dot{g}(x, r) \right] - \bar{B} \frac{u_f}{\varepsilon} \right\} \qquad (12)$$

where $\bar{A} = \begin{bmatrix} -\alpha_1 & 1 \\ -\alpha_2 & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Using (10) and (5), it can be shown that $\frac{1}{\varepsilon} u_f$ is locally Lipschitz uniformly in its arguments because

$$\frac{u_f}{\varepsilon} = \frac{1}{\varepsilon} K \left[\int e_2 dt, \quad e_2 \right]^T = K \left[\int \eta_1 dt, \quad \eta_1 \right]^T. \qquad (13)$$

Moreover, $\bar{A}$ is Hurwitz by designed constants α_1 and α_2. This means that the origin is an exponentially stable equilibrium point of the boundary-layer model [18]. Therefore, from Tikhonov's theorem, $\eta = O(\varepsilon)$ for $t \in [t_0, T(\varepsilon)]$ where $\lim_{\varepsilon \to 0} T(\varepsilon) = 0$.

Eventually, (9) can be rewritten as

$$\dot{e} = A e + B [u_f + O(\varepsilon)] = (A - BK) e + O(\varepsilon). \qquad (14)$$

This shows that $\|e\|$ is uniformly ultimately bounded and that the bound can be made arbitrarily small by choosing small ε. This means in turn that (e, η) will approach a neighborhood of the origin N_ε where N_ε can be made arbitrarily small by choosing small enough ε. Then, in N_ϵ, (9) and (12) can be rewritten as follows:

$$\dot{e} = (A - BK) e + \delta_1(\eta), \qquad (15)$$
$$\varepsilon \dot{\eta} = \bar{A} \eta + \varepsilon \delta_2(e, \eta) \qquad (16)$$

where

$$\delta_1(\eta) = B(\eta_2 - \alpha_1 \eta_1), \qquad (17)$$

$$\delta_2(e, \eta) = B \left(\dot{\eta}_2 + \frac{\alpha_2}{\varepsilon} \eta_1 \right) + \frac{1}{\varepsilon}(BKe - \bar{B}_2 Ke). \qquad (18)$$

The functions δ_1 and δ_2 are locally Lipschitz in (e, η) and vanish at the origin. Then, in N_ε,

$$\|\delta_1(\eta)\| = \|\delta_1(\eta) - \delta_1(0)\| \leq k_1 \|\eta\|$$
$$\|\delta_2(e, \eta)\| = \|\delta_2(e, \eta) - \delta_2(0, \eta) + \delta_2(e, \eta) - \delta_2(e, 0)\|$$
$$\leq \|\delta_2(e, \eta) - \delta_2(0, \eta)\| + \|\delta_2(e, \eta) - \delta_2(e, 0)\|$$
$$\leq k_2 \|e\| + k_3 \|\eta\| \qquad (19)$$

with nonnegative constants k_1, k_2, and k_3 that are independent of ε.

If $\delta_1 = 0$, the origin is an exponentially stable and the existence of a Lyapunov function V_1 that satisfies

$$\frac{\partial V_1}{\partial e} e\left[(A - BK)e\right] \le -k_4 \|e\|^2, \quad \|\frac{\partial V_1}{\partial e}\| \le k_5 \|e\| \quad (20)$$

is guaranteed by the converse Lyapunov theorem [18].

Choose the Lyapunov function candidate as $V = V_1(e) + \eta^T P \eta$. By differentiating the Lyapunov function, we have

$$\begin{aligned}
\dot{V} =& \frac{\partial V_1}{\partial e} e\left[(A - BK)e + \delta_1(\eta)\right] + \dot{\eta}^T P \eta + \eta^T P \dot{\eta} \\
\le& -k_4 \|e\|^2 + k_5 k_1 \|e\|\|\eta\| + \dot{\eta}^T P \eta + \eta^T P \dot{\eta} \\
\le& -k_4 \|e\|^2 + k_5 k_1 \|e\|\|\eta\| + \eta^T \left(\frac{1}{\varepsilon} \bar{A}^T P + \frac{1}{\varepsilon} P \bar{A}\right) \\
& + 2k_2 \lambda_{max}(P)\|e\|\|\eta\| + 2k_3 \lambda_{max}(P)\|\eta\|\|\eta\| \\
\le& \begin{bmatrix} \|e\| \\ \|\eta\| \end{bmatrix}^T \Phi \begin{bmatrix} \|e\| \\ \|\eta\| \end{bmatrix}
\end{aligned} \quad (21)$$

where

$$\Phi = \begin{bmatrix} -k_4 & \frac{1}{2}k_1 k_5 + k_2 \lambda_{max}(P) \\ \frac{1}{2}k_1 k_5 + k_2 \lambda_{max}(P) & -\frac{1}{\varepsilon}\lambda_{min}(Q) + 2k_3 \lambda_{max}(P) \end{bmatrix}, \quad (22)$$

$$Q = -\left(\bar{A}^T P + P \bar{A}\right), \quad Q = Q^T > 0. \quad (23)$$

If ε is sufficiently small, Φ is negative definite. Therefore, (e, η) converges to zero as time t goes to infinity. Finally, we can conclude that $\lim_{t \to \infty} e_1 = \lim_{t \to \infty}(x_{d1} - x_1) = \lim_{t \to \infty}(y_d - y) = 0$. This means that the output of actual piezoelectric actuator system approaches to the output of the desired system. ∎

IV. NUMERICAL SIMULATION

In this section, numerical simulation results for a piezoelectric actuator system are presented to illustrate the performance of the proposed method. We choose the parameters of the piezo system (1) as follows:

$$\begin{aligned}
m &= 0.28kg, \quad b = 1302.28Nsm^{-1}, \\
k &= 53,452N/m, \quad d_e = 0.1027nm/V.
\end{aligned}$$

The parameters adjusting the shapes of hysteresis loop in (2) are chosen as

$$\alpha = 0.5136, \quad \beta = 0.124, \quad \gamma = -0.073.$$

A sinusoidal waveform with 30 V amplitude, 30 V biased voltage, and 2 Hz frequency is applied as the input voltage r to both (3) and (4) system. Using the parameters and input voltage, the hysteresis effect between the applied input voltage and the output displacement is shown in Fig. 1. This hysteresis affects the output of the piezoelectric system. The output of (3) and (4) with $u = 0$ is presented in Fig. 2. Our proposed method suppresses this hysteresis effect and forces the output of (4) to track the output of (3). Fig. 3 shows the tracking error $y_d - y$. The used parameters are $\alpha_1 = 1$, $\alpha_2 = 0.01$, $K = [4900, 140]$,

and $\varepsilon = 0.00005$. In Fig. 4, we compare the effectiveness of the proposed method with uncontrolled system. From these results, we can see that the proposed method suppresses the hysteresis and achieves good tracking performance.
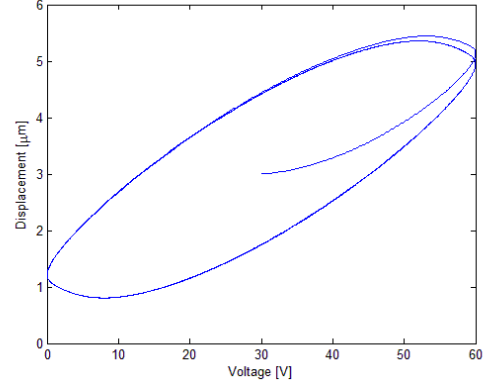

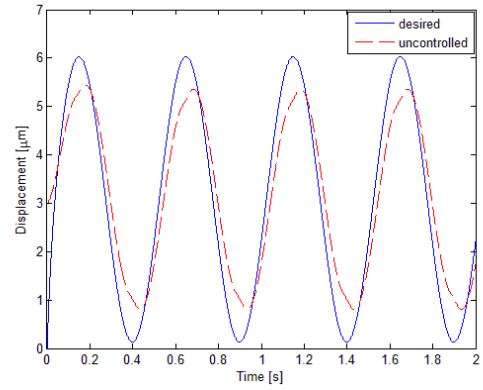Fig. 1. Hysteresis loop with a sinusoidal voltage input

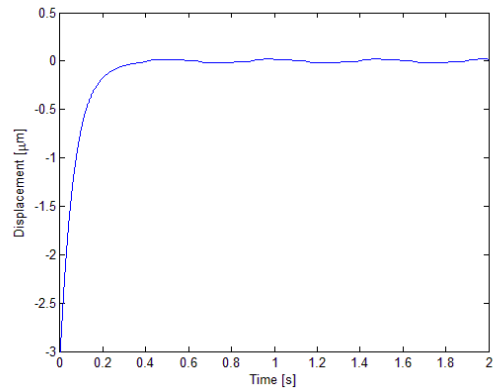Fig. 2. Outputs of the desired system (3) and actual system (4) with $u = 0$

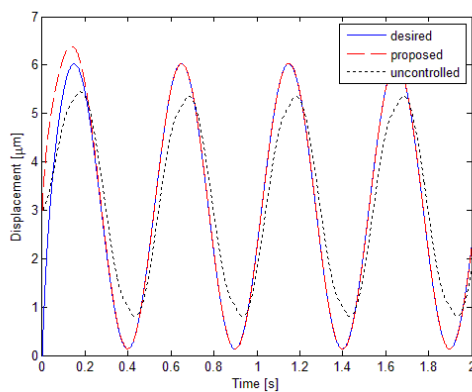Fig. 3. The tracking error $e_1 = y_d - y$

Fig. 4. Output trajectories of the desired, uncontrolled, and proposed controlled system

V. CONCLUSIONS

In this paper, an integral tracking control method for piezo-electric actuator system was proposed. The proposed method successfully achieved the tracking control without requiring any hysteresis observer or schemes to compensate the hysteresis effect. Using Tikhonov's theorem for the system converted into the standard singularly perturbed model, we guarantees that the tracking error can be reduced to arbitrarily small bound by choosing a design constant as sufficiently small value. Numerical simulation results were given to illustrate the effectiveness of our proposed method.

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