

# A Method for Modeling Multiple Antenna Channels

S. Rajabi, M. ArdebiliPoor, and M. Shahabadi

**Abstract**—In this paper we propose a method for modeling the correlation between the received signals by two or more antennas operating in a multipath environment. Considering the maximum excess delay in the channel being modeled, an elliptical region surrounding both transmitter and receiver antennas is produced. A number of scatterers are randomly distributed in this region and scatter the incoming waves. The amplitude and phase of incoming waves are computed and used to obtain statistical properties of the received signals. This model has the distinguishable advantage of being applicable for any configuration of antennas. Furthermore the common PDF (Probability Distribution Function) of received wave amplitudes for any pair of antennas can be calculated and used to produce statistical parameters of received signals.

**Keywords**—MIMO (Multiple Input Multiple Output), SIMO (Single Input Multiple Output), GBSBEM (Geometrically Based Single Bounce Elliptical Model).

## I. INTRODUCTION

THE modeling of the correlation at receivers of multiple antenna systems was mainly motivated by the need to quantify the effect of fading correlation on the performance of diversity reception systems. Another motivation is using multiple antennas at both ends of radio links, known as MIMO (Multiple Input Multiple Output) systems, for high data rate wireless systems. Recent studies by Foschini and Gans [1] and separately by Telatar [2] showed that by using multiple antennas at both ends of a radio link, its capacity increase linearly with minimum of the transmit and receive antennas numbers. They supposed independent paths between the transmit and receive antennas. Obviously, different paths in multiple antenna channels are correlated. There are two approaches to investigate this correlation. One is through field measurements or ray-tracing simulations. For field measurements in systems with receive diversity one can refer to [3] and in MIMO (Multiple Input Multiple Output) systems to [4-12]. Another approach is to construct scatterer models that include the effect of physical parameters of channels [13]. All of these models have been exploited for receive diversity

in SIMO (Single Input Multiple Output) channels. All of these models except GBSBEM (Geometrically Based Single Bounce Elliptical Model) are used for outdoor channels, that the receiver antennas are surrounded by objects in the environment and the transmitter antenna is deployed higher than the surrounding objects. GBSBEM is useful for channels that both the transmitter and receiver are mounted at the same height as surrounding objects known as indoor channels.

The correlation in MIMO systems has been first investigated in [14]. This paper has used an abstract model that dates back to the early 1970s [15, 16] and have been introduced in [13] as Lee's model. This model is useful for outdoor channels and its analytical relation for receiver correlation simplifies to famous Bessel function similar to [15] and [16]. [17, 18] use other abstract models, useful for outdoor channels and [9], [19-22] use models that are useful for indoor channels. [23-26] do not emphasize on special environments for their analytical results. [27] considers both channel types. There is a considerable difference between the results of these abstract model and the measurements about correlation coefficient at receiver side.

In this paper we introduce a method for modeling the correlation between the received signals by two or more receiver antennas operating in a multipath environment to decrease the mentioned difference. This model uses an abstract model known as GBSBEM introduced previously. We utilize ray-tracing simulations to compute the correlation coefficient. This model has two special characteristics that distinguish it from other models. First, this model can be used for any configurations of receiver and transmitter antenna arrays. Second it can produce any statistical properties of the received waveforms.

In section II the indoor multipath channel environment for computation of the correlation is constructed. The ray-tracing simulation method is presented in section III. In section IV the results of the proposed method is demonstrated and its dependence to different parameters is examined.

## II. CONSTRUCTING MULTIPATH CHANNEL MODEL

In this part we construct the multipath channel that we want to use for computing the correlation coefficient. We focus on the indoor channels. One of the most used forms for the indoor channels is the GBSBEM first introduced by Liberti and Rappaport in [28]. In this model the transmitter and receiver antennas and scatterers are distributed in surface of an ellipse. Diameters of the ellipse are determined by the maximum excess delay in the channel by the following relations,

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$$\begin{aligned} a &= \frac{c\tau_m}{2} \\ b &= \frac{1}{2}\sqrt{c^2\tau_m^2 - D^2} \end{aligned} \quad (1)$$

Where  $c$  is the speed of light  $\tau_m$  is the maximum excess delay and  $D$  is the distance between the centers of transmitter and receiver antenna arrays. The transmitter and receiver are at the foci of the ellipse. If we have an array of antenna at transmitter or receiver or both, we place the center of the array at the related foci.

Now considering a certain number of scatterers ( $N_s$ ), distribute them uniformly at the surface of the ellipse (figure 1).

Uniformly placing scatterers in an ellipse can be accomplished by first uniformly placing the scatterers in a unit circle in the center of Cartesian coordinates and then scaling each  $x$  and  $y$  coordinate by  $a$  and  $b$ , respectively [13].

If we want to distribute some scatterers at the surface of a circle it can be proved that the radius and angle of the place of scatterers must have the following pdfs [29],

$$\begin{aligned} f_r(r) &= \frac{2r}{R^2} & 0 \leq r \leq R \\ f_\varphi(\varphi) &= \frac{1}{2\pi} & 0 \leq \varphi \leq 2\pi \end{aligned} \quad (2)$$

Considering these pdfs, we place  $N_s$  scatterers at the surface of the ellipse. To this point we have constructed the medium to be modeled.

### III. COMPUTING THE CORRELATION COEFFICIENT

The second step in this model is propagating a stream of data in this medium and computing the statistical properties of the received data.

For this, we suppose that the transmitter has sent a stream of baseband symbols. We suppose that there is no line of sight between transmitter and receiver. Each symbol propagates in the medium and interacts with the scatterers and reflects and/or diffracts from them. In the other word the symbols of stream are scattered from the scatterers in the channel.

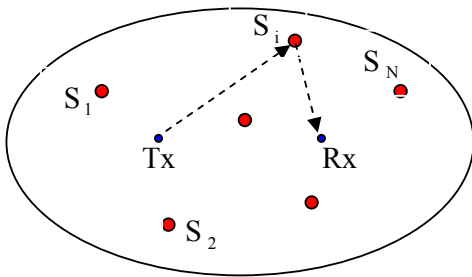


Fig. 1 Shape of multipath environment

We suppose that the scatterers are propagating the scattered stream towards the receivers. It means that each scatterer acts like a transmitter. For the sake of simplicity we suppose that

the mean power of the transmitted symbols from each scatterer is the same.

According to the central limit theorem the complex baseband signals are Gaussian distributed. Or the amplitude of them is Rayleigh and phase of them uniformly distributed in  $[0, 2\pi)$ . In addition for each scatterer we compute the distance  $d_{si}$  from the  $i$ th received antenna. And multiple the complex Gaussian signals by the  $e^{-j\beta d_{si}}$ . Then compute the sum of these  $N_s$  numbers in each antenna. Now we have one complex number for each transmitted symbol in each receiver antenna. These complex numbers are correlated because of the  $e^{-j\beta d_{si}}$  phrases.

By sending a stream of data with length  $N$  and repeating this process  $N_i$  times, we compute the joint PDF of the received signals amplitude. By these common pdfs we can compute all statistical parameters in the receivers, like mean SNR, bit error rate in each antenna or a combination of antenna, correlation coefficient between each pair of antenna and etc.

### IV. SIMULATION RESULTS

In this section we exhibit results of this model in a system with one transmitter and two receivers. We produce the correlation coefficient between two received antennas in Fig. 2. We suppose that the power of noise is constant. And compute it as the signal to noise ratio throughout the program. We supposed that signal to noise ratio is 0dB at transmitter.  $\tau_m$  is equal to 1μsec. The frequency of transmitter is 2GHz. Number of scatterers is 20 and distance between transmitter and receiver is  $D=10$ m.

In addition to this we test the effect of two import parameters on the correlation of received waveforms. These parameters are number of scatterers ( $N_s$ ) and signal to noise ratio at transmitter.

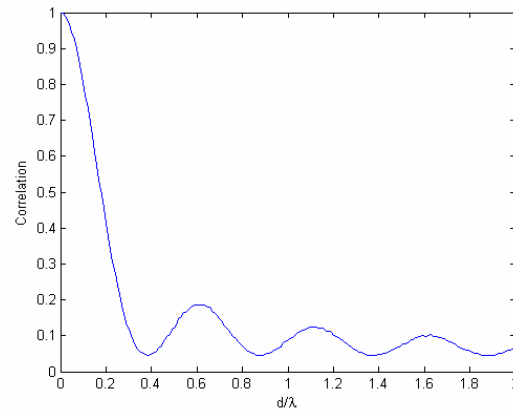


Fig. 2 Correlation vs. distance between antenna at receiver (SNR=20 dB)

We increased the number of scatterers from 20 to 40, 80 and 120. Fig. 3 shows the effect of this parameter on the correlation coefficient.

Fig. 2 shows that as the number of the scatterers increases the correlation coefficient decreases. It is compatible with the results of other papers.

The second parameter of interest is the SNR of the transmitted signals. We choose two SNR, 20dB and 100dB. The correlation coefficient of these two SNR is showed in Fig. 4. In this figure sigma is the mean SNR.

As you see the correlation coefficient is independent of the mean SNR of the received signals.

## V. CONCLUSION

One of the most important parameters in multiple antenna systems is correlation coefficient between different links of channel. The important function for computing this parameter is the joint pdf of the received signals. In this paper we proposed a way for computing this parameter at multiple antenna channels. Computing this function, we can calculate any statistical parameters at receivers of multiple antenna systems.

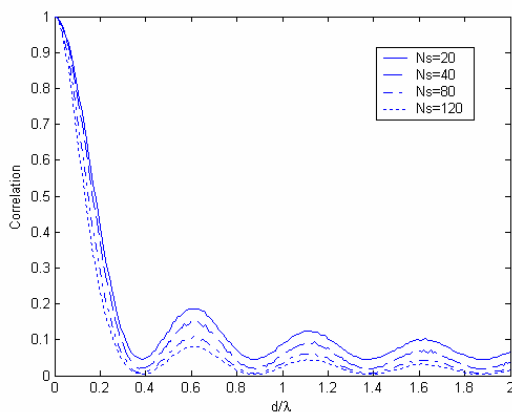


Fig. 3 The correlation coefficient vs normalized distance between antennas for different number of scatterers

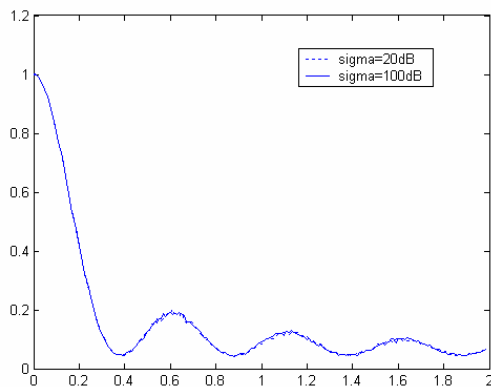


Fig. 4 The correlation coefficient vs normalized distance between antennas for different SNRs

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