

# Numerical Simulation of Convection Heat Transfer in a Lid-Driven Cavity with an Open Side

M.Jafari\*, M.Farhadi, K.Sedighi, E.Fattahi

**Abstract**—In this manuscript, the LBM is applied for simulating of Mixed Convection in a Lid-Driven cavity with an open side. The cavity horizontal walls are insulated while the west Lid-driven wall is maintained at a uniform temperature higher than the ambient. Prandtl number (Pr) is fixed to 0.71 (air) while Reynolds number (Re), Richardson number (Ri) and aspect ratio (A) of the cavity are changed in the range of 50-150, of 0.1-10 and of 1-4, respectively. The numerical code is validated for the standard square cavity, and then the results of an open ended cavity are presented.

Result shows by increasing of aspect ratio, the average Nusselt number (Nu) on lid-driven wall decreases and with same Reynolds number (Re) by increasing of aspect ratio (A), Richardson number plays more important role in heat transfer rate.

**Keywords**—Lattice Boltzmann Method- Open ended cavity- Mixed convection- Lid-driven cavity

## I. INTRODUCTION

THE Lattice Boltzmann Method in last decade has gained much attention for its ability to simulate fluid flows and heat transfer problems. Some of main reasons for using Lattice Boltzmann Method in researches are: easy implementation, ability to simulate multiphase flow and complex geometries, ability to use parallel computations and finally no need to use time-consuming Poisson equation for pressure. There are many researches using Lattice Boltzmann Method (LBM). [1-6]

Lid-driven flow in a square cavity and its convection heat transfer is a classical problem in Mechanical engineering. Hou et al. [7] presented a detailed analysis of this problem to demonstrate the abilities of the LBM method. They did expanded comparison of Lattice Boltzmann Method with Navier-stokes solution results of Ghia et al. [8]. Nemati et al simulated mixed convection in a Lid-driven cavity using Nanofluids. They analyzed the effect of existence of Nano-particle on heat transfer rate [9].

Open cavities are 2-D cavity that has an open side. These kinds of cavities have special physics in open side because of going out of flow from this side. Some research papers have been published on experimental studies of buoyant flow in open cavities [10-12].

Javam et al [13], analysis stability of stratified natural convection flow in open cavities. Mohamad[14,15] et al

presented natural convection in an open ended cavity and slots they analyzed the effect of aspect ratio of cavity on heat transfer rate. They presented a good procedure for simulating open boundaries in Lattice Boltzmann Method (LBM) [15].

## II. LATTICE BOLTZMANN METHOD

The thermal LB model utilizes two distribution functions,  $f$  and  $g$ , for the flow and temperature fields, respectively. In this approach the fluid domain is discretized in uniform Cartesian cells. Each cell holds a fixed number of distribution functions. For this work the D2Q9 model has been used. This model is shown in Fig.1 and values of  $w_0 = 4/9$  for  $|c_0| = 0$ ,  $w_{1-4} = 1/9$  for  $|c_{1-4}| = 1$  and  $w_{5-9} = 1/36$  for  $|c_{5-9}| = \sqrt{2}$  are assigned.

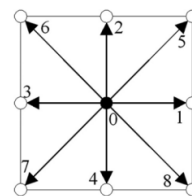


Fig. 1 D2Q9 model

The  $f$  and  $g$  are calculated by solving the Lattice Boltzmann equation (LBE). By using BGK model, the general form of lattice Boltzmann equation with an added force term can be written as:

For the flow field:

$$f_i(x + c_i \Delta t, t + \Delta t) = f_i(x, t) + \frac{\Delta t}{\tau_v} [f_i^{eq}(x, t) - f_i(x, t)] + \Delta t c_i F_k \quad (1)$$

For the temperature field:

$$g_i(x + c_i \Delta t, t + \Delta t) = g_i(x, t) + \frac{\Delta t}{\tau_D} [g_i^{eq}(x, t) - g_i(x, t)] \quad (2)$$

Where  $\Delta t$  denotes lattice time step,  $c_k$  is the discrete lattice velocity in direction  $k$ ,  $F_k$  is the external force in direction of lattice velocity,  $\tau_v$  and  $\tau_D$  denotes the lattice relaxation time for the flow and temperature fields. The kinetic viscosity  $\nu$  and the thermal diffusivity  $\alpha$ , are defined in terms of their respective relaxation times, i.e.  $\nu = c_s^2(\tau_v - 1/2)$  and  $\alpha = c_s^2(\tau_D - 1/2)$ , respectively. The local equilibrium distribution for flow and temperature fields is as follows respectively.

$$f_i^{eq} = w_i \rho \left[ 1 + \frac{c_i \cdot u}{c_s^2} + \frac{1}{2} \frac{(c_i \cdot u)^2}{c_s^4} - \frac{1}{2} \frac{u^2}{c_s^2} \right] \quad (3)$$

$$g_i^{eq} = w_i T \left[ 1 + \frac{c_i \cdot u}{c_s^2} \right] \quad (4)$$

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Where  $w_k$  is a weighting factor,  $\rho$  is the lattice fluid density. To model buoyancy force, the force term in the Eq. (1) need to be assumed as below in needed direction:

$$F = 3w_y g_y \beta T \quad (5)$$

In lattice Boltzmann Method Eqs. (1) and (2) are solved in two important step that are called collision and streaming step. *Collision step is as follows for flow field and temperature field respectively:*

$$f_\alpha(x, t + \Delta t) = -\frac{1}{\tau_m} [f_\alpha(x, t) - f_\alpha^{eq}(x, t)] - f_\alpha(x, t) \quad (6)$$

$$g_\alpha(x, t + \Delta t) = -\frac{1}{\tau_t} [g_\alpha(x, t) - g_\alpha^{eq}(x, t)] - g_\alpha(x, t) \quad (7)$$

Streaming step can be written as follows:

$$f_\alpha(x + e_\alpha \Delta t, t + \Delta t) = f_\alpha(x, t + \Delta t) \quad (8)$$

$$g_\alpha(x + e_\alpha \Delta t, t + \Delta t) = g_\alpha(x, t + \Delta t) \quad (9)$$

Where  $f_\alpha$  and  $g_\alpha$  denotes the post-collision distribution function.

Macroscopic variable can be calculated in terms of these variables, with the following formula.

Flow density:

$$\rho = \sum_i f_i$$

Momentum:

$$\rho u_j = \sum_i f_i c_{ij} \quad (10)$$

Temperature:

$$T = \sum_i g_i$$

### III. PROBLEM DESCRIPTION

In this work the problem of mixed convection heat transfer in an open ended cavity is simulated. The cavity has two horizontal insulated walls, a lid-driven west wall and an open east side. The velocity of lid-driven wall is assumed to be fixed 0.01 and its temperature is equal to unity that is more than the ambient zero temperature. One point must be noticed that these velocity and temperature are dimensionless in Lattice scale.

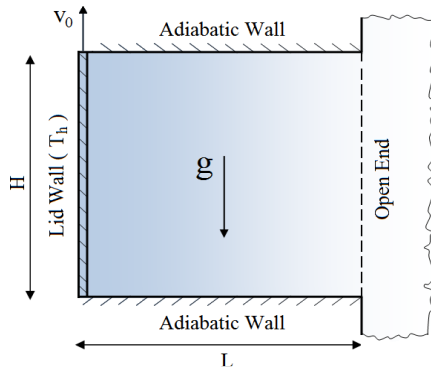


Fig. 2 Streamline and temperature contours

The motion of lid-driven wall and buoyant force causes to have mixed convection in this open ended cavity. For simulating of mixed convection in this geometry, the Reynolds (Re) and Richardson (Ri) numbers are applied in the range of 50-150 and 0.1-10 respectively. The effect of these parameters on the fluid flow and heat transfer rate are investigated when aspect ratio (A) of cavity change from 1 to 4.

To validate the numerical simulation, the results for mixed convection flow in a standard cavity with two vertical adiabatic walls and two horizontal walls with constant temperature that top wall is hot and bottom wall is cold are compared with those presented by Moallemi[16]. Streamline and temperature contours are presented in Fig.2 this comparison reveals good agreements between the present result and those published by moallemi. For more consistency, the local Nusselt number (NU) is too. (Fig.3and4.).

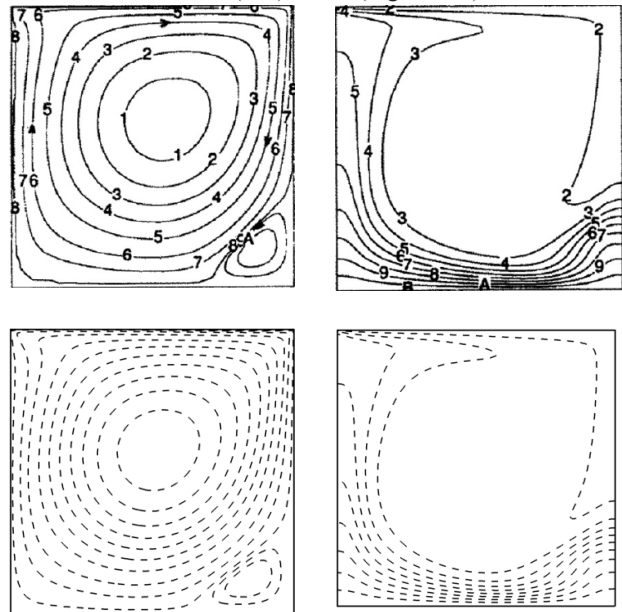


Fig. 3 Streamline and Temperature contours for Re=100 Ri=0.4 Pr=1. Top: Moallemi[16] bottom: Present study

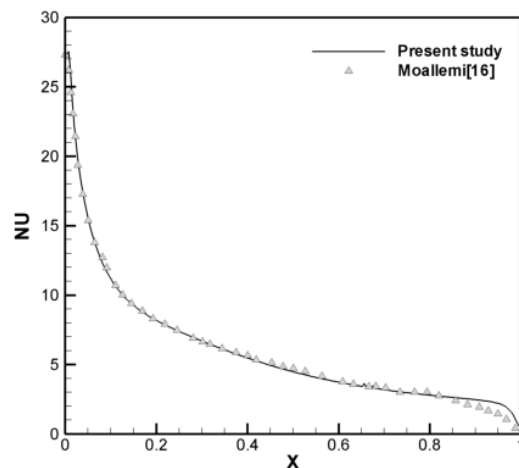


Fig. 4 Nusselt local on Lid-driven wall

To simulate the open ended boundary applied procedure is same as those presented by Mohamad [15]. In applied numerical code to model various aspect ratio of cavity 100 lattice nodes in y direction is used and number of nodes in x direction is of 100 to 400 for aspect ratio of 1 to 4 respectively.

Nusselt number is defined on the Lid-driven wall of the cavity.

$$NU = -\frac{\partial T}{\partial x} \quad (11)$$

Average Nusselt number is calculated by integrating eq. (11) along the length of the Lid-driven wall and dividing by number of lattices along the height.

$$NU_{ave} = \frac{1}{L} \int_0^L NU \, dx \quad (12)$$

Where, L is the length of lid driven wall.

#### IV. RESULTS AND DISCUSSION

The problem of mixed convection heat transfer in Lid-driven cavity with an open side was solved for different Reynolds numbers, Richardson numbers and aspect ratios when Prandtl number (Pr) is fixed to 0.71 and Lid-driven velocity is fixed to 0.01. Streamline and Temperature contours are plotted and average Nusselt number on Lid-driven wall is investigated in various conditions.

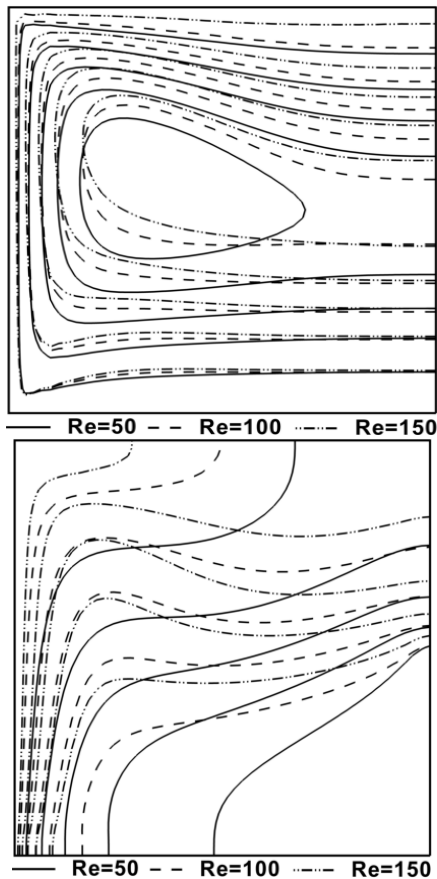


Fig. 5 Streamline and Temperature contour for A=1, Ri=1, Pr=0.71

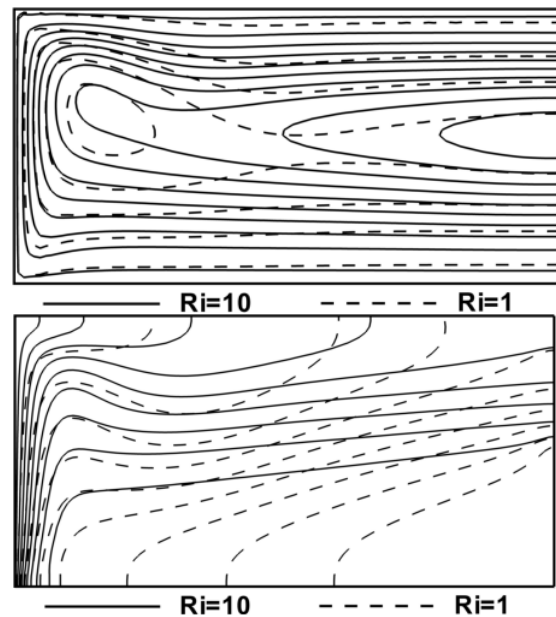


Fig. 6 Streamline and Temperature contour for A=2, Re=100, Pr=0.71

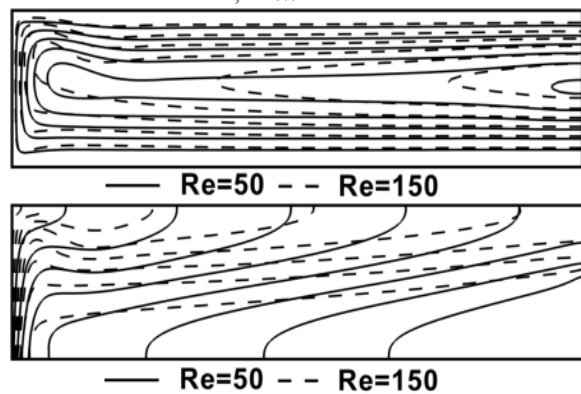


Fig. 7 Streamline and Temperature contour for A=4, Re=10, Pr=0.71

Results reveals by increasing of the aspect ratio of cavity the average Nusselt number decreases (Fig.8).

Fig.9 shows increasing of Reynolds number in constant Richardson number cause to the average Nusselt number increase and this matter can be seen too that in larger aspect ratio, increasing of Reynolds number play more important role in improvement of heat transfer rate.

Fig.10 shows that the effect of Richardson number on heat transfer rate at different aspect ratio is same as the effect of Reynolds number on heat transfer rate.

The effect of increasing aspect ratio on Nusselt average increasing in low Richardson numbers is more than larger aspect ratios (Fig.10).

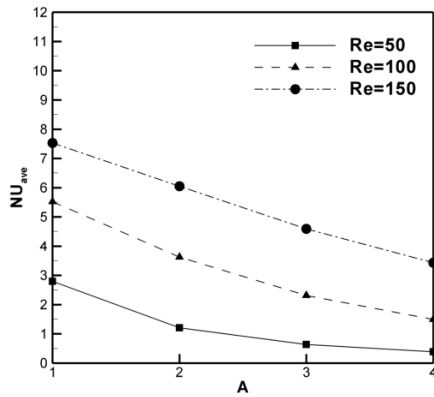


Fig. 8 Nusselt average for Ri=1, Pr=0.71

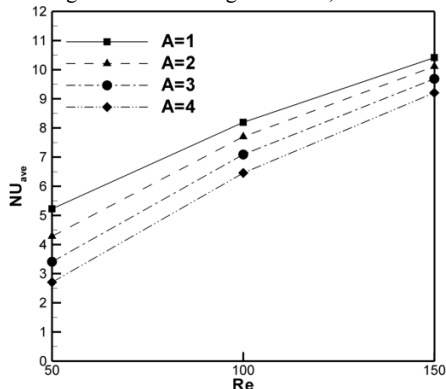


Fig. 9 Nusselt average for Ri=10, Pr=0.71

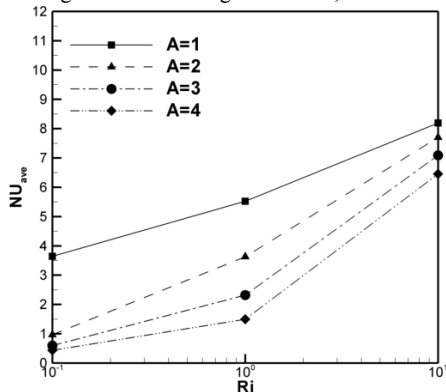


Fig. 10 Nusselt average for Re=100, Pr=0.71

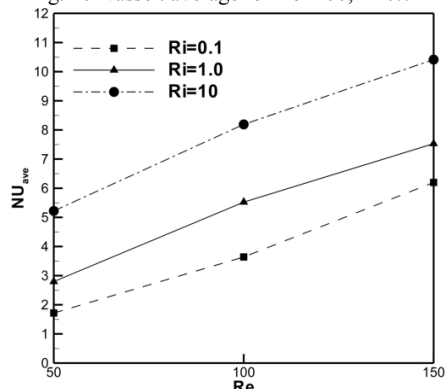


Fig. 11 Nusselt average for A=1, Pr=0.71

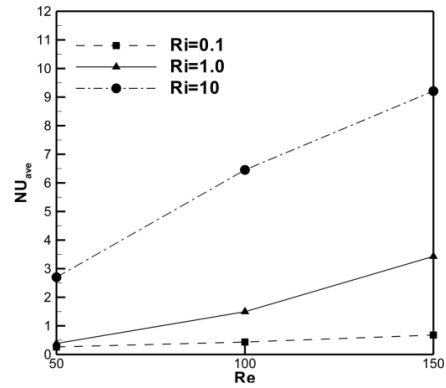


Fig. 12 Nusselt average for A=4, Pr=0.71

At low Richardson numbers the effect of Reynolds number on heat transfer rate decreases by increasing of aspect ratio, as can be seen in Fig.11 for A=1 the rate of changing in average Nusselt is same in Ri=0.1 with other Richardson numbers but Fig.12 reveals in A=4 increasing of Re has different effect on average Nusselt when Ri vary of 0.1 to 10.

#### NOMENCLATURE

$c_i$	Discrete lattice velocity in direction $i$
$c_s$	Speed of sound in Lattice scale
$C_p$	specific heat at constant pressure (kJ/kg K)
$F$	External force in direction of lattice velocity
$f_i^{eq}$	Equilibrium distribution of flow field.
$\bar{f}_\alpha$	Post-Collision Equilibrium distribution of flow field.
$g$	Acceleration due to gravity (m/s <sup>2</sup> )
$g_i^{eq}$	Equilibrium distribution of Temperature field
$\bar{g}_\alpha$	Post-Collision Equilibrium distribution of flow field.
$i$	Discrete lattice direction
$k$	Thermal conductivity (W/m K)
$L$	Length of Lid-driven wall
$NU$	Nusselt number
$NU_{ave}$	Average Nusselt number
$Pr$	Prandtl number ( $\nu / \alpha$ )
$Re$	Reynolds Number

$Ri$	Richardson Number
$v$	Vertical velocity of Lid-driven wall (m/s)
$w_i$	Weighting factor in direction $i$

**Greek symbols**

$\beta$	Thermal expansion coefficient ( $1 / K$ )
$\Delta t$	Lattice time step
$\rho$	Density (kg/m <sup>3</sup> )
$\tau_v, \tau_D$	Relaxation time for fluid and Temperature field

**REFERENCES**

- [1] Y.H. Qian, D. d'Humieres, P. Lallemand, Lattice BGK models for Navier-Stokes equation, *Europhys. Lett.* 17 (6) (1992) 479-484.
- [2] S. Chen, G.D. Doolen, Lattice Boltzmann method for fluid flows, *Annu. Rev. Fluid Mech.* 30 (1998) 329-364.
- [3] D. Yu, R. Mei, L.S. Luo, W. Shyy, Viscous flow computations with the method of lattice Boltzmann equation, *Progr. Aerospace Sci.* 39 (2003) 329-367.
- [4] S. Succi, *The Lattice Boltzmann Equation for Fluid Dynamics and Beyond*, Clarendon Press, Oxford, London, 2001.
- [5] E. Fattahi, M. Farhadi, K. Sedighi, Lattice Boltzmann simulation of natural convection heat transfer in eccentric annulus, *International Journal of Thermal Sciences*, 49 (2010) 2353-2362.
- [6] M.A. Delavar, M. Farhadi, K. Sedighi, Numerical simulation of direct methanol fuel cells using lattice Boltzmann method, *international journal of hydrogen energy*, 35 (2010) 9306-9317.
- [7] He X, Luo LS, A priori derivation of the lattice Boltzmann equation. *Phys Rev E*, 55(1997),R6333.
- [8] U. Ghia, K.N. Ghia, C.T. Shin, High-resolutions for incompressible flow using Navier-Stokes equations and a multigrid method, *Comput. Phys*, 48 (1982) 387-411.
- [9] H. Nemat, M. Farhadi, K. Sedighi, E. Fattahi, A.A.R. Darzi, Lattice Boltzmann simulation of nanofluid in lid-driven cavity, *International Communications in Heat and Mass Transfer*, 37 (2010) 1528-1534.
- [10] Y.L. Chan, C.L. Tien, Laminar natural convection in shallow open cavities, *J. Heat Transfer* 108 (1986) 305-309.
- [11] E. Bilgen, Passive solar massive wall systems with fins attached on the heated wall and without glazing, *J. Sol. Energ. Eng.* 122 (2000) 30-34.
- [12] S.S. Cha, K.J. Choi, An interferometric investigation of open cavity natural convection heat transfer, *Exp. Heat Transfer* 2 (1989) 27-40.
- [13] A. Javam, S.W. Armfield, Stability and transition of stratified natural convection flow in open cavities, *J. Fluid Mech.* 44 (2001) 285-303.
- [14] A.A. Mohamad, Natural convection in open cavities and slots, *International journal of Heat Transfer* 27 (1995) 705-716.
- [15] A.A. Mohamad, M. El-Ganaoui, R. Bennacer, Lattice Boltzmann simulation of natural convection in an open ended cavity, *International Journal of Thermal Sciences* 48 (2009) 1870-1875.
- [16] M.K. Moallemi, K.S. Jang, Prandtl number effects on laminar mixed convection heat transfer in a lid-driven cavity, *Int. J. Heat Mass Transfer* 35 (1992) 1881-1892.