

# Effect of Cyclotron Resonance Frequencies in Particles Due to AC and DC Electromagnetic Fields

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**Abstract**—A fundamental model consisting of charged particles moving in free space exposed to alternating and direct current (AC-DC) electromagnetic fields is analyzed. Effects of charged particles initial position and initial velocity to cyclotron resonance frequency are observed. Strong effects are observed revealing that effects of electric and magnetic fields on a charged particle in free space varies with the initial conditions. This indicates the frequency where maximum displacement occur can be changed. At this frequency the amplitude of oscillation of the particle displacement becomes unbounded.

**Keywords**—Cyclotron resonance, electromagnetic fields, particle displacement

## I. INTRODUCTION

A cyclotron is a particle accelerator which accelerates charged particles using high frequency alternating voltage. Specific biological interaction with extremely low frequency (ELF) magnetic fields and ion cyclotron resonance (ICR) frequencies which is derived from ionic charge to mass ratio is evident from experimental results reported in studies on a wide range of biological systems. In 2005, Liboff [1] observed low frequency magnetic fields interact with biological systems when the fields are adjusted to cyclotron resonance frequencies of ions.

In Section II we explain some existing Magnetic Resonance Models and in Section III we illustrate the particles behavior using Maxwell's equations and Lorenze model. In Section IV, we show numerical results and then discuss the behavior of the particle with different initial positions and velocities and in Section V we conclude the paper.

## II. MAGNETIC RESONANCE MODELS

Both Blackman et al. and Liboff [2], [3] observed the dependency between the frequency and the magnitude of the static geomagnetic fields and amplitude windows in 1985. These observations led to the formulation of the Ion Cyclotron Resonance (ICR) theory [4]. One basic model which explains the occurrence of resonance frequencies on an ion to the strength of the geomagnetic field is the ion cyclotron resonance (ICR) model as proposed by [4]. The solution can be written as

$$\omega = \left(\frac{q}{m}\right) B_{DC},$$

where  $\omega$  is the circle frequency of the circular motion around the direction of the static field,  $m$  is the particle mass,  $q$  is the charge of the particle and  $B_{DC}$  is static magnetic

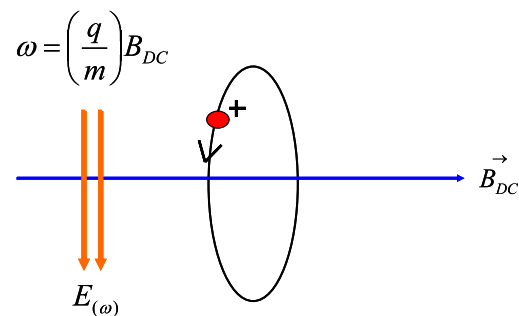


Fig. 1. The force exerted on a charged particle is perpendicular to both the magnetic field and velocity of the particle. This is the centripetal force. This results in a spiral motion around the magnetic field at the cyclotron frequency. In other words, cyclotron motion of an ion around the direction of a static (earth) magnetic field,  $B_{DC}$  with mass  $m$  and charge  $q$ , is accelerated by an electric field  $E_{\omega}$  tuned to the cyclotron frequency [5]. Cyclotron resonance means an external AC electric field is tuned to the cyclotron frequency of a specific ion moving in a static magnetic field  $B_{DC}$ . Under this condition, the ion is accelerated by the time-varying electric field.

field, as in Fig. 1. When an electric field is applied with the same frequency, the ion can be accelerated. Associated with cyclotron resonance, the ion is accelerated by the time-varying electric field  $E$ , which is supposed to facilitate different biological processes such as ion transport through channel proteins.

Both classical and quantum mechanical models have been proposed [6], [7]. Both resonance frequencies and amplitude windows can be explained by this model. Adair [8] criticised the quantum mechanical model presented by Lednev. Engström [9] extended these models and also predicts the occurrence of superharmonic resonances. He showed that Adair criticisms were not valid and that Lednev's model is physically feasible. Experimental data by Blackman [10] and Eberhardt [11] support the model. Biological effects were observed for magnetic field strength  $> 10 \mu\text{T}$ . In a newer model, Lednev [12] considered the polarisation of proton spins of water by weak time varying magnetic fields and how this influence biological systems. In this model, biological effects can be expected for all frequencies of the time varying magnetic field, but the magnitude of the biological response depends on the ratio of the amplitude of the magnetic field to its frequency.

For the reason that the basic signal to noise issues could not be resolved in the ICR model, a new model was proposed

by Lednev, the Ion Parametric Resonance model (IPR) [6]. When combination of static and alternating magnetic fields are applied to a biosystem biological effects can be observed at the cyclotron frequency  $\omega_c$  and is given by  $\omega_c = (q/m) B_{stat}$ , where  $B_{stat}$  is a combination of static and alternating magnetic fields. In the next section we present Lorentz model for the motion of a charged particle with electromagnetic fields and effect of cyclotron resonance frequency.

### III. LORENTZ MODEL ANALYSIS

The main aim in this section is to observe nonlinearity of the motion of charged particles. In early studies [13] a non-linear Lorentz model describing interaction between charged particles and combined AC-DC magnetic fields is investigated for different combinations of field strengths, frequencies and relative angle between AC and DC magnetic fields. This paper has investigated the problem of effect of charged particles initial position and velocity to cyclotron resonance frequency.

The equation of motion of a particle characterized by a charge  $q$  and mass  $m$  moving with velocity  $\nu$  at a time  $t$  in the existing of electric field  $\mathbf{E}$  and a magnetic field  $\mathbf{B}$  is given by

$$m \frac{d\nu}{dt} + m\nu\nu = q(\mathbf{E} + \nu \times \mathbf{B}), \quad (1)$$

where  $\nu$  is a coefficient of drag and right side of this equation is the Lorentz force, therefore Lorentz model. Here we did not consider restoration force. The drag coefficient shows the drag or resistance of an particle in a moving fluid such as air or water. This depends on the particle size, shape as well as the fluid viscosity. We choose  $E$  and  $B$  arbitrarily to be at  $90^\circ$  with

$$\begin{aligned} \mathbf{E} &= (E_x, 0, 0) \\ \mathbf{B} &= (0, B_{oy}, B_{oz}), \end{aligned} \quad (2)$$

where  $B$  is a constant and  $E$  is directed along x-axis and varies sinusoidally with time as  $E = E_o \sin \omega t$ . When drag coefficient is zero ( $\nu = 0$ ),  $x = \{qE/m\omega^2(\omega_0^2 - 1)\} \sin \omega t$  and  $\omega_0 = q\sqrt{(B_{oy}^2 + B_{oz}^2)}/m\omega$  where  $\omega_0$  is a dimensionless number. As in (2),  $B_{oy}$  and  $B_{oz}$  are horizontal and vertical elements of the DC magnetic field. Moreover, when  $\omega_0 = 1$  then  $\omega = \omega_c$  and amplitude oscillation of the particle displacement approaches unconstrained. Cyclotron resonance frequency is a unique function of charge to mass ratio ( $q/m$ ) and intensity of magnetic field as  $\omega_c = (q/m)B$ . Further this can be written by

$$\omega_c = \frac{q\sqrt{(B_{oy}^2 + B_{oz}^2)}}{m}.$$

Lorentz model is linear with static or DC fields such as earth's magnetic fields. However, for AC fields,  $\mathbf{E}$  and  $\mathbf{B}$  fields are coupled together via well known Maxwell's equations [13] and are given by  $\nabla \cdot \mathbf{E} = \rho$ ,  $\nabla \cdot \mathbf{B} = 0$ ,  $\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$ , and  $c^2 \nabla \times \mathbf{B} = -\partial \mathbf{E} / \partial t + \mathbf{J}$ , where  $\rho$  is the charge density,  $c$  is the velocity of light, and  $\mathbf{J}$  is the conductive current density. As in [13], we use  $B_{1z} = B_1 \cos kx \cos ky \cos \omega t$  where  $k = \omega/c\sqrt{2}$  and also by solving Maxwell's equations first and then

using Lorentz model equations we obtained by replacing  $kx$  by  $x$ ,  $ky$  by  $y$  and  $\omega t$  by  $t$  the dimensionless form as

$$\begin{aligned} x'' + \tilde{\nu}x' &= -(\omega_1/2) \sin y \cos x \sin t \\ &+ (\omega_1 \cos x \cos y \cos t + \omega_{0z})y' - \omega_{0y}z', \end{aligned} \quad (3)$$

$$\begin{aligned} y'' + \tilde{\nu}y' &= (\omega_1/2) \sin x \cos y \sin t \\ &- (\omega_1 \cos x \cos y \cos t + \omega_{0z})x', \end{aligned} \quad (4)$$

$$z'' + \tilde{\nu}z' = \omega_{0y}x', \quad (5)$$

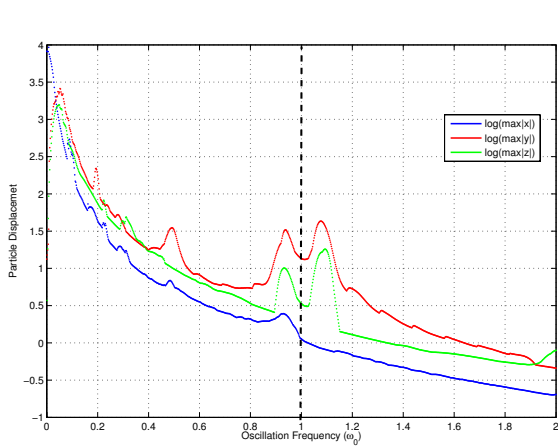
where  $\omega_1 = qB_1/m\omega$ ,  $\omega_{0y} = qB_{oy}/m\omega$ ,  $\omega_{0z} = qB_{oz}/m\omega$  and  $\tilde{\nu} = \nu/\omega$ , where  $\tilde{\nu}$  is dimensionless drag coefficient. From equation (2) we can get  $\omega_{0y} = \omega_0 \sin \theta$ ,  $\omega_{0z} = \omega_0 \cos \theta$  and also  $\nu = 0$ .

### IV. NUMERICAL RESULTS AND DISCUSSION

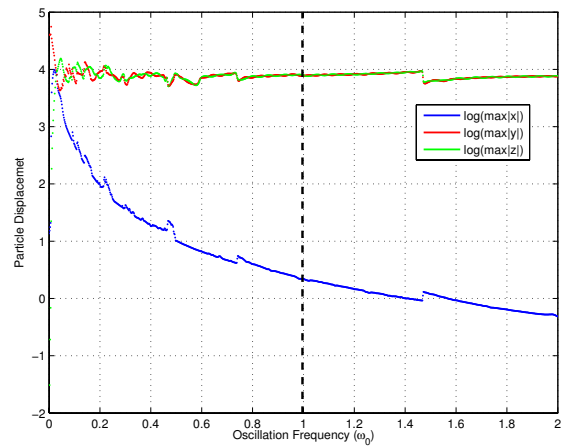
Amplitude of oscillation is maximum at the cyclotron resonance frequency [13]. According to theory, the cyclotron resonance occurs at  $\omega_0 = 1$ . Therefore, we observed displacement values when reaching cyclotron resonance frequency (when  $\omega_0 = 1$ ) and the frequency which caused the maximum amplitude of oscillation. In this analysis we changed initial position and the velocity of the particles and integrated equations (3), (4) and (5) numerically over 1000 time steps. The initial conditions are defined as below:

$x(0)$  : Initial position in  $x$  direction  
 $y(0)$  : Initial position in  $y$  direction  
 $z(0)$  : initial position in  $z$  direction  
 $x'(0)$  : initial velocity in  $x$  direction  
 $y'(0)$  : initial velocity in  $y$  direction  
 $z'(0)$  : initial velocity in  $z$  direction.

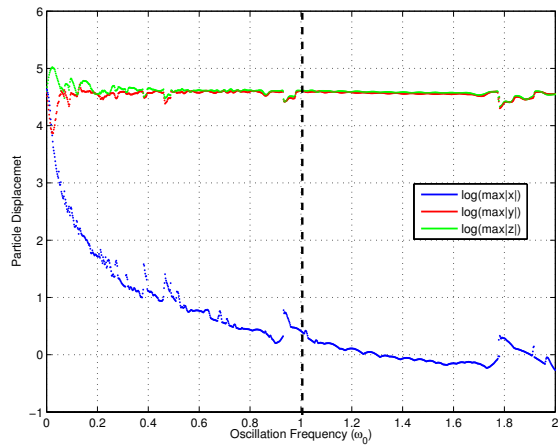
Here we considered only particles moving in free space, therefore the starting position and the velocity of charged particle can be changed. The following graphs describe the variation of displacement of the charged particle according to its initial conditions of the position and velocity. When  $x'(0) = 1$ , the cyclotron resonance occurs around  $\omega_0 = 1$  while other initial conditions are zero. Furthermore the particle displacements in all three directions are decreasing with increasing  $\omega_0$  amplitude. However, when  $y'(0) = 1$  or  $z'(0) = 1$  with other zero initial conditions the amplitude of oscillation in  $x$  direction is not a maximum at  $\omega_0 = 1$  where as in  $y$  and  $z$  direction the particle has the maximum amplitude of displacement with very small oscillation. Moreover, in this case the particle displacement in  $x$  direction decreases towards zero and in  $y$  and  $z$  directions move away from zero magnitude. When we consider  $x(0) = x'(0) = 1$  the cyclotron resonance occurs at  $\omega_0 = 1$ . In this case also displacement of the particle in all three directions decay with increasing  $\omega_0$ . Furthermore, when  $\omega_0 < 1.2$  the particle oscillates randomly. When  $y(0) = y'(0) = 1$  or  $z(0) = z'(0) = 1$  while  $x(0) = x'(0) = 0$  the figure illustrates that maximum amplitude of displacement in  $y$  and  $z$  directions around  $\omega_0 = 1$ . In this situation also the particles in  $y$  and  $z$  directions displace away from zero while in  $x$  direction it moves towards zero. When all initial positions are one or all initial conditions are one the particle behaviour is similar to previous case. Furthermore when considering  $x'(0) = 5$  and  $x'(0) = 15$  while other initial



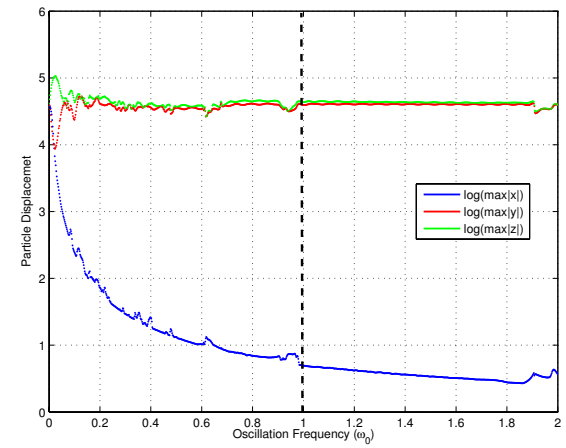
(a) when  $x'(0) = 1$



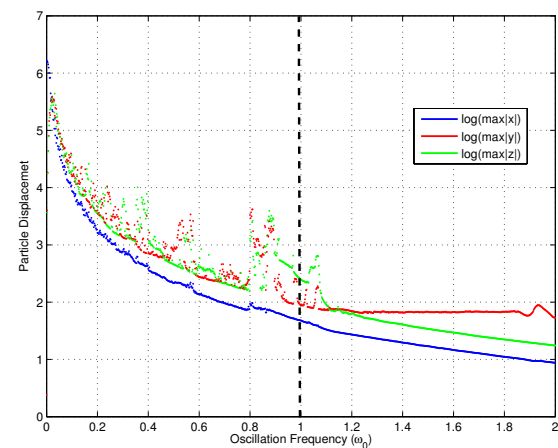
(b) when  $y'(0) = 1$



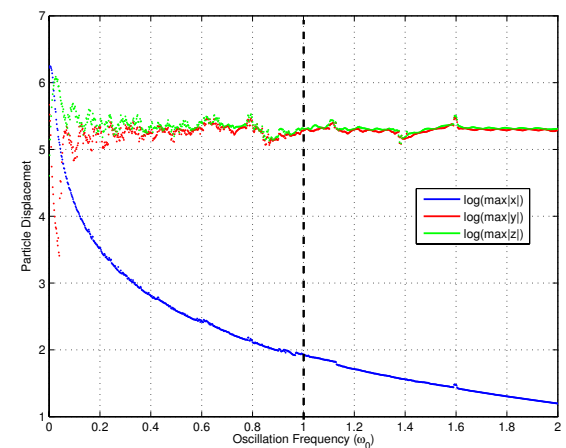
(c) when  $x'(0) = 1, y'(0) = 1, z'(0) = 1$



(d) when  $x(0) = 1, y(0) = 1, z(0) = 1, x'(0) = 1, y'(0) = 1, z'(0) = 1$



(e) when  $x'(0) = 5$



(f) when  $x'(0) = 5, y'(0) = 3, z'(0) = 1$

Fig. 2. Comparing effect different initial velocity and displacement of with cyclotron resonance verses particle displacement where zero drag ( $\nu = 0$ ),  $\omega_1 = 0.1$  and  $\theta = \pi/4$  for the non-linear model.

TABLE I  
CYCLOTRON RESONANCE VALUES

Initial Conditions	Cyclotron Resonance			Amplitude of Displacement (when $\omega_0 = 1$ )		
	$x$ direction	$y$ direction	$z$ direction	$x$ direction	$y$ direction	$z$ direction
$x'(0) = 1$	0.93	1.08	1.09	0.04	1.12	0.54
$y'(0) = 1$	0.75	1	1	0.35	3.82	3.82
$z'(0) = 1$	0.69	1	1	0.36	3.94	3.94
$x(0) = 1, x'(0) = 1$	0.92	1.09	1.09	0.71	1.13	0.73
$y(0) = 1, y'(0) = 1$	0.77	1	1	0.35	3.82	3.82
$z(0) = 1, z'(0) = 1$	0.68	1	1	0.32	3.82	3.82
$x(0) = y(0) = z(0) = 1$	0.95	1	1	0.35	4.53	4.53
All conditions = 1	0.95	1	1	0.75	4.5	4.5
$x(0) = 5$	0.8	0.88	0.88	1.65	1.94	2.41
$x'(0) = 5, y'(0) = 3, z'(0) = 1$	0.8	0.8	0.8	1.9	5.2	5.2
$x(0) = 15$	0.8	0.93	0.93	2.77	3.08	3.08
$x(0) = 15, y'(0) = 10, z'(0) = 5$	0.8	1	1	2.95	6.55	6.55

conditions are zero, the particle displacement in all 3 directions decreases towards zero whereas cyclotron resonance occurs between  $\omega_0 = 0.8$  and  $0.9$ . Moreover if we take  $x'(0) = 5$ ,  $y'(0) = 3$ ,  $z'(0) = 1$  or  $x'(0) = 15$ ,  $y'(0) = 10$ ,  $x(0) = 5$  when other initial positions are zero the movement of the particle in  $y$  and  $z$  directions are away from zero while  $x$  moves towards zero with increasing  $\omega_0$ . The magnitude of movement in  $y$  and  $z$  directions are higher than that of in  $x$  direction. When we compare the maximum amplitude of displacement at  $\omega_0 \simeq 1$ , we can see that as the value of the initial conditions increases, amplitude also increases as in Fig. 2 and Table I.

Several important facts can be drawn from above analysis. (i) When there is only an initial position or an initial velocity or both in  $x$  direction the particle displacement in all 3 directions decrease toward zero and cyclotron resonance occurs approximately at  $\omega_0 \simeq 1$ . That means having an initial position or velocity only in  $x$  direction (direction of electric field) gives the maximum amplitude of oscillation around  $\omega_0 \simeq 1$ . (ii) If one or many initial conditions exist independent of the initial position and the velocity in  $x$  direction the particles movement in  $y$  and  $z$  direction is away from zero and in  $x$  direction it decreases toward zero. In this situation the maximum amplitude of oscillation in  $x$  direction does not occur at  $\omega_0 = 1$  while in other directions it occurs at  $\omega_0 \simeq 1$  with very small oscillations. (iii) When the values of initial conditions increase the maximum amplitude of displacement also increases.

## V. CONCLUSION

It is realistic to formulate a model to explain the cyclotron resonance response of a charged particle moving in free space interacting with a combined AC-DC electromagnetic field. This paper has investigated the effect of charged particles interacting with combined AC-DC electromagnetic fields. Our main concern is on the effect of charged particles' initial position and speed on cyclotron resonance. Results indicate the frequency where maximum displacement occurs can be changed. Our results show that the particles' displacement in free space due to electric and magnetic fields is sensitive to initial conditions.

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