Theory of Nanowire Radial p-n-Junction

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Abstract—We have developed an analytic model for the radial pn-junction in a nanowire (NW) core-shell structure utilizing as a new building block in different semiconductor devices. The potential distribution through the p-n-junction is calculated and the analytical expressions are derived to compute the depletion region widths. We show that the widths of space charge layers, surrounding the core, are the functions of core radius, which is the manifestation of so called classical size effect. The relationship between the depletion layer width and the built-in potential in the asymptotes of infinitely large core radius transforms to square-root dependence specific for conventional planar p-n-junctions. The explicit equation is derived to compute the capacitance of radial p-n-junction. The current-voltage behavior is also carefully determined taking into account the "short base" effects.

Keyword—Snanowire, p-n- junction, barrier capacitance, high injection.

I. INTRODUCTION

THE semiconductor nanowires have the potential to impact L many different technologies either through the improved material parameters or by offering a new geometry not possible with bulk or thin film structures. Modern advances in nanotechnology allow to incorporate several material layers with the same or different type of conductivity into a single nanowire (nanorod). Coaxial core/shell nanowire (NW) structures with built-in radial p-n-junctions recently have been reported [1]. This new type of structures represents an important class of nanoscale building blocks with potential for exploring new device concepts, e.g., for photovoltaic applications [2]-[4] or field-effect transistors [5]. The array of NWs in which each wire has a p-n- junction in the radial direction may provide an interesting application in the third generation solar cells technology [6], [7]. The advantage of such solar cells is that the directions of light absorption and carrier collection can be orthogonal, which allows to provide efficient carrier separation in the radial direction for the optically thick NW arrays, even when the minority carrier diffusion lengths are shorter than the optical absorption length.

Therefore it is important issue to develop the theoretical model of the nanowire radial p-n- junctions, hence the good understanding of the device performance will guide designer to choose the best components for optimization. The model for

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S. Nersesyan is with the Russian-Armenian (Slavonic) University, 123 Hovsep Emin str., 0051, Yerevan, Armenia. the NW radial p-n-junction will be constructed by extending the analyses of the conventional planar p-n-junction geometry [8] to a cylindrical geometry, taking into account that a considerable portion of core and (or) shell can be covered by space charge and the core radius (shell thickness) can be less or comparable to the diffusion length of minority carriers. The analytical relationships for the depletion region widths versus applied voltage, the maximum electric field across the junction, the capacitance-voltage and the current-voltage characteristics are presented and analyzed as functions of material properties and radius of NW.

II. ELECTROSTATIC CHARACTERISTICS OF RADIAL P-N-JUNCTION

We have focused on NW p-n- coaxial homo-junction, consisting of a p-type inner "core" with acceptor concentration N_A and radius R_1 , capped by n- type outer "shell" with donor concentration N_D and thickness $(R_2 - R_1)$. The considered structure is illustrated in Fig. 1 (a).

Considering relatively thick NWs, with a diameter of several 100 nm, we neglect the quantum size effects. The p-njunction in NW is assumed to be abrupt and the depletion approximation is assumed to be valid. In Fig. 1 (b) the *r* axis shows the schematic division of the structure into four regions: the quasi-neutral part of the p-core $(r \le R_1 - w_p)$, the depletion part of the p-core (of width w_p), the depletion part of the p-core (of width w_p), the depletion part of the n-shell (of width w_n) and the quasi-neutral part of the n-shell ($r \le R_2 - R_1 - w_n$). If the temperature is sufficiently high and all impurities are ionized, the majority charge carriers in the core and shell quasi-neutral regions are: $p_p = N_A$ for $r \le R_1 - w_p$ and $n_n = N_D$ for $R_1 + w_n \le r \le R_2$.

A. Potential Distribution and Depletion Region Width

When p-n-juncion is formed along core/shell interface, a fraction of the electrons pass from the n region into the p region, while the holes, on the contrary, pass from the p region into the n region. The space-charge layers are created on both sides of the core/shell interface and potential energy barrier is established, (see Fig. 1 (b)). The electric field arises across the junction and the energy bands bent until the establishment of equilibrium state and the alignment of Fermi energy levels (see Fig. 2 (c)). The "built-in" potential energy barrier that forms along the p-n-junction and has the height defined by work function difference of the core and shell, can be written as usual:



Fig. 1 Cross section of core/shell NW (a), profile of potential energy across the junction (b), energy band diagram (c)

$$\Phi_0 = kT \ln \frac{n_n p_p}{n_i^2},\tag{1}$$

where kT is the thermal energy, n_i is the intrinsic carrier concentration of the semiconductor. This potential barrier opposes to electron diffusion from the n-type shell to the core and to hole diffusion from the core to the shell, such as the total electron current and the total hole current are both zero at thermal equilibrium.

Close to the junction the concentration of mobile charge sharply decreases and the space charge is mainly formed by ionized donors and acceptors. In complete-depletion approximation the space charge density is:

$$\rho = \begin{cases} -eN_A = -e \cdot p_p, & R_1 - w_p \le r \le R_1 \\ eN_D = e \cdot n_n, & R_1 \le r \le R_1 + w_n \end{cases}$$
(2)

where e is the electronic charge.

To determine the potential distribution across the junction, we should solve the Poisson equation in cylindrical coordinates.

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\Phi}{dr}\right) = \begin{cases} -\frac{e^2p_p}{\epsilon\epsilon_0}, & R_1 - w_p \le r \le R_1 \\ \frac{e^2n_n}{\epsilon\epsilon_0}, & R_1 \le r \le R_1 + w_n \end{cases}$$
(3)

where Φ is the potential energy of electrons, ϵ is the permittivity of the semiconductor, ϵ_0 is the electric constant. The potential reference is chosen at the border of depletion region in the p-type core. Thus the boundary conditions at $r = R_1 - w_p$ and $r = R_1 + w_n$ present the variation of potential energy across the junction and the requirement that the radial component of the electric field should be zero outside the space charge region:

$$\Phi\left(R_1 - w_p\right) = 0, \left. \frac{d\Phi}{dr} \right|_{r=R_1 - w_p} = 0 \tag{4}$$

$$\Phi(R_1 + w_n) = -\Phi_0, \left. \frac{d\Phi}{dr} \right|_{r=R_1 + w_n} = 0.$$
 (5)

It is easy to see that the solution for (3)-(5) has the form

$$\begin{split} \Phi(r) &= -\frac{e^2 p_p}{4\epsilon\epsilon_0} \bigg[r^2 - (R_1 - w_p)^2 - 2(R_1 - w_p)^2 ln \frac{r}{R_1 - w_p} \bigg], \\ \text{for } R_1 - w_p &\leq r \leq R_1, \end{split}$$

and

$$\Phi(\mathbf{r}) = -\Phi_{o} + \frac{e^{2}n_{n}}{4\epsilon\epsilon_{0}} \Big[\mathbf{r}^{2} - (\mathbf{R}_{1} + \mathbf{w}_{n})^{2} - 2(\mathbf{R}_{1} + \mathbf{w}_{n})^{2} \ln \frac{\mathbf{r}}{\mathbf{R}_{1} + \mathbf{w}_{n}} \Big],$$
(6)

for $R_1 \le r \le R_1 + w_n$.

In our model we ignore the charge on the core/shell interface states, so the electric field continuity at $r=R_1\, gives$

$$n_n (R_1^2 - (R_1 + w_n)^2) = p_p ((R_1 - w_p)^2 - R_1^2)$$
(7)

which represents the charge neutrality of the junction. Using (6), (7) and demanding the continuity of the potential at the homo-junction interface $r = R_1$ we get the following equation for w_p

$$\left(\frac{n_n + p_p}{p_p} - \left(1 - \frac{w_p}{R_1}\right)^2\right) \cdot \ln\left(\frac{n_n + p_p}{n_n} - \frac{p_p}{n_n}\left(1 - \frac{w_p}{R_1}\right)^2\right) + \left(1 - \frac{w_p}{R_1}\right)^2 \ln\left(1 - \frac{w_p}{R_1}\right)^2 = \frac{4\varepsilon\varepsilon_0}{e^2 p_p R_1^2} \Phi_o .$$

$$(8)$$

Thus for the given n_n , p_p , Φ_o and R_1 the depletion layer widths in the core (w_p) and in the shell (w_n) can be calculated from (7), (8).

It is worth to note that in the radial p-n-junction the width of space charge layer is a function of core radius, which is the manifestation of the classical size effect. This means that for the NWs with the same doping level but different radius the depletion layer has different width. For sufficiently large NW radius, when $R_1 \gg w_p$ we compute from (8) the standard relationship for depletion width, which various with Φ_o by the square root law as in conventional p-n-junctions: $w_p =$

$$\frac{2\epsilon\epsilon_0}{e^2} \frac{n_n}{p_p} \frac{\Phi_0}{p_p+n_r}$$

As it follows from (7) the ratio w_p/w_n depends not only from the doping concentrations but also on NW radius:

$$\frac{w_{p}}{w_{n}} = \frac{n_{n}}{p_{p}} \frac{2 + \frac{w_{n}}{R_{1}}}{2 - \frac{w_{n}}{R_{1}}}$$
(9)

It is evident from (9) that at equal core and shell doping levels $(n_n = p_p)$ the depletion width in the core is always larger than in the shell. This is a specific feature of radial p-njunctions, whereas in planar geometry at equal donor and acceptor concentrations the junction is symmetric $(w_p = w_n)$. In the NW p-n-junctions the widths of depletion layers are limited by the core radius: $w_p < R_1$ and by the shell thickness: $w_n < R_2 - R_1$.

To find w_p , w_n under the external bias (V), we should replace the built-in energy barrier Φ_0 with ($\Phi_0 - eV$) in the right hand side of (7), and (8), where V > 0 means p-core is positively biased with respect to the n-shell. Let us define the critical value of reverse bias (V_c), when the nanowire core is totally depleted of holes. By substituting $w_p = R_1$ in (8) we obtain:

$$\frac{e|V_c|}{\Phi_0} = \frac{e^2(n_n + p_p)R_1^2}{4\epsilon\epsilon_0 \Phi_0} \ln \frac{n_n + p_p}{n_n} - 1 \quad . \tag{10}$$

However, even in the absence of external bias for a given core radius R_1 there is a certain ratio of doping concentrations n_n and p_p and, hence, a certain built-in potential barrier Φ_{o_c} at which the NW core is totally covered by the space charge. From (8) we get

$$\Phi_{0_{c}} = \frac{e^{2} (n_{n} + p_{p}) R_{1}^{2}}{4\epsilon\epsilon_{0}} ln \frac{n_{n} + p_{p}}{n_{n}}.$$
 (11)

Fig. 2 presents the depletion width in the NW core at different core radiuses as a function of applied voltage. It is seen that for a given value of applied voltage, the width of depletion region is smaller for larger radiuses R_1 and for



Fig. 2 The depletion region width in p-region versus the applied voltage at NWs with different core diameter. Core-shell NW structure (solid lines), planar p-n-junction (dash line)

sufficiently thick NW it tends to the square root dependence on applied voltages, which is depicted in Fig. 2 by dashed line.

For sufficiently long NW the junction electric field has only radial component and its distribution across the space charge region is different from that in the conventional p-n-junction. In general, it may be expressed by the sum of linear and hyperbolic terms as:

$$\mathcal{E}(\mathbf{r}) = \begin{cases} \frac{\mathrm{ep}_{\mathrm{p}}}{2\varepsilon\varepsilon_{0}} \left(\mathbf{r} - \frac{(R_{1} - w_{p})^{2}}{\mathbf{r}}\right) & R_{1} - w_{p} \leq \mathbf{r} \leq R_{1} \\ \frac{\mathrm{en}_{\mathrm{n}}}{2\varepsilon\varepsilon_{0}} \left(\frac{(R_{1} + w_{n})^{2}}{\mathbf{r}} - \mathbf{r}\right) & R_{1} \leq \mathbf{r} \leq R_{1} + w_{n} \end{cases}$$
(12)

The maximum of electric field falls at core/shell interface $(r = R_1)$ and is given as

$$\mathcal{E}_{\mathrm{m}} = \frac{\mathrm{en}_{\mathrm{n}} \, \mathrm{w}_{\mathrm{n}}}{2 \varepsilon \varepsilon_{0}} \left(1 + \frac{\mathrm{w}_{\mathrm{n}}}{2 \mathrm{R}_{1}} \right) = \frac{\mathrm{ep}_{\mathrm{p}} \, \mathrm{w}_{\mathrm{p}}}{2 \varepsilon \varepsilon_{0}} \left(1 - \frac{\mathrm{w}_{\mathrm{p}}}{2 \mathrm{R}_{1}} \right). \tag{13}$$

Thus the maximum value of the electric field depends not only on the doping concentrations and applied voltage but also on NW radius.

B. Symmetric Radial p-n-Junction

To form the symmetric p-n-junction, such that $w_p = w_n$, at a given radius R_1 , the proper ratio of doping concentrations $y = \frac{N_A}{N_D} = \frac{p_p}{n_n}$ should be chosen. It is evident from (7) that it is possible to have the symmetric radial p-n junction if y > 1, (i.e. the core should be more heavily doped than the shell).

From (7) and (8) we get the equation for y:

$$\frac{1}{y}\left(\frac{3y-1}{y+1}\right)^2 \ln\left(\frac{3y-1}{y+1}\right) + \left(\frac{3-y}{y+1}\right)^2 \ln\left(\frac{3-y}{y+1}\right) = A \cdot \left(\ln\left(\frac{p_p}{n_i}\right)^2 - \ln(y)\right), \quad (14)$$



Fig. 3 The ratio of doping concentrations $\binom{p_p}{n_n}$ computed from (14) for the symmetric radial p-n-junction versus core radius

where $A = \frac{2\varepsilon\varepsilon_0}{e^2 R_1^2 p_p} kT$.

From (14) it follows that for asymptotic case $(R_1 \rightarrow \infty)$ y = 1 as it should be in the case of conventional planar p-njunction.

The dependence of y on the core radius at different shell doping concentrations is illustrated in Fig. 3. It is seen that the smaller is the NW core radius the higher has to be the doping concentration in the core to form a symmetric p-n-junction. In the radial p-n-junction the depletion width in the core increases with decreasing the core radius. The variable y approaches to its possible maximum value when the core radius is so small, that NW core becomes fully depleted.

III. CAPACITANCE OF RADIAL P-N-JUNCTION

The p-n- junction barrier capacitance dominates when the applied voltage is reverse biased or the forward bias is much smaller than the built-in potential. For the given applied voltage the total charge in the depletion layer is easy to calculate if w_p and w_n are defined from (8) and (9).

The depletion charge in p-region is defined by the following relationship

$$Q = e p_p \pi R_1^2 L \left(1 - \left(1 - w_p / R_1 \right)^2 \right).$$
(15)

Thus for the junction capacitance we get

$$C = \left| \frac{dQ}{dv} \right| = 2ep_p \pi R_1 L \left(1 - w_p / R_1 \right) \frac{dw_p}{dv} = 4\varepsilon \varepsilon_0 \pi L \cdot ln \left| \left(1 + \frac{n_n}{p_p} \frac{w_p}{R_1} \left(2 - \frac{w_p}{R_1} \right) \right) \right/ \left(1 - \frac{w_p}{R_1} \right)^2 \right|^{-1}.$$
 (16)



Fig. 4 The dependence of $(1/C^2)$ on the applied voltage for the NWs with different core diameter

We verify that the expression for specific capacitance of radial p-n-junction at $R_1 \rightarrow \infty$, coincides with that for planar p-n-junction. When $w_p \ll R_1$ we calculate from (16):

$$\frac{c}{2\pi L} \approx \frac{\varepsilon \varepsilon_0}{w_p + w_n} \Big\{ 1 - \frac{w_p}{2R_1} \frac{1 - p_p / n_n - 2(p_p / n_n)^2}{1 + p_p / n_n} \Big\}$$
(17)

where $w_n \approx w_p \cdot p_p / n_n$.

The relationship (17) makes evident that the specific capacitance of radial p-n-junction at doping concentrations $p_p < n_n$ is smaller than that of corresponding planar junction, and vice versa at $p_p > n_n$. Close to the critical value of reverse bias, $w_p \approx R_1$, as following from (16), the barrier capacitance of the radial p-n-junction tends to zero in analogy with cylindrical capacitor, when a decrease of the inner plate radius leads to the logarithmical decrease of capacitance $C = 2\epsilon\epsilon_0\pi L/\ln (R_2/R_1)$.

It is seen from (16) that the capacitance of nanowire radial p-n-junction per unit length is determined by the fundamental constant ε_0 (*F*/*m*). Thus for the NW with $L = 10 \,\mu\text{m}$ length the junction capacitance is order of 1÷10 fF.

The Fig. 4 presents the dependence of $1/C^2$ on applied reverse bias for the NWs with different diameter. It is seen that this dependence has the nonlinear form and for extremely large diameters becomes linear as in the case of planar junctions.

IV. CURRENT-VOLTAGE CHARACTERISTICS

In calculations of current-voltage characteristics of radial pn-junction, as a specific case, we assume that the core doping level is much higher than that of the shell, so we consider $p_p \gg n_n$. Most experimentally realized and studied NWs have diameter of the order of 100 nm, and so for many material systems the thickness of quasi-neutral region in the shell $(d = R_2 - (R_1 + w_n))$ is much smaller than the minority carriers diffusion length L_p . Thus the NW radial p-n- junction will operate in so-called *short-base diode* conditions [7]. In such junctions minority carriers injected from the core to the shell will reach the metallic contact, made on external surface of the shell, without significant recombination in the volume. Consequently, the conditions of the current passage through the shell/metal interface plays important role in the current characteristics. In our analyzes we will follow the standard current transport model of planar short-base diode [7]. We will consider an ideal (ohmic) metallic contact, wrapping the shell, such that the built-in potential barrier, possibly formed at the metal and shell contact, in both accumulation or depletion modes is not higher than several kT. Under these conditions we can write [7]:

$$p(R_2) - p_n = \Delta p(R_2) = 0, \ n(R_2) = n_n, \tag{18}$$

where $p(R_2)$ and $n(R_2)$ are, respectively, the concentrations of holes and electrons at the surface of the shell, p_n is the minority carrier concentration in the volume of the *n*-shell.

If $d \ll L_p$ then (18) means that the holes injected from the p-n-junction do not accumulated at the semiconductor-metal interface and freely pass to the metal, recombining there with electrons. Therefore the hole recombination process in n-region can be ignored. So the net electron and hole currents remain constant throughout the n-region:

$$I_p = 2\pi r Le D_p \left(\frac{e}{kT} E(r) \cdot p(r) - \frac{dp(r)}{dr}\right) = const \qquad (19)$$

$$I_n = 2\pi r Le D_n \left(\frac{e}{kT} E(r) \cdot n(r) + \frac{dn(r)}{dr} \right) = const \qquad (20)$$

where D_n and D_p are the diffusion constants of electrons and holes respectively, E(r) is the radial component of electric field.

In this case, when the carrier recombination does not take place in the shell, the current density passing through the metal-semiconductor contact is $\frac{R_2}{R_1}$ times less than the one crossing the core/shell interface. From other side, it is known that for the same value of applied voltage and for not very large built-in potential barrier, the density of the current passing through the semiconductor/metal contact is much higher than that through the p-n-junction. In the radial p-n-junction, as we mentioned, the density of the current through the metallic contact is only $\frac{R_2}{R_1}$ times less than through the p-n-junction, thus the main voltage drop occurs along the p-n-junction:

$$\Delta p(R_1 + w_n) = p_n \left(e^{\frac{eV_1}{kT}} - 1 \right)$$
(21)

The minority carriers injected into the n-shell will induce the redistribution of majority carriers, so that they will quickly compensate the charge of minority carriers resulting in an increase of electron concentration on the same value: $\Delta n(r) - n_n = p(r) - p_n = \Delta p(r)$. Depending on injection level the concentration of non-equilibrium charge carriers at the depletion layer edge in the n-region can be comparable to n_n or even higher, i.e. $\Delta n(R_1 + w_n) = n_n + \Delta p(R_1 + w_n)$ (22) therefore the concentration of injected electrons at the end of depletion layer in the p-core should be:

$$\Delta n \left(R_1 - w_p \right) = n_p \left| \left(1 + \frac{\Delta p (R_1 + w_n)}{n_n} \right) e^{\frac{eV_1}{kT}} - 1 \right|$$
(23)

It should be noted that by writing the boundary condition for the injected holes in the form of (21) we have assumed that $\Delta n (R_1 - w_p) \ll p_n$, i.e. in the presence of external voltage the height of junction potential barrier still remains larger than kT: $\Phi_o - eV_1 \gg kT$. In such conditions, when $p_p \gg n_n$, the electron current (I_n) through the p-njunction is very small compared with the hole current (I_p) . As the bulk recombination is assumed to be very small, we can consider $I_n \approx 0$, then it is easy to find the radial component of electric field in the n-region:

$$E(r) \approx -\frac{kT}{e} \frac{1}{n} \frac{dn}{dr}.$$
 (24)

By substituting (24) into (19) and by taking into account the quasi-neutrality of n-region we can express the net hole current as:

$$I_p = -2\pi r Le D_p \left(2 - \frac{n_n}{n_n + \Delta p(r)}\right) \frac{dp(r)}{dr}.$$
 (25)

Then multiplying (25) by r^{-1} and by integrating it between $(R_1 + w_n)$ and R_2 , for $I_p = const$ we get:

$$I_{p} = \frac{2\pi e D_{p} p_{n} L}{\ln(R_{2}/R_{1} + w_{n})} \Big\{ \frac{2\left(e^{\frac{eV_{1}}{kT}} - 1\right) - }{-\frac{n_{n}}{p_{n}} \ln\left|1 + \frac{p_{n}}{n_{n}}\left(e^{\frac{eV_{1}}{kT}} - 1\right)\right| \Big\}.$$
 (26)

Using the boundary conditions (18) and (22) we can find the potential drop in quasi-neutral part of the shell:

$$V_2 = \int_{R_1 + w_n}^{R_1 + w_n + d} E \, dr = \frac{kT}{e} \ln \left[1 + \frac{p_n}{n_n} \left(e^{\frac{eV_1}{kT}} - 1 \right) \right]. \tag{27}$$

Thus for the given applied voltage $V = V_1 + V_2$, by using the relationship (27) we can define the voltage drops on p-n junction and on its base (shell).

At first we consider the case of small applied voltages corresponding to the weak injection regime of holes, when $\Delta p(R_1 + w_n) \ll n_n$. As it follows from (27) the voltage drop in the shell is very small: $eV_2 \ll kT$, and $V_1 \approx V$ so for the junction current we have:

$$I_p = \frac{2\pi e D_p \, p_n L}{\ln[R_2/R_1 + w_n]} \left(e^{\frac{eV}{kT}} - 1 \right). \tag{28}$$

It is seen from the last relationship, that at small forward bias the current increases as $e^{\frac{e_V}{kT}}$. In this case the concentration of non-equilibrium charge carriers decreases in n-region very

slowly: logarithmically, and not linearly as in conventional planar junctions,

$$\Delta p(r) = \Delta p(R_1 + w_n) \frac{\ln \frac{R_2}{r}}{\ln \frac{R_2}{R_1 + w_n}}.$$
 (29)

Therefore, as it follows from (28) the magnitude of junction net current is defined by the value of $2\pi(R_1 + w_n)\frac{d\Delta p}{dr}$ at the edge of depletion layer, which is:

$$\frac{d\Delta p(R_1+w_n)}{dr} = -\frac{\Delta p(R_1+w_n)}{(R_1+w_n)ln(\frac{R_2}{R_1+w_n})}.$$
(30)

The depletion width w_n increases at high reverse biases and therefore the width of quasi-neutral part of the shell decreases, then, due to the increase of the hole concentration gradient the junction current, according to (28), continues to slowly increase instead to become saturated, which is a specific feature of devices with planar geometry and thick base.

The situation with current transport is quite different when high forwarding bias is applied and the minority carrier injection is enhanced, such that, $p(R_1 + w_n) \gg n_n$, or $eV_1 \gg kT ln \frac{n_n}{n_i}$. Then from (27) it follows that the voltage drops on the shell volume and on the junction are:

$$\begin{cases} V_2 = V_1 - \frac{kT}{e} ln \frac{n_n}{p_n} = \frac{V}{2} - \frac{kT}{2e} ln \frac{n_n}{p_n} = \frac{V}{2} - \frac{kT}{e} ln \frac{n_n}{n_i} \\ V_1 = \frac{V}{2} - \frac{kT}{2e} ln \frac{n_n}{p_n} = \frac{V}{2} + \frac{kT}{e} ln \frac{n_n}{n_i}. \end{cases}$$
(31)

Substituting V_1 into the (26) for the current we can write

$$I_p \approx \frac{2\pi e D_p n_i L}{\ln[(R_2/R_1 + w_n]]} exp\left(\frac{eV}{2kT}\right).$$
(32)

Thus, if an applied voltage is in the range: $\frac{2kT}{e} ln \frac{n_n}{n_i} < V < \frac{2kT}{e} ln \frac{p_p}{n_i}$, the main part of the voltage drop occurs on the quasi-neutral region of n-shell and the junction current starts to increase less sharply, proportional to $exp\left(\frac{eV}{2kT}\right)$.

Such behavior of the current-voltage characteristics is illustrated in Fig. 5. The calculations are done by solving (26) and (27) and the results are shown in linear and logarithmic scales. It is seen, that the slope of $ln(I/I_o)$, where $I_o = 2\pi e D_p p_n L$, decreases almost two times with an increase of applied voltage, which means that the forward current of the NW radial p-n-junction starts to increase with significantly reduced slope at high applied voltages. Such specific feature of current-voltage characteristics was observed experimentally for gallium arsenide NW radial p-n-junctions [4].



Fig. 5 The current versus applied forward bias in linear scale (right axes) and in logarithmic scale (left axes)

It should be noted that even at high injection level the distribution of non-equilibrium charge carriers stays logarithmical along the NW's radius. Indeed, for $\Delta p(r) \gg n_n$ it follows from (25)

$$I_p \approx -2eD_p \left(2\pi rL\frac{dp}{dr}\right) = const$$
, (33)

i.e. the hole ohmic current is exactly equal to the diffusion current, and their distribution is described by the relationship (29).

V. CONCLUSION

The theoretical analysis is performed for the radial p-njunction. The final sizes of the NW core and shell are accounted in the model. The analytical expressions are derived to calculate the widths of depletion layers, the barrier capacitance and the volt-ampere characteristics. The developed model is a good base for further studies of built-in junctions in the core/shell NWs, particularly its photovoltaic applications.

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