

# Acoustic Source Localization Based on the Extended Kalman Filter for an Underwater Vehicle with a Pair of Hydrophones

ByungHoon Kang, Jeawook Shin, Ju-man Song, Hyun-Taek Choi, PooGyeon Park

**Abstract**—In this study, we consider a special situation that only a pair of hydrophone on a moving underwater vehicle is available to localize a fixed acoustic source of far distance. The trigonometry can be used in this situation by using two different DOA of different locations. Notice that the distance between the two locations should be measured. Therefore, we assume that the vehicle is sailing straightly and the moving distance for each unit time is measured continuously. However, the accuracy of the localization using the trigonometry is highly dependent to the accuracy of DOAs and measured moving distances. Therefore, we proposed another method based on the extended Kalman filter that gives more robust and accurate localization result.

**Keywords**—Localization, acoustic, underwater, extended Kalman filter.

## I. INTRODUCTION

**L**OCALIZATION and navigation for an underwater vehicle are important tasks and have been studied widely in several decades [1]- [6]. Acoustic signal has a special position in the tasks because it spreads much farther than other signals such as sonar signal and vision signal. If the acoustic source is localized, the acoustic source can be a target or a landmark for an underwater vehicle and help the vehicle find its position and a direction to move. An array of hydrophones is mostly used to localize the acoustic source [9]- [10]. A pair of hydrophones can give direction of arrival (DOA) based on time difference of arrival (TDOA) between the two hydrophones [11]- [18]. TDOA is generally obtained by the method of general cross-correlation (GCC) which compute cross-correlation of the signals received by each hydrophones and find the time where a maximum peak arise. By the trigonometry, the location in two-dimensional space can be specified at least three hydrophone. More accurate localization is also possible by the method of beamforming using several hydrophones. Generally speaking, the more hydrophones are used, the more accurate localization is possible.

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However, the use of sensors are mostly restricted in underwater vehicle because the power and space of the vehicle are limited and the cost should be also considered. In this study, we consider a special situation that only a pair of hydrophone on a moving underwater vehicle is available to localize a fixed acoustic source of far distance. The trigonometry can be used in this situation by using two different DOA of different locations. Notice that the distance between the two locations should be measured. Therefore, we assume that the vehicle is sailing straightly and the moving distance for each unit time is measured continuously. However, the accuracy of the localization using the trigonometry is highly dependent to the accuracy of DOAs and measured moving distances. Therefore, we propose another method based on the extended Kalman filter that gives more robust and accurate localization result.

The paper is organized as follows: in Section II, the underwater acoustic source localization problem of consideration is formally stated and deterministic localization using the trigonometry is presented; in Section III, the methods of determining the location of an acoustic source using the extended Kalman filter is formulated; in Section IV, the simulation results are presented and two methods are compared; finally concluding remarks are given in Section V.

## II. PROBLEM STATEMENT

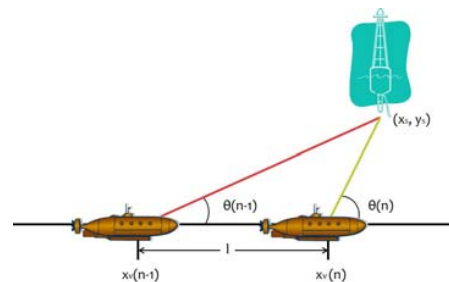


Fig. 1. Acoustic source localization for an underwater vehicle

The considered acoustic source localization problem is illustrated on Fig.1. The underwater vehicle is moving straightly and the direction of moving is set as x-axis. The location of the vehicle at current time  $n$  is denoted by  $(x_v(n), 0)$  and a fixed acoustic source is denoted by  $(x_s, y_s)$ . The DOA on current time  $n$  and the moving distance between current and prior measurement time is obtained with measurement noises and denoted by  $\theta(n)$  and  $l(n)$ , respectively.

From the definition of tangent function, three triangular equations are made by tangent function to  $(\frac{\pi}{2} - \theta(n-1))$  and  $(\frac{\pi}{2} - \theta(n))$ :

$$\tan\left(\frac{\pi}{2} - \theta(n-1)\right) = \frac{x_s}{y_s}, \quad (1)$$

$$\tan\left(\frac{\pi}{2} - \theta(n)\right) = \frac{x_s - l}{y_s}, \quad (2)$$

$$x(n) - x(n-1) = l \quad (3)$$

The location of the acoustic source can be given by using Equation (1)-(3) as

$$x_s = \frac{l \tan\left(\frac{\pi}{2} - \theta(n-1)\right)}{\tan\left(\frac{\pi}{2} - \theta(n-1)\right) - \tan\left(\frac{\pi}{2} - \theta(n)\right)}, \quad (4)$$

$$y_s = \frac{l}{\tan\left(\frac{\pi}{2} - \theta(n-1)\right) - \tan\left(\frac{\pi}{2} - \theta(n)\right)} \quad (5)$$

However, this calculated equations can be useful solution when the noise of observer is exact or near zero. A little bit more noise can make large error. Thus different methods should be applied to estimate accurate position information in noisy environment, and one of those methods is the extended Kalman filter.

### III. LOCALIZING AN ACOUSTIC SOURCE USING EXTENDED KALMAN FILTER

Consider a discrete nonlinear system given by

$$\mathbf{x}(n+1) = \mathbf{F}(n, \mathbf{x}(n)), \quad (6)$$

$$\mathbf{y}(n) = \mathbf{C}(n, \mathbf{x}(n)) + \mathbf{v}_1(n) \quad (7)$$

where  $n$  is the time step,  $\mathbf{x}(n)$  is the state vector,  $\mathbf{y}(n)$  is the measurement vector,  $\mathbf{v}_1(n)$  is the measurement noise. We assume that  $\mathbf{v}_1(n)$  have a zero-mean with covariance  $Q(n)$ .

To estimate the location of the acoustic source, we use a model of the extended Kalman filter as follows. The state vector is  $\mathbf{x}(n) = [x_1 \ x_2 \ x_3]^T$ , where  $x_1$  is the distance between acoustic source ( $x_s$ ) and vehicle ( $x_v(n)$ ) on the x-axis,  $x_2$  is the position of acoustic source on y-axis ( $y_s$ ), and  $x_3$  is the distance between previous position and current position of robot on x-axis ( $l(n)$ ). The measurement vector is  $\mathbf{y}(n) = [y_1 \ y_2 \ y_3]^T$ , where  $y_1$  and  $y_2$  are the angle between acoustic source and vehicle at previous time step and current time step, respectively  $(\theta(n-1), \theta(n))$ , and  $y_3 = l(n)$ . The linear system and nonlinear measurement functions are used as follows:

$$\mathbf{F}(n, \mathbf{x}(n)) = \begin{bmatrix} x_1 - x_3 \\ x_2 \\ x_3 \end{bmatrix}, \quad (8)$$

$$\mathbf{C}(n, \mathbf{x}(n)) = \begin{bmatrix} \tan^{-1}\left(\frac{x_2}{x_1}\right) \\ \tan^{-1}\left(\frac{x_2}{x_1 - x_3}\right) \\ x_3 \end{bmatrix}. \quad (9)$$

The state and the measurement functions are linearized according to

$$\mathbf{F}(n+1, n) = \left. \frac{\partial \mathbf{F}(n, \mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}(n|y_n)}, \quad (10)$$

$$\mathbf{C}(n) = \left. \frac{\partial \mathbf{C}(n, \mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}(n|y_{n-1})}. \quad (11)$$

Then, the system model can be approximated as

$$\mathbf{x}(n+1) = \mathbf{F}(n+1, n)\mathbf{x}(n), \quad (12)$$

$$\mathbf{y}(n) = \mathbf{C}(n)\mathbf{x}(n) + \mathbf{v}_1(n) \quad (13)$$

where

$$\mathbf{F}(n+1, n) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (14)$$

and

$$\mathbf{C}(n) = \begin{bmatrix} \frac{-\hat{x}_2(n|y_{n-1})}{\hat{x}_1^2(n|y_{n-1}) + \hat{x}_2^2(n|y_{n-1})} \\ \frac{-\hat{x}_2(n|y_{n-1})}{(\hat{x}_1(n|y_{n-1}) - \hat{x}_3(n|y_{n-1}))^2 + \hat{x}_2^2(n|y_{n-1})} \\ 0 \\ \frac{\hat{x}_1(n|y_{n-1})}{\hat{x}_1^2(n|y_{n-1}) + \hat{x}_2^2(n|y_{n-1})} \\ \frac{\hat{x}_1(n|y_{n-1}) - \hat{x}_3(n|y_{n-1})}{(\hat{x}_1(n|y_{n-1}) - \hat{x}_3(n|y_{n-1}))^2 + \hat{x}_2^2(n|y_{n-1})} \\ 0 \\ \frac{\hat{x}_2(n|y_{n-1})}{(\hat{x}_1(n|y_{n-1}) - \hat{x}_3(n|y_{n-1}))^2 + \hat{x}_2^2(n|y_{n-1})} \\ 1 \end{bmatrix} \quad (15)$$

Mean and covariance of the prior density are as follows:

$$\begin{aligned} E(\mathbf{x}(n)|\mathbf{y}(1) \cdots \mathbf{y}(n-1)) &= \hat{\mathbf{x}}(n|\mathbf{y}(n-1)) \\ &= \mathbf{F}(n, n-1)\hat{\mathbf{x}}(n-1|\mathbf{y}(n-1)), \end{aligned} \quad (16)$$

$$\begin{aligned} V(\mathbf{x}(n)|\mathbf{y}(1) \cdots \mathbf{y}(n-1)) &= \mathbf{K}(n|n-1) \\ &= \mathbf{F}(n, n-1)\mathbf{K}(n-1|n-1)\mathbf{F}^T(n, n-1) \end{aligned} \quad (17)$$

where  $E(\cdot)$  is expectation operator, and  $V(\cdot)$  is covariance operator. The posterior density is now Gaussian with mean and covariance as follows:

$$\begin{aligned} E(\mathbf{x}(n)|\mathbf{y}(1) \cdots \mathbf{y}(n)) &= \hat{\mathbf{x}}(n|\mathbf{y}(n)) \\ &= \hat{\mathbf{x}}(n|\mathbf{y}(n-1)) + \mathbf{G}_f(n)(\mathbf{y}(n) - \mathbf{C}(n)\hat{\mathbf{x}}(n|\mathbf{y}(n-1))), \end{aligned} \quad (18)$$

$$\begin{aligned} V(\mathbf{x}(n)|\mathbf{y}(1) \cdots \mathbf{y}(n)) &= \mathbf{K}(n|n) \\ &= (\mathbf{I} - \mathbf{G}_f(n)\mathbf{C}(n))\mathbf{K}(n|n-1). \end{aligned} \quad (19)$$

where  $\mathbf{I}$  is identity matrix, and  $\mathbf{G}_f(n)$  is the Kalman gain matrix

$$\mathbf{G}_f(n) = \mathbf{K}(n|n-1)\mathbf{C}^T(n)(\mathbf{C}(n)\mathbf{K}(n|n-1)\mathbf{C}^T(n) + Q(n))^{-1} \quad (20)$$

The proposed algorithm is summarized in TABLE I

TABLE I  
THE PROPOSED ALGORITHM FOR ACOUSTIC SOURCE LOCALIZATION

$$\begin{aligned}\hat{\mathbf{x}}(n|\mathbf{y}(n-1)) &= \mathbf{F}(n, n-1)\hat{\mathbf{x}}(n-1|\mathbf{y}(n-1)) \\ \mathbf{K}(n|n-1) &= \mathbf{F}(n, n-1)\mathbf{K}(n-1|n-1)\mathbf{F}^T(n, n-1) \\ \mathbf{G}_f(n) &= \mathbf{K}(n|n-1)\mathbf{C}^T(n) (\mathbf{C}(n)\mathbf{K}(n|n-1)\mathbf{C}^T(n) + \mathbf{Q}(n))^{-1} \\ \hat{\mathbf{x}}(n|\mathbf{y}(n)) &= \hat{\mathbf{x}}(n|\mathbf{y}(n-1)) + \mathbf{G}_f(n) (\mathbf{y}(n) - \mathbf{C}(n)\hat{\mathbf{x}}(n|\mathbf{y}(n-1))) \\ \mathbf{K}(n|n) &= (\mathbf{I} - \mathbf{G}_f(n)\mathbf{C}(n)) \mathbf{K}(n|n-1)\end{aligned}$$

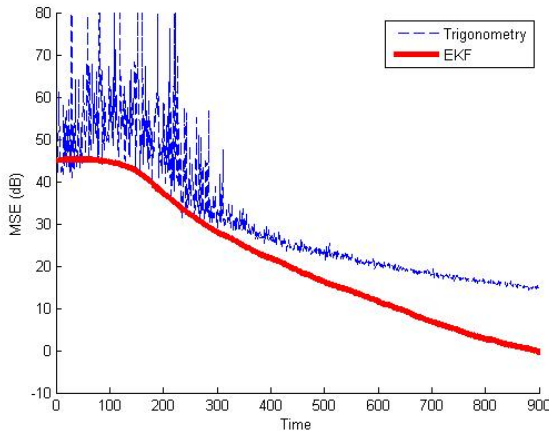


Fig. 2. MSE of Acoustic Source Localization with errors of  $2^\circ$  in DOA and 20% in moving distance

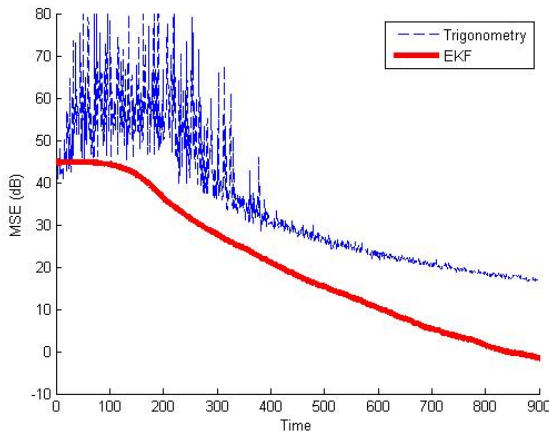


Fig. 3. MSE of Acoustic Source Localization with errors of  $2^\circ$  in DOA and 40% in moving distance

#### IV. SIMULATION RESULTS

The acoustic source localization scenario is as follows: The location of the source is 1 kilometers apart along the moving direction of vehicle and 500 meters apart along the perpendicular direction from the location of the vehicle at time

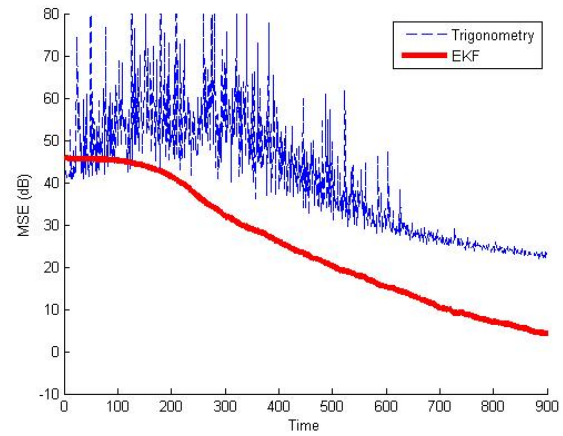


Fig. 4. MSE of Acoustic Source Localization with errors of  $5^\circ$  in DOA and 20% in moving distance

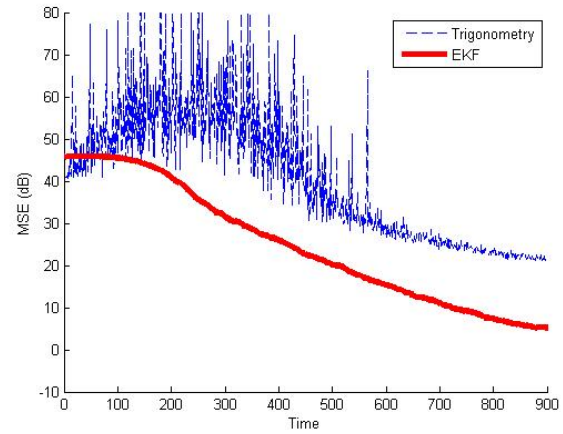


Fig. 5. MSE of Acoustic Source Localization with errors of  $5^\circ$  in DOA and 40% in moving distance

0, that is,  $(x_s, y_s) = (1000, 500)$  and  $x_v(0) = 0$ . The vehicle is moving with the speed of  $1m/s$ . The DOA of each time is measured but have error which has normal distribution with 2 or  $5^\circ$  of standard deviation. The distance between the current and prior location of the vehicle is also measured with an error of less than 20 or 40%. The simulation results are illustrated in Fig2, Fig3, Fig4, and Fig5 with errors of  $(2^\circ, 20\%)$ ,  $(2^\circ, 40\%)$ ,  $(5^\circ, 20\%)$ , and  $(5^\circ, 40\%)$ , respectively. The mean square error (MSE) is defined by

$$MSE(n) = E(x_s - (x_v(n) + \mathbf{x}(1)))^2 + (y_s - (\mathbf{x}(2)))^2, \quad (21)$$

that is the distance between the source location and estimated source location.

#### V. CONCLUDING REMARKS

In this study, we considered a special situation that only a pair of hydrophone on a moving underwater vehicle is

available to localize a fixed acoustic source of far distance. The trigonometry can be used in this situation by using two different DOA of different locations. Notice that the distance between the two locations should be measured. Therefore, we assumed that the vehicle is sailing straightly and the moving distance for each unit time is measured continuously. However, the accuracy of the localization using the trigonometry is highly dependent to the accuracy of DOAs and measured moving distances. Therefore, we proposed another method based on the extended Kalman filter that gives more robust and accurate localization result.

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#### REFERENCES

- [1] Z. Zhou, J. Cui, and A. Bagatzoglou, Scalable Localization with Mobility Prediction for Underwater Sensor Networks, in 2008 *IEEE INFOCOM Conf.*, pp.2198-2206.
- [2] Z. Zhou, J. Cui, S. Zhou, Localization for large-scale underwater sensor networks, *Proc. the 6th international IFIP-TC6 Conf. on Ad Hoc and sensor networks, wireless networks, next generation internet*, May 14-18, 2007, Atlanta, GA, USA, pp.108-119.
- [3] P. Corke, C. Detweiler, M. Dumbabin, M. Hamilton, D. Rus, and I. Vasilescu, Experiments with Underwater Robot Localization and Tracking, *Proc. IEEE Int. Conf. Robotics and Automation*, pp.4556-4561, April 2007.
- [4] N. Y. Ko, T. G. Kim, and S. W. Noh, Monte Carlo Localization of Underwater Robot Using Internal and External Information, *IEEE Conf. Asia-Pacific Services Computing*, pp.410-415, Dec 2011.
- [5] B.A.A.P. Balasuriya, M. Takai, W.C. Lam, T. Ura, and Y. Kuroda, Vision based autonomous underwater vehicle navigation: underwater cable tracking, *Proc. OCEANS 97. MTS/IEEE Conf.*, 14181424 vol.2, Oct 1997.
- [6] L. Stutters, H. Liu, C. Tiltman, and D. J. Brown, Navigation technologies for autonomous underwater vehicles, *IEEE Trans. Syst. Man Cybern. C, Appl. Rev.*, vol. 38, no. 4, pp. 581589, Jul. 2008.
- [7] K. T. Wong and M. D. Zoltowski, Closed-form underwater acoustic direction-finding with arbitrarily spaced vector hydrophones at unknown locations, *IEEE J. Oceanic Eng.*, vol. 22, no. 3, pp. 566575, July 1997.
- [8] K. T. Wong and M. D. Zoltowski, Extended-aperture underwater acoustic multi-source azimuth/elevation direction-finding using uniformly but sparsely spaced vector hydrophones, *IEEE J. Oceanic Eng.*, vol. 22, no. 4, pp. 659672, Oct. 1997.
- [9] K. T. Wong, and M. K. AWAD, Source Tracking with Multiple-Forgetting-Factor RLS Using a Vector-Hydrophone Away From or Near a Reflecting Boundary, *IEEE Conf. OCEANS*, pp. 1-4, May 2007.
- [10] N. Zou, C.C. Swee, B.A.L Chew, Vector Hydrophone Array Development and its Associated DOA Estimation Algorithms, in the Proceedings of OCEANS Asia Pacific, pp. 1-5, Singapore, May 2006.
- [11] J. Li, and X. Zhang, Improved Joint DOD and DOA Estimation for MIMO Array With Velocity Receive Sensors, *IEEE Signal Processing Lett.*, vol. 18, issue. 12, pp/ 717-720, Dec, 2011.
- [12] H. Lai and K. Bell and H. Cox, DOA Estimation using Vector Sensor Arrays, 42nd Asilomar Conference on Signals, Systems and Computers, 2009.
- [13] Q. Zhang, and J. Huang, N. Zou, Joint estimation of DOA and time-delay in underwater localization, *Proc. IEEE International Conf. on Acoustic, Speech, and Signal Processing*, vol. 5, pp. 2817-2820, 1999.
- [14] D. Carevic, Automatic estimation of multiple target positions and velocities using passive TDOA measurements of transients, *IEEE Trans.Signal Process.*, vol. 55, no. 2, pp. 424436, Feb. 2007.
- [15] L. Yang and K. C. Ho, An Approximately Efficient TDOA Localization Algorithm in Closed-Form for Locating Multiple Disjoint Sources With Erroneous Sensor Positions, *IEEE Transactions on Signal Processing*, vol. 57, pp. 45984615, Dec. 2009.
- [16] D. Carevic, Detection and Tracking of Underwater Targets Using Directional Sensors, *Proc. of Intelligent Sensors, Sensor Networks and Information, 2007 3rd International Conf.*, 2007, pp. 143-148.
- [17] Z. Yang, W. Xu, Z. Xiao, and X. Pan, Passive localization of an Autonomous Underwater Vehicle with periodic sonar signaling, *IEEE Conf. OCEANS*, pp. 1-4, May 2010.
- [18] M. Cai, B. Bingham, Passive acoustic detection of a small remotely operated vehicle, *IEEE Conf. OCEANS*, pp. 1-7, June 2011.