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Unit Commitment Solution Methods

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***Abstract*—**An effort to develop a unit commitment approach capable of handling large power systems consisting of both thermal and hydro generating units offers a large profitable return. In order to be feasible, the method to be developed must be flexible, efficient and reliable. In this paper, various proposed methods have been described along with their strengths and weaknesses. As all of these methods have some sort of weaknesses, a comprehensive algorithm that combines the strengths of different methods and overcomes each other's weaknesses would be a suitable approach for solving industry-grade unit commitment problem.

***Keywords*—**Unit commitment, Solution methods, and Comprehensive algorithm.

I. Introduction

THE unit commitment problem determines the combination of available generating units and scheduling their respective outputs to satisfy the forecasted demand with the minimum total production cost under the operating constraints enforced by the system for a specified period that usually varies from 24 hours to one week. Attempts to develop rigid unit operating schedules more than one week in advance are extremely curtailed due to uncertainty in hourly load forecasts at lead times greater than one week.

Besides achieving minimum total production cost, a generation schedule needs to satisfy a number of operating constraints. These constraints reduce freedom in the choice of starting up and shutting down generating units. The constraints to be satisfied are usually the status restriction of individual generating units, minimum up time, minimum down time, capacity limits, generation limit for the first and last hour, limited ramp rate, group constraint, power balance constraint, spinning reserve constraint, hydro constraint, etc.

The high dimensionality and combinatorial nature of the unit commitment problem curtail attempts to develop any rigorous mathematical optimization method capable of solving the whole problem for any real-size system. Nevertheless, in the literature, many methods using some sort of approximation and simplification have been proposed. The available approaches for solving unit commitment problem can usually be classified into heuristic methods and mathematical programming methods. The proposed mathematical programming approaches are dynamic programming, Lagrangian relaxation, Benders decomposition and mixed integer programming [1]-[3].

In the literature, dynamic programming and Lagrangian relaxation have been used extensively to develop industry-grade unit commitment programs. Their major advantage seems to be the requirement of reasonable computation time when compared to other mathematical approaches.

In dynamic programming, it is relatively easy to add constraints that affect operations at an hour (such as power balance constraints) since these constraints mainly affect the economic dispatch and solution method. However, the dynamic programming suffers from the curse of dimensionality. Hence, it is required to limit the commitments considered at any hour through some simplification techniques such as truncation and fixed priority ordering. This simplification, particularly for large scale systems, can lead to suboptimal schedules.

The utilization of Lagrangian relaxation in production unit commitment problems is much more recent than the dynamic programming methods. It has the advantage of being easily modified to model characteristics of specific utilities. Lagrangian relaxation method is more advantageous due to its flexibility in dealing with different types of constraints. It is relatively easy to add unit constraints. Lagrangian relaxation is flexible to incorporating additional coupling constraints that have not been considered so far. The only requirement is that constraints must be additively separable in units. Such constraints could be area reserve constraint, area interchange constraint, etc. To incorporate such constraint into the framework of Lagrangian relaxation, a Lagrangian multiplier is defined for each constraint for each time period and the constraints are adjoined into the objective function of the relaxed problem. The Lagrangian relaxation method is also more flexible than dynamic programming because no priority ordering is imposed. The amount of computation varies linearly with the number of units. Hence, it is computationally much more attractive for large systems.

One weakness of the Lagrangian relaxation method is that the dual optimal solution seldom satisfies the once relaxed coupling constraints. Another weakness is the sensitivity problem that may cause unnecessary commitments of some units. Therefore only a near optimal feasible solution can be expected. However, the degree of suboptimality decreases as the number of units increases.

Aoki et al. [4] presented the most promising Lagrangian relaxation approach developed so far. Tong and Shahidehpour [2] improved it further through the inclusion of post-processor. Two new efficient techniques namely non-discretization of generation levels in the solution process of

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single unit dynamic programming and inclusion of ramp rate constraints are proposed by Guan et al. [5]. Kuloor et al. [6] show environmental constraint incorporation technique in unit commitment problem formulation.

Due to the imperfections of the dynamic programming algorithm, the application of a unit commitment expert system [7], [8] has been proposed to supplement this method. The constraints, which are difficult or impractical to be implemented in unit commitment algorithm, can be handled by this expert system. In this paper, various proposed methods for unit commitment have been described. Finally a comprehensive algorithm [9], [10] for solving industry-grade unit commitment problem is described.

II. Heuristic Methods

Heuristic methods are non-rigorous computer aided empirical methods, which make the unit commitment decisions according to a pre-calculated priority list and incorporate all the operating constraints heuristically.

Baldwin et al. [11] have used a heuristic approach for unit commitment. All units are shut down and started up in strict priority order. The priority list is prepared on the basis of the average full load cost of each unit, where the average full load cost is the net heat rate at full load times the fuel cost.

Kerr et al. [12] have proposed heuristic approach that begins with an initial feasible schedule and then follows the sequence of steps for adjusting the starting and stopping times accordingly to reduce the cost of operation.

The heuristic method proposed by Happ et al. [13] uses a sub-optimizer to get a feasible and near optimal commitment. Then that method employs an optimizer to further optimize the schedule. The optimizer uses a sequence of steps repeatedly until no further reduction in cost is observed.

Heuristic approach to the short-term unit commitment problem has also been implemented using an expert system [14], [15].

Heuristic methods have the following advantages [16]:

- are flexible and allow for the consideration of practical operating constraints
- feasible solutions if there are any are usually obtained
- computational requirements in terms of memory and running time are modest

The main shortcoming of heuristic methods is that they cannot guarantee the optimal solutions or even furnish an estimate of the magnitude of their sub-optimality. This aspect becomes rather significant in large-scale power systems, as a small percentage, e.g. 0.5%, in the costs of unit commitment schedules represents a substantial financial annual saving. Therefore, it is advantageous to employ more rigorous methods compared with heuristics methods to generate more economical solutions as the size of a system grows, despite the requirement of comparatively large computational efforts.

More recently, metaheuristic approaches have been used such as simulated annealing [17], tabu search [18], genetic algorithms [19]-[21], and greedy randomized adaptive search procedure [22].

III. Mixed Integer Programming

Integer programming optimizes integer function of integer variables. A modification of standard integer programming that allows non-integer function is known as mixed-integer programming (MIP). MIP treates the objective and constraint functions as continuous and the variables as integers.

Branch and bound is one of the techniques used for the solution of the integer problem. It is a technique to solve a discrete variable problem by solving a sequence of simpler problems derived from the original problem. In solving using the branch and bound method, one needs to define 1) the problems in the branch and bound three, 2) the method of solving each problem on the tree, and 3) the method of searching the tree. The branch and bound tree is fully described if one defines the problem corresponding to the top node of the tree and the method of obtaining the children of any node [1].

Dillon et al. [23] have formulated the unit commitment problem as a linear MIP problem. Then they have used standard integer programming algorithm for solving the commitment schedule.

One of the proposed MIP methods [24] transforms the linear optimization problems that arise during the search procedure in the branch and bound algorithm into capacitated transhipment problems. These are then solved efficiently by a network-based solution procedure.

Bond and Fox [25] presented an algorithm based on a combination of mixed integer-linear and dynamic programming. Mixed integer-linear programming is used to determine feasible combinations of units at each scheduling point, while a novel dynamic programming approach identifies promising scheduling routes in the time domain.

MIP models proposed so far have employed linear cost functions although more accurate cost models are available. To date, branch and bound techniques have only been employed on moderate sized systems using linear models [1]. On the contrary, many available economic dispatch algorithms use quadratic cost curves. Moreover, the present trend is toward improved modeling of unit input/output characteristics with more detailed non-linear models. The MIP based approach, using only linear models, was found to take long computation time. The MIP formulation of unit commitment would become a very large problem demanding extremely long computation time if applied to a typical generation mix with more detailed non-linear models.

IV. Benders Decomposition

In Benders Decomposition method [26]-[29], the unit commitment problem is decomposed into a master problem involving only the discrete commitment variables and a subproblem involving the continuous generation variables. The subproblem corresponds to the economic dispatch problem with a given commitment. The marginal costs, for each hour, from the subproblem are used to constrain the allowed commitments in the master problem. The master problem supplies commitments to the economic dispatch problem. The master problem and the economic dispatch subproblem are solved iteratively until the solution converges. Any economic dispatch routine can be applied for solving the subproblem.

The major difficulty in Benders decomposition approach is the determination of the solution of the master problem, which is still regarded as a large scale integer optimization problem. Turgeon [26] has solved the master problem by a variational approach and a branch and bound algorithm whereas Baptistella and Geromel [27] have solved it by relaxation in the master level of Benders decomposition approach. In order to improve the efficiency, some of the constraints which are difficult to handle, such as nonlinear minimum up and down time constraints, are replaced by simpler constraints in the actual formulation of the scheduling algorithm. For instance, Habibollahzadeh and Bubenko [28] did not use the minimum up and down time constraints in their mathematical model but, instead, included a constraint that allowed only one commitment per day for each unit. Ma and Shahidehpour [29] dealt with transmission-constraint by introducing a proper constraint called Benders cut.

V. Dynamic Programming

Dynamic programming acts as an important optimization technique with broad application in many areas [30]. Dynamic programming decomposes a problem into a series of smaller problems, solves the small problems, and develops an optimal solution to the original problem step-by-step. The optimal solution is developed from the subproblem recursively.

In its fundamental form, the dynamic programming algorithm for unit commitment problem examines every possible state in every interval. Some of these states are rejected instantly because they are found infeasible. But even, for an average size utility, a large number of feasible states will exist and the requirement of execution time will stretch the capability of even the largest computers. Hence many proposed techniques used some sort of simplification and approximation to the fundamental dynamic programming algorithm [31].

The approach, first used by Lowery [32], and later refined by Ayoub and Patton [33], selected unit generation output as a state variable and on-line capacity as the stages. Ayoub and Patton included probabilistic techniques for reserve determination in the developed code.

A typical approach [34], [35] determines some nominal commitment, which is determined to be good for each hour. Choices that have to be considered are minimum number of units, in the priority ordering, needed to meet the reserve constraints and the result of the above priority list optimization. A set of units in the priority list about the nominal commitment are then chosen for optimization - the units below that set are assumed to be committed while the units above that set are assumed to be off. If this set contains 5 units, then the state space size is 31 which is a reasonable number.

The approaches [31], [36] used selection techniques for choosing the most promising states from all possible states and implemented approximate economic dispatch subroutines to reduce computer running time requirement. Variable window truncated dynamic programming that adjusts the window size according to the incremental load demands in adjacent hours and controls the program execution time have been proposed. Kumar and Palanisamy [37] proposed a two-step process that uses a direct computation Hopfield neural network to generate economic dispatch. Then using dynamic programming the generator schedule is produced.

The approaches [7], [8] have an integrated expert system into the truncated dynamic programming based unit commitment program to check and modify commitment results.

VI. Lagrangian Relaxation

In Lagrangian relaxation approaches, unit commitment problem is formulated in terms of 1) a cost function, that is the sum of terms each involving a single unit, 2) a set of constraints involving a single unit, and 3) a set of coupling constraints (the generation and reserve requirements), one for each hour in the study period, involving all units. Cohen and Sherkat [1] have reported that an approximate solution to this problem can be obtained by adjoining the coupling constraints onto the cost using Lagrangian multipliers. The cost function (primal objective function) of the unit commitment problem is relaxed to the power balance and the generating constraints via two sets of Lagrangian multipliers to form a Lagrangian dual function. The dual problem is then decoupled into small subproblems which are solved separately with the remaining constraints. Meanwhile, the dual function is maximized with respect to the Lagrangian multipliers, usually by a series of iterations.

Unfortunately, duality theory has shown that for nonconvex problems there will be a duality gap between the cost obtained by solving the relaxed problem and the optimal cost of the original problem. Since the commitment decision variables are discrete, the unit commitment problem is nonconvex. Due to the nonconvexity of the unit commitment problem, the dual solution seldom satisfies the power balance constraints and the reserve constraints. Hence, in addition to solving the dual problem, a suboptimal feasible solution is usually searched near the dual optimal point. Even though the dual optimal solutions may violate the feasibilities of the original problem, they may usually provide sharp lower bounds.

Muckstadt and Koenig [38] employed Lagrangian relaxation to replace the common linear programming relaxation approach, which dropped the integrality requirements of the variables, in the fathoming process of branch and bound algorithms. This causes a significant improvement of computational efficiency compared with previous branch and bound algorithms.

Based on the sharp bound provided by the Lagrangian dual optimum, it is expected that a suboptimal feasible solution near the dual optimal point can be accepted as a proper solution for the primal problem. Merlin and Sandrin [39] presented a more direct and fairly efficient technique based on this idea. In this algorithm, a modified subgradient method was used which incorporated the search of the suboptimal feasible solution along the direction for maximizing the dual function.

Some methods have been suggested which can be implemented in systems with fuel constraints. An additional set of multipliers has been introduced to associate the fuel constraints with the primal function. Cohen and Wan [40] proposed a successive approximation approach for altering the three sets of multipliers. Three iteration loops were constructed to update these multipliers independently, according to their corresponding constraints. The iteration of these three sets of multipliers was done by enclosing the three iteration loops with an external loop. Once the dual optimal had been found, the commitment of the fuel-constrained units would be fixed. Then, any violations of the constraints were adjusted by changing the commitment schedule and the outputs of the remaining generating units, which included thermal units operating without fuel constraints. These adjustments were within an iteration loop that successively solved the primal subproblems and modified the set of multipliers associated with reserve constraints.

Aoki et al. [4] presented an algorithm which also dealt with the fuel-constrained unit commitment problem. In the algorithm, the dual optimal solution was determined by an iteration loop which updated the multipliers associated with power balance constraints and fuel constraints simultaneously. The multipliers associated with fuel constraints were updated by introducing an additional iteration loop within the previous one. Compared with Cohen and Wan's work, this method introduced a more appropriate coupling between the three sets of multipliers. In addition, the variable metric method was employed instead of the subgradient method for updating the multipliers. These two modifications improved the efficiency of the process for finding the dual optimal. The feasible solution was determined by successive modification of the three sets of multipliers near the dual optimal and the solution of the primal subproblem.

Tong and Shahidehpour [2] have developed an algorithm to deal with limitations of the available Lagrangian relaxation methods for scheduling large scale systems which consist of thermal, fuel constrained and hydro generating units. The basic building block of the algorithm follows the Lagrangian relaxation approach directly [4]. The Lagrangian dual optimal is first solved and a feasible solution is searched by modifying the multipliers. Once a feasible solution is obtained, a new post-processor based on the application of linear programming is applied to find the final dispatch of units and improve the schedule by decomitting unnecessary units for further saving in cost.

In many approaches [2], [4], [39], the thermal subproblem is solved by using dynamic programming that discretizes generation levels and the ramp rate constraint is not considered. The discretization of generation levels causes a trade-off between computational requirements and the accuracy. On the other hand, if the generation levels are not discretized, the ramp rate constraint that couples the generation levels of two consecutive hours is difficult to deal with. A straightforward application of dynamic programming technique may lead to suboptimal results as illustrated by Guan et al. [5].

Guan et al. [5] have presented an approach that does not discretize generation levels and handles ramp rate constraints systematically. The thermal subproblem without ramp rate constraint is solved by first constructing a state transition diagram where the optimal generation levels of all up states are computed without discretizing generation levels. Dynamic programming technique is then applied with only a few well-structured states. This eliminates the difficult trade-off between computational requirements and the accuracy as needed by most approaches that discretize generation levels. Ramp rate constraints are relaxed by introducing an additional set of multipliers for a unit with the constraints. The subproblem is then solved as if there were no ramp rate constraint. An intermediate level is introduced to update this set of multipliers.

The algorithm presented by Yan et al. [41] follows the Lagrangian relaxation approach [5]. But, in addition to thermal units, hydro units have been included in the problem formulation. Given the set of Lagrange multipliers, a hydro unit subproblem is solved by a merit order allocation method, and a thermal unit subproblem is solved by using dynamic programming without discretizing generation levels.

Kuloor et al. [6] describe a method of solving environmentally constrained thermal unit commitment. Environmental consideration is added as a second objective function to the conventional unit commitment. The problem is thus converted from this dual-objective minimization model into a single-objective model.

VII. Comprehensive Algorithm

As the size of a power system increases, the savings potential of unit commitment in absolute terms increases although in percentage terms it remains the same. The effort to develop a unit commitment approach that is able to handle large systems consisting of both thermal and hydro generating units therefore offers a large profitable return. In order to be feasible, the method to be developed must be flexible, efficient and reliable. This requires that nonlinear functions such as thermal generation cost, water discharge characteristics, transmission loss and emission constraints must be incorporated into the method. Furthermore, the proposed method must be capable of dealing with other operational constraints.

The potential of unit commitment can be realized by employing a more rigorous mathematical programming technique. Of the four techniques surveyed, Benders decomposition is the least promising and is reflected by the lack of published work reporting its success. Even though mixed integer programming is rigorous, its demand on computing resources is prohibitive. It may be feasible if parallel computing using a fast processor is employed. Dynamic programming also suffers the same problem as MIP, but the truncated version reduces true computing requirement albeit at the possible loss of accuracy. Lagrangian relaxation is the least rigorous of the mathematical programming techniques but offers the best performance in computing requirements. Under the current computing technology, it seems that Lagrangian relaxation is the viable mathematical programming technique to solve large-scale unit commitment. This is reflected by the fact that they are widely reported in the literature.

To get an industry-grade efficient algorithm based on Lagrangian relaxation, the techniques like non-discretization of generation levels, handling of ramp rate and environmental constraints [5], [6] can be incorporated in the algorithm [2], [4]. Transmission loss can be incorporated using a general transmission loss formula [42] whose expression has a similar quadratic form to the B matrix loss formulation. The generation cost and water discharge rate can be represented as continuously increasing quadratic function of the generation. Finally, an efficient unit commitment expert system [8] can be developed as a supplement to the Lagrangian relaxation method.

To fulfill the aim cited above, hydrothermal scheduling based Lagrangian relaxation (HTSLR) approach to solve the unit commitment problem for a large practical system comprising both thermal and hydro generating units is proposed [9]. Commitment states of thermal units are obtained by solving thermal subproblems only. To achieve the output levels of hydro units, the hydrothermal scheduling is performed with a thermal unit commitment schedule obtained by solving only thermal subproblems. Extensive constraints are taken into account such as status restriction of individual generating units i.e. must run, must out, base load, cycling and peaking, power balance, spinning reserve, minimum up/down time, capacity limits, ramp rate, limited generation for the first and last hour, sulfur oxide emission and hydro constraints. Non-linear functions are employed for thermal generation cost and water discharge rate. A general transmission loss formula [42] whose expression has a similar quadratic form to the B matrix loss formulation has been utilized for incorporating transmission loss.

In the HTSLR approach, the commitment schedule may be so sensitive to the variations of the Lagrange multipliers that a slight modification of the multipliers may change the status of several units. This sensitivity problem is more serious for systems having several groups of identical units. Even though fuel costs of identical units can be slightly modified to make small differences among cost characteristics, this sensitivity problem still exists. In other words, unnecessary commitment of some units may be possible in the solution given by this method. In order to overcome this difficulty, a refinement process similar to that proposed by Tong and Shahidehpour [2] has been developed for HTSLR approach. This refinement process inspects some candidate units whose shutdown may result in additional reduction of the operating cost.

A unit commitment expert system [8] has also been developed [10]. It was employed as a preprocessor as well as a postprocessor to the HTSLR based unit commitment program to check and alter commitment results by adjusting the input data if necessary. It handles constraints which are difficult or impractical to be implemented in commitment algorithm such as cycling of gas turbine and steam turbine units, group constraints, etc.

VIII. Conclusion

The savings potential of unit commitment in absolute terms increases with the increase of power system size. The effort to develop a unit commitment approach that is able to handle large systems consisting of both thermal and hydro generating units therefore offers a large profitable return. In order to be feasible, the method to be developed must be flexible, efficient and reliable. Many proposed methods have been described along with their strengths and weaknesses. It seems that the comprehensive algorithm that combines the strengths of different methods and overcomes each other's weaknesses would be a suitable approach for solving industry-grade unit commitment problem.

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