Weight functions for signal reconstruction based on level crossings

Nagesha, G. Hemantha Kumar

Abstract—Although the level crossing concept has been the subject of intensive investigation over the last few years, certain problems of great interest remain unsolved. One of these concern is distribution of threshold levels. This paper presents a new threshold level allocation schemes for level crossing based on nonuniform sampling. Intuitively, it is more reasonable if the information rich regions of the signal are sampled finer and those with sparse information are sampled coarser. To achieve this objective, we propose non-linear quantization functions which dynamically assign the number of quantization levels depending on the importance of the given amplitude range. Two new approaches to determine the importance of the given amplitude segment are presented. The proposed methods are based on exponential and logarithmic functions. Various aspects of proposed techniques are discussed and experimentally validated. Its efficacy is investigated by comparison with uniform sampling.

Keywords—speech signals, sampling, signal reconstruction, asynchronous delta modulation, non-linear quantization.

I. Introduction

IN recent years, there has been considerable interest in level crossing algorithms for sampling continuous time signals. Driven by a growing demand for intelligent and high speed analog-to-digital converter (ADC) with low-power processor, increasing efforts have been made to improve level crossing based sampling techniques. An asynchronous level crossing sampling scheme records a new sample whenever the source signal crosses a threshold level. Consequently, more samples are recorded during fast changing intervals and fewer samples are recorded during relatively quiescent intervals. As a result, the signal is sampled nonuniformly. If the quiescent intervals are long and the number of these long intervals is large, then the average number of samples recorded would be relatively low. However, the recorded samples contain sufficient information that enables a fairly accurate reconstruction of the source signal. The recorded samples can be represented with very high accuracy; essentially because highly accurate clocks are much easier to build than circuits that quantize amplitudes very accurately. Also, asynchronous level crossing sampling is attractive because it can be implemented with a single-comparator circuit [8].

Several case studies in ADC's show that level crossing based on asynchronous sampling technique can be more effective than synchronous ADCs. The 1-bit ADC (bipolar) is optimized improving the dynamic range such that quantization error effectively decreases [9], [20]. The level crossing sampling scheme has been demonstrated for speech applications using CMOS technology and a voltage mode approach for the analog

Dept. of Studies in Computer Science, University of Mysore, Mysore, India, email: nagesh.bk@gmail.com

parts of the converter. Electrical simulations prove that the Figure of Merit of asynchronous level crossing converters increased compared to uniform sampling ADCs [4], [11]. Level crossing sampling scheme have also been suggested in literature for non-bandlimited signals [5], random processes [10], band limited gaussian random processes [6], reconstruction from nonuniform sampling [8], [21] and for monitoring and control systems [12], [13], [14], [15], [16]. The level crossing sampling strategy is also known as an event-based sampling [17], [18], Lebesgue sampling [19], send-on-delta concept [14] or deadband concept [16].

In general, conventional uniform sampling is with uniform time-step and variable amplitude. There is a trade-off between the requirements of bandwidth and the dynamic range to obtain a certain resolution. Sampling at the Nyquist rate requires smallest bandwidth but large number of quantization levels to achieve high resolution. Increasing the bandwidth decreases the need for large number of quantization levels, thus reducing the quantization error power and increasing the number of samples. At the extreme, the signal can be sampled capturing its characteristics using level crossing concept. Several signals have interesting statistical properties, but uniform sampling does not take advantage of them. Signals such as electro cardiograms, speech signals, temperature sensors, pressure sensors, seismic signals are almost always constant and may vary significantly during brief moments. In level crossing, the characteristics of the waveform play a vital role in approximation of the input signal. It has been proved in [1], [4] that level crossing sampling approach can lead to reduction in number of samples. The other advantage of level crossing sampling is that sampling frequency and quantization levels are decided by the signal itself. However, the methods developed for various cases use either constant threshold step size quantization levels (linear levels) or manually determined levels. The problem of primary interest is to determine statistical information on automatic distribution of quantization levels based on the characteristics of the input signal. Linear threshold level allocation scheme is simple but not efficient in terms of data bit usage for the following reason. The linear threshold allocation will result in a higher SNR at the region of higher amplitude than the region of lower amplitude. This increased SNR at the higher signal amplitude does not increase the perceived audio quality because humans are most sensitive to lower amplitude components [23]. To overcome these problems, we propose a non-linear quantization approach based on exponential and logarithmic functions which dynamically assign the number of quantization levels exploiting this auditory motivation.

The paper is organized as follows. In section II level cross-

ing based sampling approach with the proposed nonuniform threshold allocation scheme is described. The incorporation of exponential and logarithmic functions to formulate a rule for allocation of nonuniform threshold levels in multi level crossing is discussed in Section III. In section IV experimental setup for testing the proposed approach and results are discussed. Also, the performance and analysis of the proposed method is discussed. Section VI is devoted to conclusions as well as indicate directions for future lines of work.

II. LEVEL CROSSING BASED IRREGULAR SAMPLING MODEL

Level Crossing Analysis represents an approach to interpretation and characterization of time signals by relating frequency and amplitude information. Measurement of level crossing of a signal is defined as the crossings of a threshold level l by consecutive samples.

Definition 2.1 Let $w(x)$ be a deterministic weight function and $p(x)$ be the probability density function of a source signal. The level sampler $L_{f(.)}$ density distribution with a deterministic level allocation weight function $f(.)$ is a mapping

$$L_{f(.)}: R \longrightarrow f(p(x), w(x), Z): L_{f(.)} = (p(x) \otimes w(x)) \times N$$

where N is the total number of nonuniform levels. R and Z denotes the set of real and integer numbers respectively. The $\otimes$ symbol represents convolution.

Since the quantization levels are irregularly spaced across the amplitude range of the signal, it increases the efficiency of bit usage. The spacing of the levels is decided by the importance of the amplitude segments which is discussed in section III. A sample is recorded when the input signal crosses one of the nonuniformly spaced levels. The precession of time of the recorded sample is decided by the local timer τ.

Definition 2.2 Let $L_{f(.)} = \{l_1, l_2, ...l_N\}$ be the set of nonuniformly spaced levels and $2^b = N$ quantization levels with b bit resolution. The level crossing of the threshold level l_i by a signal $s(t)$ with period T is given by

$$L_{f(.)}(I_{ni}) = l_i \;\; iff \left(s\left(\frac{i-1}{N}T\right) - l_i \right) \times \left(s\left(\frac{i}{N}T\right) - l_i \right) < 0 \tag{1}$$

where n sub intervals are defined by $I_{ni} = \left(\frac{i-1}{N}T, \frac{i}{N}T\right)$, $i = 1, 2, ...n$

The level crossing problem is depicted in Figure 1 where the samples are recorded whenever the input signal crosses the threshold levels. If a sample is recorded and transmitted every time a level crossing occurs, the encoding procedure is called asynchronous delta modulation [2].

III. WEIGHT FUNCTIONS FOR IRREGULAR SAMPLING

Determining the positions of threshold levels on an amplitude scale is very important as it has a huge impact on the performance of coding. Unfortunately there is no theory available to determine the locations of threshold levels which exploit the statistics of a random variable under a particular distribution. Furthermore, the uniform threshold levels are not the efficient coding of the levels because they do not
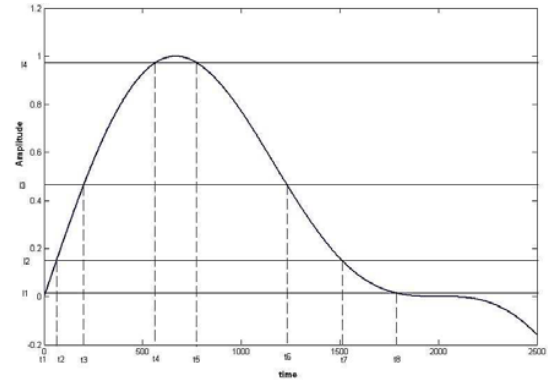

Fig. 1. Level Crossing sampling. $t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8$ denotes the recorded samples due to levels l_1, l_2, l_3, l_4 which are nonuniformly spaced.

take advantage of the statistical properties of the signal. The basic idea behind the weight functions is to emphasize the amplitude regions where (speech) signal is dominant, and to attenuate the amplitude regions which are less important considering auditory properties. As a result, signals with lesser activity in higher amplitude regions compared to the lower amplitude regions, will have less number of levels at higher amplitude region. Hence, basic methodology in level crossing based irregular sampling is to choose a weight function which encourages the important amplitude regions. The present study discusses distribution of nonuniform threshold levels based on the two weight functions namely exponential and logarithmic, to study and analyze the characteristics of proposed approach.

Our sense of hearing perceives equal ratios of frequencies as equal differences in pitch. Representation of importance of amplitude on a logarithmic scale can be helpful when the importance of regions varies monotonically. Logarithmic rule assigns less number of levels to the corner amplitude regions and more levels are assigned logarithmically in important amplitude regions. The center amplitude regions (near zero amplitude regions) are considered to be important amplitude regions. This issue, however is not whether to accept or reject logarithmic rule but to appreciate where it fits in, and where it does not.

A. LOGARITHMIC FUNCTION

A logarithm of a number x in base b is a number n such that $x = b^n$, where the value b must be neither 0 nor a root of 1. It is usually written as

$$log_b(x) = n$$

When x and b are further restricted to positive real numbers, the logarithm is a unique real number.

B. EXPONENTIAL FUNCTION

Take $e > 0$ and not equal to 1 . Then, exponential function is defined as a mapping

$$f: R \longrightarrow R: x \longrightarrow e^x$$

where e is know as base of exponent function.

C. LEVEL ESTIMATION

In a deterministic environment, the accuracy of the signal reconstruction depends on several parameters such as positioning of the levels, total number of levels, statistical properties of the signal etc. If weight functions are directly applied for level estimation, amplitude activity information of a given signal will not be used, which results in biased level estimation. Hence, level distribution PDF is convolved with signal PDF to correct for the biased distribution of levels. This ensures that level distribution is unbiased. Specifically, for a given signal we analyze its structural behavior by estimating its PDF. The signal histogram is approximated to obtain the signal PDF $p(x)$.

Now, consider a signal with amplitude PDF $p(x)$ and weight function $w(x)$. Let N be the total number of levels. The locations of N levels are estimated by the distribution

$$L_{f(.)}(x) = p(x) \otimes w(x) \qquad (2)$$

$L_{f(.)}(x)$ gives the probability distribution of levels and guides the distribution of N levels over the amplitude range. As expected, the spacing of N levels are not uniform and they are nonuniformly spaced over the amplitude range. Each level can be represented with $log_2(N)$ bits. Since the levels are nonuniformly spaced depending on the importance of the amplitude segment, we efficiently utilize the quantization levels by ignoring the amplitude regions with less activity. Hence only amplitude regions with higher activity and important lower amplitude regions will be allocated more number of levels using the weight function $w(x)$ and signal amplitude PDF $p(x)$. The histogram of sample speech signal is shown in Fig.2(a), along with plot of PDF of logarithmic weight function(Fig.2(b)) and exponential weight function (Fig.2(c)). The steps employed for the proposed approach are summarized as follows.

1) Input signal s[n] is normalized to lie within [-1, 1] and made zero mean.
2) Find the signal histogram. Approximate the signal histogram to find the PDF of the speech signal.
3) For each weight function and for varying number of bins(used to compute the weight function)
   a) Find the distribution of quantization levels
   b) Find the level crossings of the input signal. Store the level crossed sample value and its position.

IV. EXPERIMENTAL EVALUATION

In this section, the performance of the proposed approach is evaluated for speech signals. We have run simulations for the level crossing based sampling of speech signals from TIMIT database [22]. The TIMIT speech signals are sampled at 16 KHz sampling rate and each sample size is 16 bit. Speech signals are chosen from TEST/DR1 folder which contains seven male and four female adult speakers thereby yielding a total of 100 signals. The PDF of the speech signal is estimated by computing the amplitude histogram of the signal with 100 bins. The total number of quantization levels required
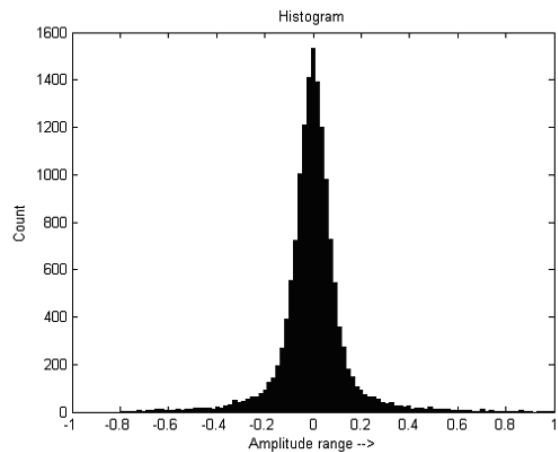
(a)
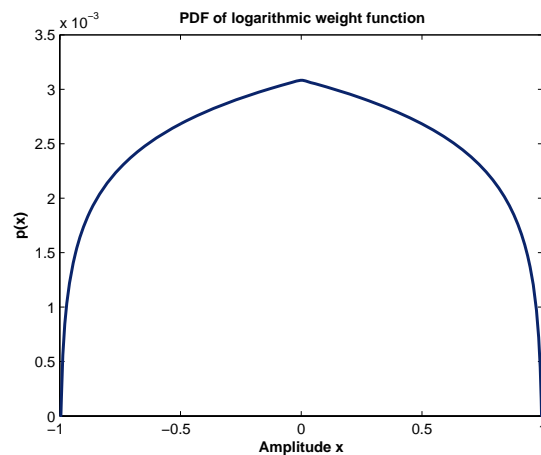
(b)
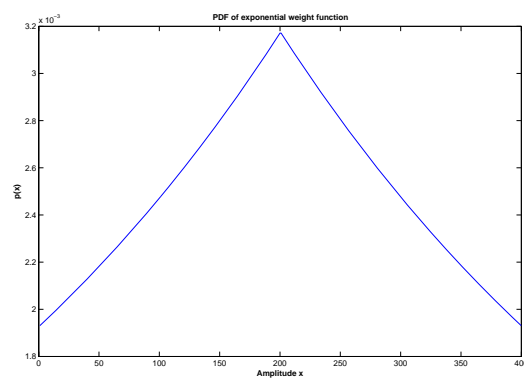
(c)

Fig. 2. (a) Signal histogram of a clean speech signal. (b) PDF of logarithmic weight function (c) PDF of exponential function

to sample the given signal are set to 16, 32, 64 and 128. The accuracy of distribution of the levels computed from equation 6 also depends on the number of bins used to compute convoluted PDF of the signal with weight function. The levels are estimated for 20, 40, 60, 80, 100 bins for comparison and analysis. We evaluated the system with proposed logarithmic and exponential weight functions. The performance of the proposed method is evaluated computing SNR and compression ratio. The performance measure SNR can be interpreted as

$$SNR = 10log_{10}\left(\frac{\frac{1}{N}\sum_{i=1}^{N}s\left(i\right)^2}{\frac{1}{N}\sum_{i=1}^{N}s\left(i\right)^2 - s'\left(i\right)^2}\right)$$

where $s\left(i\right)$ represents the original speech signal and $s'\left(i\right)$ denotes the reconstructed signal. Computation of SNR can be interpreted as the speed-up factor by which level crossing sampler achieves the same precision as the uniform sampling method. The ability to recover the uniform samples from its data representation of unequal sample values is also important. In our study, we applied direct interpolation scheme, polynomial curve fitting to approximate original signal from level crossed signal. Compression ratio is used to quantify the reduction in data-representation size produced by the proposed method and is defined as the ratio between the uncompressed size (original signal size) and the compressed size (level crossed sample size).

$$compression\ ratio = \frac{number\ of\ samples\ in\ original\ signal}{number\ of\ samples\ used\ in\ reconstruction\ of\ signal}$$

The simulation results are approximated analytically using quadratic polynomial.

By comparing exponential and logarithmic rule results, we analyze the performances. We investigated relationship between SNR and the histogram bins used to compute the signal amplitude histogram. The simulation results are depicted in Fig. 3(a) and Fig. 4(a). SNR of the resampled signal generally improves as the histogram bins increase for all the levels. This shows that increasing the resolution of the amplitude scale helps in accurate distribution of the levels thereby increasing the SNR. The exponent rule gives high SNR consistently compared to the logarithmic and linear rule at all bins. The characteristic graph appears convex for 128 levels in Fig. 4(a) whereas characteristic graph appears concave for 128 levels in Fig. 3(a). This is due to the distribution of quantization levels on amplitude range by the weight functions. Exponential weight function assignes more levels near zero zero amplitude regions compared to logarithmic weight function. Hence, as the number of quantization levels increase, performance of the exponential weight function better than logarithmic weight function. From Fig. 2(b), it can be observed that near zero amplitude regions are given equal importance whereas distribution levels keep increasing towards the near zero amplitude regions in case of exponential weight function. Hence, level crossing based sampling process results in poor performance for logarithmic weight function. As the number of histogram bins and quantization levels increase, more quantization levels
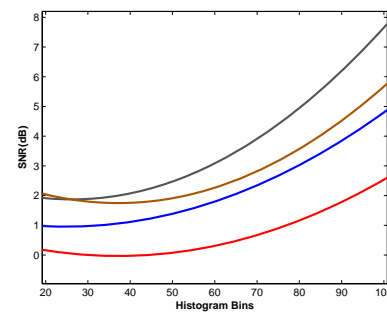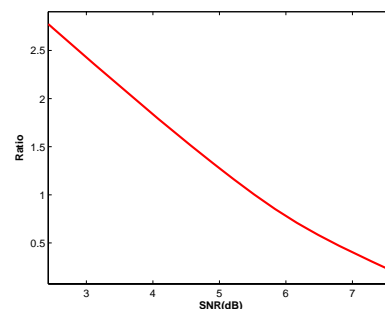

(a)
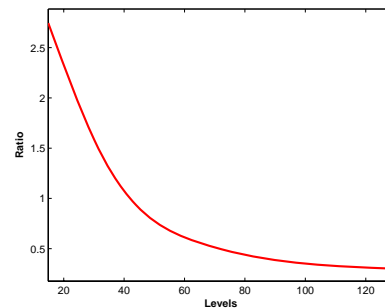
(b)
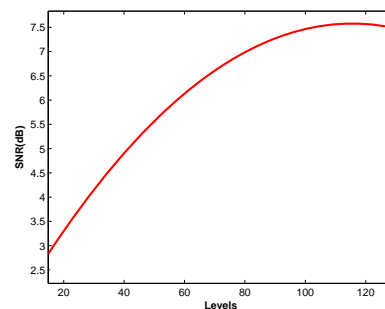
(c)

(d)

Fig. 3. Performance of logarithmic weight function (a) Histogram bin versus SNR. (b) Quantization level versus Compression ratio (c) Quantization level versus SNR. (d) SNR versus Compression ratio.
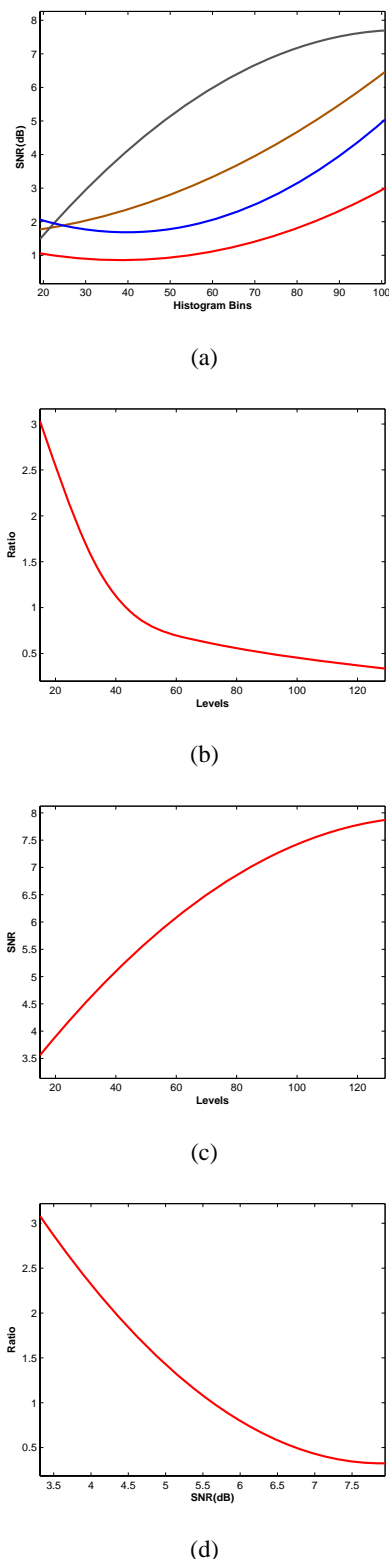
(a)

(b)

(c)

(d)

Fig. 4. Experimental results for exponential weight function (a) Histogram bin versus SNR. (b) Quantization level versus Compression ratio (c) Quantization level versus SNR. (d) SNR versus Compression ratio.

are spread across amplitude range. Hence, SNR increases as the number of histogram bins and quantization levels increases. The performance of logarithmic rule (Fig. 3(a)) is slightly less than that of exponential rule (Fig. 4(a)) for all the levels. The best performance is observed for exponential rule with 128 levels. In case of exponential rule, 1 dB increment in SNR is observed compared to logarithmic for 128 levels. This proves that increasing the quantization levels increases the SNR.

However, increasing the quantization levels considerably decreases the compression ratio. The comparison of compression ratio at various levels for the two rules is shown in Fig. 3(b) and Fig. 4(b). We observe that exponential rule slightly outperforms exponential rule. The exponential rule gives higher SNR for lesser levels and the ratio decreases as the levels are increased. For higher levels all the rules give similar results. Since, the logarithmic weight function forces quantization levels to spread equally near zero amplitude segments, the compression ratio considerably decreases compared to exponential weight function. The results of SNR versus levels (Fig. 3(c) and Fig. 4(c)) show that exponential weight function performance is superior to logarithmic weight function at all levels. Minimum SNR for 16 levels is near 3.5 dB in exponential weight function whereas minimum SNR in logarithmic weight function is 2.8 dB for 16 levels.

Figure 3(d) and 4(d) shows the plot of SNR versus compression ratio. The characteristic curve appears to be concave for exponential rule and linear for logarithmic rule. Increasing SNR decreases the compression ratio rapidly due to increased number of level crossings. Compression ratio of greater than 3 is achieved in exponential rule (Fig. 4(d)) with SNR 3.4 dB. From Fig. 3(d), we see that compression ratio less than 3 is achieved with SNR 2.4dB resulting in a poor performance for logarithmic rule. As the SNR increases, exponential weight function achieve good performance compared to logarithmic weight function due to the spread of quantization levels. Furthermore for higher SNR values the compression ratio drops drastically for exponential and logarithmic rule. Performance of exponential weight functions is considerably better than logarithmic weight function, with higher SNR for higher compression ratio, which is nonetheless better performance. Fig. 5 compares the plot of input signal(speech signal from TIMIT database) with reconstructed signal and plot of error in signal reconstruction. It can be seen that, reconstruction of signal with exponential weight function is more closer to the original signal than the reconstruction with logarithmic weight function. This argument is further supported by the error graphs. The error graphs shows the error in reconstruction using logarithmic weight function is more than exponential weight function.

The behavioral patterns of logarithmic and exponential weight function appear to be similar except in SNR versus ratio analysis. Both weight functions try to exploit the auditory motivation and try to assign dynamic nonuniform quantization levels. Exponential weight function distributes more levels in the critical amplitude regions. However, the priority of the amplitude regions varies logarithmically from corner amplitude regions to near zero value amplitude regions in case of
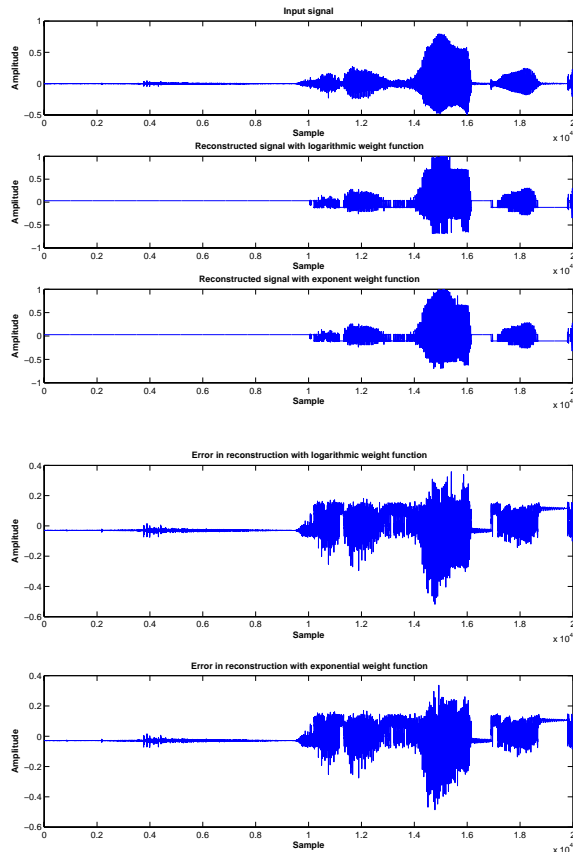
Fig. 5.   Comparison of input signal and reconstructed signal with logarithmic and exponential weight functions. In this example, thirty two quantization levels are distributed using deterministic weight functions. The reconstructed signals along with error in reconstruction is shown in figure.

logarithmic weight function. Exponential distribution of the quantization levels assignes more levels to near zero aplitude regions than logarithmic weight function. Hence, the SNR of the resampled signal using exponential weight function remains consistently superior to logarithmic weight function. Lack of levels at critical amplitude regions of the signal decreases the SNR of the resampled signal. The performance of the proposed approaches is fairly consistent with that of Sayiner[3]. This experimental analysis illustrates that signal with special statistical behavior such as speech, medical signals are not suitable for uniform sampling. These types of signals can be more efficiently reconstructed using a level crossing scheme.

V. CONCLUSION

This paper presents a new threshold level allocation schemes for level crossing based on nonuniform sampling which dynamically assigns the number of quantization levels depending on the importance of the given amplitude range of the input signal. Proposed methods take the advantage of statistical properties of the signal and allocate the nonuniformly spaced quantization levels across the amplitude range. The proposed

level allocation scheme for nonuniform sampling based on level crossing may motivate directed attempts to augment traditional methods that will improve their ability. Overall, these results motivate continued work on level crossing based on nonuniform sampling for improving sampling performance and analyzing the signals. Simplicity but significantly good performance of logarithmic weight function is what is observed. In general logarithmic is best because implementation complexity of logarithmic is much lesser than IBF wight function. Further investigation could look at this level crossing problem for 2-dimensional signals. This is much more challenging and also not a simple extension of 1-dimensional solution.

VI. CONCLUSION

This paper presents a new threshold level allocation schemes for level crossing based on nonuniform sampling which dynamically assigns the number of quantization levels depending on the importance of the given amplitude range of the input signal. Proposed methods take the advantage of statistical properties of the signal and allocate the nonuniformly spaced quantization levels across the amplitude range. The proposed level allocation scheme for nonuniform sampling based on level crossing may motivate directed attempts to augment traditional methods that will improve their ability. Overall, these results motivate continued work on level crossing based on nonuniform sampling for improving sampling performance and analyzing the signals. Simplicity but significantly good performance of exponential weight function is what is observed. Further investigation could look at this level crossing problem for 2-dimensional signals. This is much more challenging and also not a simple extension of 1-dimensional solution.

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Nagesha graduated from Mangalore University, India in 2000, and received MS degree from University of Mysore, India in 2003. Since 2005 he has been with the Department of studies in Computer Science at University of Mysore, India, as a researcher. His research interests are signal processing, speech and speaker recognition, image processing and analysis and neural networks.

G. Hemantha Kumar received the B.Sc in computer science in 1980, M.Sc and Ph.D. degrees in computer science in 1984 and 1988, respectively, from the University of Mysore, India. He joined the University of Myore as a lecturer in 1988. From 1998 to present he is working as a Reader with the Department of studies in Computer Science, University of Mysore, India. His research focuses on Pattern Recognition and Image Processing. His current area of research includes Document Analysis and Biometrics including Face and Speech recognition.