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# Performance of Dual MRC Receiver for M-ary Modulations over Correlated Nakagami-*m* Fading Channels with Non-identical and Arbitrary Fading Parameter

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***Abstract*—**Performance of a dual maximal ratio combining receiver has been analyzed for *M*-ary coherent and non-coherent modulations over correlated Nakagami-*m* fading channels with non-identical and arbitrary fading parameter. The classical probability density function (PDF) based approach is used for analysis. Expressions for outage probability and average symbol error performance for *M*-ary coherent and non-coherent modulations have been obtained. The obtained results are verified against the special case published results and found to be matching. The effect of the unequal fading parameters, branch correlation and unequal input average SNR on the receiver performance has been studied.

***Keywords*—** MRC, correlated Nakagami-*m* fading, non-identical fading statistics, average symbol error rate

## I. Introduction

MULTIPATH propagation is the prime barrier of wireless radio channels, produces the well known phenomena known as fading. Fading is responsible for degradation of the performance in wireless communication systems. In fading channel reliability of correct reception of the transmitted signal can be improved by receiving the signal through independent fading path and combining them efficiently. This technique exploit low probability of occurrence of deep fades in all the branches simultaneously and known as diversity combining technique. Maximal ratio combining (MRC) performs best among all the diversity combiner. Nakagami-*m* fading channels [1] receive attention of researchers due to its flexibility, simple analytical form and best fit to the practically obtained data. Performance of MRC receiver over Nakagami-*m* fading channels have been presented in many research papers [2]–[7]. A independent fading scenarios has been considered in analysis of [2]. However, correlated fading channels have been considered in the analysis of [3]–[7]. But, these results have been derived only for equal fading parameter. In [4] performance of MRC receiver is presented for equal and exponential correlated Nakagami-*m* fading channels whereas, [5] and [6] presented the performances for arbitrary co-variance matrix. In [7] performance measures for MRC receivers are presented for arbitrary fading parameter but this is valid for uncorrelated branches only. In [8], exact errors have been presented for correlated MRC receiver with non-identical fading parameter but for integer fading orders using a virtual branch technique for *M*-ary modulations. In practical scenario it is difficult to mount multiple antennas in the receiver sufficiently apart to avoid correlation and also it is difficult to have identical fading distribution in each branch. This generates a motive to know the performance of maximal ratio combining (MRC) receiver in correlated Nakagami-*m* fading channels with arbitrary and nonidentical fading parameter.

In this paper a PDF based approach have been followed to derive the performance of dual MRC receiver in correlated Nakagami-*m* fading channels with arbitrary and nonidentical fading parameter. The rest of the paper is organized as follows; In Section II introduction has been given to the channel and the diversity system. Section III presents the performance measures of diversity receiver followed by the numerical results and discussion in Section IV. The paper is concluded in section V.

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## II. Channel and Diversity Systems

The channel has been assumed Nakagami-*m* statistics with slow, frequency nonselective fading. The complex low-pass equivalent of the signal received at the $l^{th}$ input branch over one symbol duration $T_s$ can be expressed as

$$r_l(t) = \alpha_l e^{j\phi_l} s(t) + n_l(t), 0 \le t \le T_s, l = 1, 2, \tag{1}$$

where $s(t)$ is the transmitted signal with symbol energy $E_s$ and $n_l(t)$ is zero mean complex Gaussian noise with two sided power spectral density $2N_0$. Random variable $\phi_l$ represents the phase of the received signal which is uniformly distributed over $[0, 2\pi]$ while $\alpha_l$ is Nakagami-*m* distributed with density function given by [1]

$$f_{\alpha_l}(\alpha_l) = \frac{2\alpha_l^{2m_l-1}}{\Gamma(m_l)\Omega_l^{m_l}} \exp\left(-\frac{\alpha_l^2}{\Omega_l}\right), \alpha_l \ge 0 \tag{2}$$

where $\Omega_l = \frac{1}{m_l} E\left[\alpha_l^2\right]$, $E$ is the expectation operator and

$m_l$ is the fading parameter which lies in the range $\frac{1}{2} \le m_l < \infty$. We assume $\alpha_l$ is independent of $\phi_l$. Analysis have been carried out considering envelops are correlated to each other and their power correlation coefficient $\rho$ lies between $0 \le \rho \le 1$.

In MRC, the received signals are co-phased and weighted as per their SNR, combinations of which maximize the output SNR. This is followed by a detector suitable to detect the signal corresponding to the modulation scheme used at the transmitter. The instantaneous output SNR $\gamma$ of the dual MRC receiver is given by sum of their instantaneous SNR, [9]

$$\gamma = \gamma_1 + \gamma_2 = \frac{E_s}{N_0}\left(\alpha_1^2 + \alpha_2^2\right) \tag{3}$$

where $\gamma_l$ is the instantaneous SNR of $l^{\text{th}}$ branch. The average SNR of $l^{\text{th}}$ branch is given by $\bar{\gamma}_l$.

From (3) it can be observed that, it is possible to obtain the PDF of $\gamma$ from the PDF of $\alpha = \alpha_1^2 + \alpha_2^2$. A useful expression of joint characteristic function of $\alpha_1^2$ and $\alpha_2^2$ is presented in [10], from which applying the rule of inverse Fourier transform, PDF of $\alpha$ can be obtained. It is easy to obtain the PDF of $\gamma$ from the PDF of $\alpha$ using the relation given in (3) and applying the standard rule of transformation of RV. Using the explained approach an expression for PDF of $\gamma$ has been obtained by the author in [11] as

$$f_\gamma(\gamma) = \left(\frac{m_1}{\bar{\gamma}_1(1-\rho)}\right)^{m_1}\left(\frac{m_2}{\bar{\gamma}_2}\right)^{m_2}\sum_{k=0}^{\infty}\left(\frac{m_1 m_2 \rho}{\bar{\gamma}_1\bar{\gamma}_2(1-\rho)^2}\right)^k \times \frac{(m_1)_k\,\gamma^{\lambda_1-1}}{k!\,\Gamma(\lambda_1)e^{\frac{m_2}{\bar{\gamma}_2(1-\rho)}\gamma}}\,{}_1F_1\left(m_1+k,\lambda_1;\frac{\bar{\gamma}_1 m_2 - \bar{\gamma}_2 m_1}{\bar{\gamma}_2\bar{\gamma}_1(1-\rho)}\gamma\right) \tag{4}$$

where ${}_1F_1(a;b;z)$ is the Kummer confluent hypergeometric function [12, 13.1.2] and $\lambda_1 = m_1 + m_2$. It is now suitable to use (4) for analysis of MRC receiver over correlated Nakagami-*m* fading channels with non-identical and arbitrary statistics.

## III. Performance Analysis OF MRC Receiver

### A. Outage Probability:

Outage probability, as per definition, can be given as [9]

$$P_{\text{out}} = \int_0^{\gamma_{th}} f_\Gamma(\gamma)\,d\gamma, \tag{5}$$

TABLE I
Values a and b for Some Coherent Modulations

| a | b | | | |
|---|---|---|---|---|
| | 1 | 2 | $2\sin^2(\pi/M)$ | 3/(M-1) |
| 0.5 | BFSK | BPSK | -- | -- |
| *1* | QPSK | DBPSK | MPSK | |
| $\frac{4(\sqrt{M}-1)}{\sqrt{M}}$ | -- | -- | -- | Rect.QAM |

TABLE II
Values a and b for Some Noncoherent Modulations

| a | b | |
|---|---|---|
| | 0.5 | 1 |
| 0.5 | BFSK | DBPSK |
| $\frac{M-1}{2}$ | MFSK | -- |

where $\gamma_{th}$ is the threshold value of the SNR. Putting $f_\gamma(\gamma)$ from (4), the integral can be solved by expressing the hypergeometric function in infinite series form and then applying [13, (3.381.1)]. The final expression for the outage probability can be obtained as

$$P_{out} = (1-\rho)^{m_2}\left(\frac{m_1\bar{\gamma}_2}{\bar{\gamma}_1 m_2}\right)^{m_1}\sum_{k=0}^{\infty}\sum_{t=0}^{\infty}\left(\frac{\bar{\gamma}_2\rho m_1}{\bar{\gamma}_1 m_2}\right)^k\frac{(m_1+k)_t\,(m_1)_k}{t!\,k!\,(\lambda_1)_t} \times\left(\frac{\bar{\gamma}_1 m_2 - \bar{\gamma}_2 m_1}{\bar{\gamma}_1 m_2}\right)^t\frac{G\left(\lambda_1+t,\frac{m_2}{\bar{\gamma}_2(1-\rho)}\gamma_{th}\right)}{\Gamma(\lambda_1)} \tag{6}$$

where $G(a,x) = \int_0^x t^{a-1}e^{-t}dt$ is the incomplete gamma function.

For $\rho = 0$ and $m_1 = m_2 = m$ (6) becomes $P_{out} = \frac{\gamma\left(2m,\frac{m}{\bar{\gamma}}\gamma_{th}\right)}{\Gamma(2m)}$ which is matching with the result shown in [4, (48)] for $\rho = 0$ and $M = 2$.

### B. Average Symbol Error Rate:

The ASER of a digital communication system for various *M*-ary modulations can be obtained by averaging the conditional symbol error rate (SER) corresponding to the modulation over the PDF of the receiver output SNR [9]. Mathematically, ASER can be given as

$$P_e(\bar{\gamma}) = \int_0^\infty p_e(\grave{o}|\gamma)\,f_\gamma(\gamma)\,d\gamma, \tag{7}$$

where $p_e(\grave{o}|\gamma)$ is the conditional SER corresponding to the modulation scheme used. The conditional symbol error rate (conditioned on the received SNR) for different digital *M*-ary modulation schemes are available in literature.

#### 1) Coherent Modulation:

For binary coherent modulations, the expression for the conditional BER can be given as

$$p_{e,\text{coh}}(\grave{o}|\gamma) = aQ\left(\sqrt{b\gamma}\right), \tag{8}$$

where parameter $a$ and $b$ are chosen from Table I as per the modulation scheme used. Putting $p_{e,\mathrm{coh}}(\grave{o}|\gamma)$ and $f_{\gamma}(\gamma)$ into (7) and solving th integral ((using [4, A-(6), A-(8a)]), an expression for ASER can be obtained as

$$
\begin{aligned}
P_{e_{coh}} &= \frac{(1-\rho)^{m_2+0.5}\, a\sqrt{b}}{2\sqrt{2\pi}\,\Gamma(m_1)} \left(\frac{\overline{\gamma}_2 Z}{m_2}\right)^{\lambda_1+.5} \left(\frac{m_1}{\overline{\gamma}_1}\right)^{m_1} \left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2} \\
&\times \sum_{k=0}^{\infty}\sum_{t=0}^{\infty} \frac{\Gamma(m_1+k+t)\,\Gamma(\lambda_1+t+2k+.5)}{k!\,t!\,\Gamma(\lambda_1+t+2k)(\lambda_1+t+2k)} \\
&\times \left(\frac{(1-\rho)(\overline{\gamma}_1 m_2-\overline{\gamma}_2 m_1)Z}{m_2\overline{\gamma}_1(1-\rho)}\right)^{t} \left(\frac{m_1\rho\overline{\gamma}_2}{\overline{\gamma}_1 m_2}Z^2\right)^{k} \\
&\times {}_2F_1\left(1,\lambda_1+t+2k+.5,\lambda_1+t+2k+1;Z\right),
\end{aligned}
\qquad (9)
$$

where $Z \triangleq \dfrac{2m_2}{b\overline{\gamma}_2(1-\rho)+2m_2}$.

*2) Noncoherent Modulation:*

For non-coherent modulations, the conditional BER is given as

$$p_{e,\mathrm{ncoh}}(\grave{o}|\gamma) = a\exp(-b\gamma). \qquad (10)$$

The values of $a$ and $b$ depends on modulation format used and are given in Table II for convenience. By putting $p_{e,\mathrm{ncoh}}(\grave{o}|\gamma)$ and $f_{\gamma}(\gamma)$ into (7) and solving the integral (using [13, (7.621.4)] and [4, A-(5)]), an expression for ASER can be obtained as

$$P_{e_{ncoh}} = aR_1^{m_1}R_2^{m_2}\sum_{k=0}^{\infty}\frac{(m_1)_k}{k!}\left(R_1R_2\rho(1-\rho)\right)^k \qquad (11)$$

where $R_i \triangleq \dfrac{m_i}{m_i+b\overline{\gamma}_i(1-\rho)}$. The expression given in (11) can be further reduced by arranging the infinite series as

$$P_{e_{ncoh}} = \frac{aR_1^{m_1}R_2^{m_2}}{\left(1-R_1R_2\rho(1-\rho)\right)^{m_1}} \qquad (12)$$

## IV. Numerical Results and Discussion

For the purpose of investigation obtained mathematical expressions in section III have been numerically evaluated and plotted. In the numerical evaluation $\delta$ is the average fading power decay factor [9]. In Fig. 1 outage probability has been plotted for binary DPSK and CPSK modulation techniques. Threshold value for which the bit error probability (BEP) exceeds a given value can be obtained from [4 (43)] for different binary modulation techniques. Outage probability for a BEP of $10^{-3}$ is presented here for illustration. For binary, coherent and non-coherent modulations, ABER vs. $\overline{\gamma}_1$ have been plotted in Figs. 2 and 3, respectively. As expected, BER degrades with increase in $\rho$ and $\delta$.

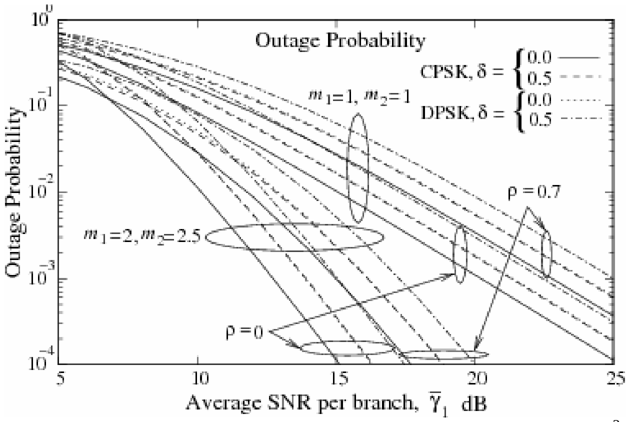


Fig. 1 Outage for CPSK and DPSK modulations for a BEP of $10^{-3}$

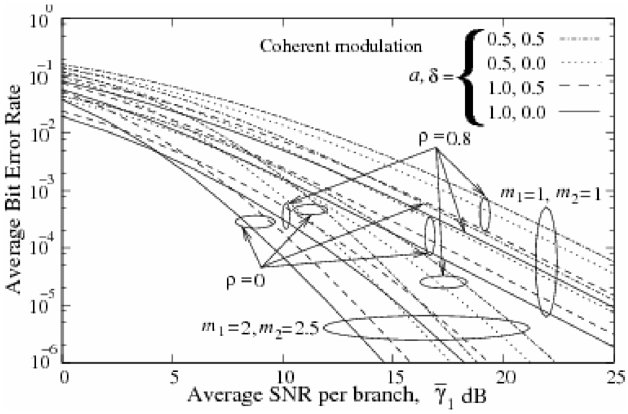


Fig. 2 ABER for binary CPSK and CFSK modulations

The convergences of infinite series present in the expressions of performance measures are also observed. The infinite series involved in (4) is convergent series and the upper bound on truncation error can be obtained in a similar approached given in [14]. It can be observed that the error bound decreases very fast with increase in number of terms in infinite series and a maximum of 25 terms is enough for numerical evaluation for accuracy of 7th place of decimal digit.

However for (6) and (9) it is difficult to obtain an upper bound of truncation error. In Table III we have illustrated the number of terms ($N$) required to achieve an accuracy of $10^{-7}$ in the evaluation of (9) as a function of $\overline{\gamma}_1$ and $\rho$.

TABLE III
NUMBER OF TERMS (N) REQUIRED FOR AN ACCURACY AT 7TH PLACE OF DECIMAL DIGIT IN THE NUMERICAL EVALUATION OF (9) WITH $\overline{\gamma}_1 = \overline{\gamma}_2$

| SNR (dB) | $\rho$ | $m_1=1, m_2=1$ | | $m_1=2, m_2=2.5$ | |
|---|---|---|---|---|---|
| | | N | BER | N | BER |
| 5 | 0.5 | 5 | 0.0174211 | 8 | 0.0041427 |
| | 0.8 | 9 | 0.0245875 | 29 | 0.0035931 |
| 0 | 0.5 | 2 | 0.0027927 | 4 | 0.0001685 |
| | 0.8 | 4 | 0.0050834 | 11 | 0.0003157 |

## V. Conclusion

In this paper analysis of the performances of a dual MRC receiver in correlated Nakagami-$m$ fading channels with arbitrary and nonidentical fading parameter has been carried out. Expressions have been obtained for outage probability and

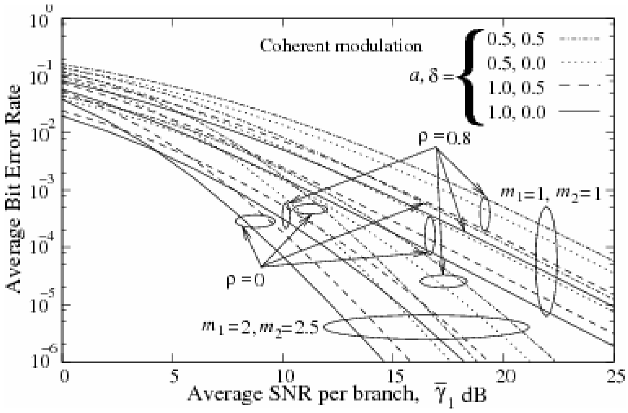


Fig. 3 ABER for binary DPSK and NCFSK modulation ASER

expressions for *M*-ary coherent and non-coherent modulations. Numerically evaluated results have been plotted to study the impact of fading correlation on the receiver performance with equal and unequal fading parameter in each branch. The obtained expressions are in the form of infinite series containing hypergeometric functions. The convergence of the infinite series present in the derived expressions has been observed. The result obtained for non-coherent modulation has been in closed form. The numerical results have been compared with available special case results.

## References

[1] M. Nakagami, "The m-distribution-A general formula of intensity distribution of rapid fading," *Statistical Methods in Radio Wave Propagation*, W. G. Hoffman, Ed. Oxford, England: Pergamon, 1960.

[2] E. K. AI-Hussaini and A. M. Albassiouni, "Performance of MRC diversity systems for the detection of signals with Nakagami fading," *IEEE Trans. Commun.*, vol. COM-33, pp. 1315-1319, Dec. 1985.

[3] P. Lombardo, G. Fedele, and M. M. Rao, "MRC performance for binary signals in Nakagami fading with general branch correlation," *IEEE Trans. Commun.*, vol. 47, pp. 44-52, Jan. 1999..

[4] V. A. Aalo, "Performance of maximal-ratio diversity systems in a correlated Nakagami-fading environment," *IEEE Trans. on Commun.* Vo. 43. No. 8 , Aug. 1995.

[5] Q. T. Zhang, "Maximal-ratio combining over Nakagami fading channels with an arbitrary branch covariance matrix," *IEEE Trans. on Veh. Technol*. Vol. 48, No. 4, pp. 1141-1150, Jul. 1999..

[6] G. C. Alexandropoulos, N. C. Sagias, F. I. Lazarakis and K. Berberidis, "New results for the multivariate Nakagami-*m* fading model with arbitrary correlation matrix and applications,"*IEEE Trans. on Wireless Commun.*, Vol. 8, No. 1, pp. 245-255, Jan. 2009.

[7] M.-S. Alouini, A. Abdi and M. Kaveh, "Sum of gamma variates and performance of wireless communication systems over Nakagami-fading channels" *IEEE Trans. on Veh. Technol*. Vol. 50, No. 6, pp. 1471-1480, Nov. 2001.

[8] M. Z. Win and J. H. Winters, "Exact error probability expressions for MRC in correlated Nakagami channels with unequal fading parameters and branch powers," in *Proc. IEEE Global Commun. Conf. (GLOBECOM' 99)*, Rio de Janeiro, Brazil, pp. 2331-2335. Dec. 1999.

[9] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, John Wiley & Sons, Inc., 2005.

[10] J. Reig, L. Rubio, and N. Cardona, "Bivariate Nakagami-*m* distribution with arbitrary fading parameters," *IEE Electron. Lett.*, vol. 38, no. 25, pp. 1715-1717, Dec. 2002.

[11] R. Subadar and P. R. Sahu , "Capacity analysis of dual -SC and -MRC systems over correlated Nakagami-*m* fading channels with nonidentical and arbitrary fading parameters," *National Conference on Communications (NCC)*, Chennai, Jan. 2010.

[12] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions*, 9th printing ed. New York: Dover, 1970.

[13] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th Ed., San Diego, CA: Academic, 2000.

[14] C. C. Tan and N. C. Beaulieu, "*Infnite Series Representations of the Bivariate Rayleigh and Nakagami-m Distributions*," IEEE Trans. on Commun., Vol. 45, No. 10, Oct. 1997.