Robust control for discrete-time sector bounded systems with time-varying delay

Ju H. Park, S.M. Lee

Abstract—In this paper, we propose a robust controller design method for discrete-time systems with sector-bounded nonlinearities and time-varying delay. Based on the Lyapunov theory, delay-dependent stabilization criteria are obtained in terms of linear matrix inequalities (LMIs) by constructing the new Lyapunov-Krasovskii functional and using some inequalities. A robust state feedback controller is designed by LMI framework and a reciprocally convex combination technique. The effectiveness of the proposed method is verified throughout a numerical example.

Keywords—Lur'e systems, Time-delay, Stabilization, LMIs.

I. INTRODUCTION

ALL physical systems are nonlinear in nature and there are various kinds of nonlinearities. Among those nonlinear systems, an important class of nonlinear systems is the feedback system whose forward path contains a linear time invariant subsystem and whose feedback path contains a memoryless nonlinearity. Especially, when the nonlinearity satisfies certain sector condition, the nonlinear systems are called Lur'e systems. Stability analysis for the Lur'e systems has been intensively studied, since Lur'e introduced the concept of the absolute stability for the Lur'e systems [1-4]. On the other hands, it is well known that time delays of many communication processes often occur due to the unavoidable signal propagation delay and cause poor performance or even instability [5-6]. In practice, the propagation delay frequently encountered in communication channel. Recently, there are much research results for the stability analysis of Lur'e systems with time varying delays [1-4,7]. However, to the best of authors' knowledge, there are only a few articles in the published literature to study the robust controller design problem of the discrete-time Lur'e system with time-varying delay [8-9], because the stabilization condition becomes the nonconvex feasibility problem.

In this paper, we propose an absolute stabilization condition for discrete-time Lur'e systems with time varying delay. The condition is derived in terms of LMIs by using equality constraints for the nonlinear function. The equality constraints are expressed with convex representation for sector bounds of the nonlinear function. Also, the proposed condition is formulated by construction of a new augmented Lyapunov-Krasovskii's functional and utilization of reciprocally convex approach introduced by [6]. In order to obtain a less conservative delay-dependent stabilization condition, an extended Lyapunov-Krasovskii functionals is considered. The

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synthesis condition is expressed in terms of LMIs so that it can be easily verified the effectiveness of the proposed method by numerical algorithms [10]. Finally, simulation result demonstrates the benefits of the proposed controller design methodology for the sector-bounded systems with time-varying delay.

Notations. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices. For a real matrix X, $X > 0$ and $X < 0$ mean that X is a positive/negative definite symmetric matrix, respectively. I is an identity matrix with appropriate dimension and 0 is a null matrix with appropriate dimension. $\| \cdot \|$ refers to the Euclidean vector norm and the induced matrix norm. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite, (respectively, nonnegative). $diag\{\cdots\}$ denotes the block diagonal matrix.

II. PROBLEM FORMULATION

Consider the following discrete-time systems with time-varying delays

$$
\begin{aligned}
x(k+1) &= Ax(k) + A_d x(k - h(k)) + \\
&\quad Ff(x(k)) + Bu(k), \qquad (1) \\
x(k) &= \psi(k), \qquad \forall\, k = -h_M, -h_M + 1, ..., 0.
\end{aligned}
$$

where $x(k) \in \mathbb{R}^n$ is state vector, A, $A_d \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $F \in \mathbb{R}^{n \times n}$ are constant matrices, $0 \leq h(k) \leq h_M$ is the time delay, $\psi(\cdot)$ is a discrete-time vector valued function for initial values. The nonlinearity of the system $f(\cdot) \in \mathbb{R}^n$ is a memoryless vector valued function with certain sector conditions such as

$$
0 \leq \frac{f_i(x_i(k))}{x_i(k)} \leq a_i, \qquad (2)
$$

where $f_i(\cdot)$ is ith element of $f(\cdot)$, a_i is an upper bound of the sector.
For simplicity, define

$$
\delta_i(x_i(k)) \equiv \frac{f_i(x_i(k))}{x_i(k)}, \qquad (3)
$$

$$
\Delta(k) \equiv diag\{\delta_1(x_1(k)), ..., \delta_n(x_n(k))\}, \qquad (4)
$$

then we have

$$
\Delta(k) \in Co\{\Delta^1, \Delta^2\} \qquad (5)
$$

where $\Delta^1 = diag\{0, .., 0\}$, $\Delta^2 = diag\{a_1, ..., a_n\}$ and Co denotes convex hull.
Thus, the nonlinear function $f(x(k))$ can be expressed as

$$
f(x(k)) = \Delta(k)x(k). \qquad (6)
$$

Here, the goal of this paper is to design a stabilizing control $u(k)$ for (1) with the following structure.

$$u(k|k) = Kx(k), \qquad (7)$$

where K is the control gain.

The following lemma is useful for deriving a stabilization criterion.

Lemma 1: [7] For any constant matrix $M \in \mathbb{R}^{n \times n} > 0$, a scalar $\gamma > 0$ and a vector function $x(k) : \mathbb{R} \to \mathbb{R}^n$ such that the following integration is well defined, then

$$\left(\sum_{l=k-\gamma}^{k-1} x(l) \right)^T M \left(\sum_{l=k-\gamma}^{k-1} x(l) \right) \leq \gamma \sum_{l=k-\gamma}^{k-1} x^T(l)Mx(l). \qquad (8)$$

III. MAIN RESULTS

In this section, we derive a stabilization criterion of the system (1) via LMI formulation.

First of all, define the augmented vectors for simplicity,

$$
\begin{aligned}
y(k) &= x(k+1) - x(k), \\
\zeta(k) &= \big[x^T(k), \ x^T(k-h(k)), \ x^T(k-h_M), \\
&\quad y^T(k), \ y^T(k-h(k)), \ y^T(k-h_M), \ f^T(x(k)) \big]^T,
\end{aligned}
$$

and define the matrices

$$e_i \in R^{7n \times n} (i = 1, ..., 7),$$
$$\text{e.g., } e_2 = \left[\begin{array}{ccccccc} 0_n & I_n & 0_n & 0_n & 0_n & 0_n & 0_n \end{array}\right]^T,$$
$$\Xi = \left[\begin{array}{cccc} AG - G + BH & A_dG & 0 & -G \\ 0 & 0 & FG \end{array}\right.,$$
$$H = KG,$$
$$\hat{\Omega}_1 = [e_1 + e_4, \ e_3 + e_6]\hat{P}[e_1 + e_4, \ e_3 + e_6]^T$$
$$\quad - [e_1, \ e_3]\hat{P}[e_1, \ e_3]^T,$$
$$\hat{\Omega}_2 = [e_1, \ e_3](\hat{Q}_1 + \hat{Q}_2)[e_1, \ e_3]^T$$
$$\quad - [e_3, \ e_6]\hat{Q}_1[e_3, \ e_6]^T - [e_2, \ e_5]\hat{Q}_2[e_2, \ e_5]^T,$$
$$\hat{\Omega}_3 = h_M^2 e_4 \hat{R}_1 e_4^T - (e_1 - e_2)\hat{R}_1(e_1 - e_2)^T,$$
$$\hat{\Omega}_4 = h_M^2 [e_1, \ e_3]\hat{R}_2[e_1, \ e_3]^T$$
$$\quad - h_M e_1 \hat{M} e_1^T - h_M e_3 \hat{N} e_3^T - h_M e_2 (\hat{M} - \hat{N})e_2^T$$
$$\quad - [e_1 - e_2, e_2 - e_3]\left[\begin{array}{cc} \hat{M} & \hat{S} \\ \hat{S}^T & \hat{N} \end{array}\right][e_1 - e_2, \ e_2 - e_3]^T,$$
$$\hat{\Omega}_5 = -2e_7 \Lambda e_7^T + e_7 \Delta^j G e_1^T + e_1 G^T \Delta^j e_7^T,$$
$$\hat{\Omega}_6 = [e_1 + \delta e_4]\Xi + \Xi^T [e_1 + \delta e_4]^T,$$
$$R_{2a} = R_2 + \left[\begin{array}{cc} 0 & M \\ M^T & 0 \end{array}\right], \ R_{2b} = R_2 + \left[\begin{array}{cc} 0 & N \\ N^T & 0 \end{array}\right].$$

Theorem 1: For given $h_M > 0$ and $\delta > 0$, the system Eq. (1) with the state feedback controller Eq. (7) is stable, if there exist positive definite matrices $\hat{P}, \hat{Q}_1, \hat{Q}_2, \hat{R}_1, \hat{R}_2, \Lambda$ and matrices $\hat{M}, \hat{N}, \hat{S}, H$ and G satisfying the following LMIs :

$$\sum_{i=1}^{6} \hat{\Omega}_i < 0, \ j = 1, 2, \qquad (9)$$

$$\left[\begin{array}{cc} M + R_1 & S \\ \star & N + R_1 \end{array}\right] \geq 0, \ R_{2a} > 0, \ R_{2b} > 0. \qquad (10)$$

Proof. Consider the following Lyapunov-Krasovskii functional :

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) \qquad (11)$$

where

$$V_1(k) = \left[\begin{array}{c} x(k) \\ x(k-h_M) \end{array}\right]^T P \left[\begin{array}{c} x(k) \\ x(k-h_M) \end{array}\right],$$

$$V_2(k) = \sum_{s=k-h_M}^{k-1} \left[\begin{array}{c} x(s) \\ y(s) \end{array}\right]^T Q_1 \left[\begin{array}{c} x(s) \\ y(s) \end{array}\right]$$
$$\quad + \sum_{s=k-h(k)}^{k-1} \left[\begin{array}{c} e(s) \\ y(s) \end{array}\right]^T Q_2 \left[\begin{array}{c} e(s) \\ y(s) \end{array}\right],$$

$$V_3(k) = h_M \sum_{s=-h_M}^{-1} \sum_{u=k+s}^{k-1} y^T(u)R_1y(u),$$

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The difference of $V_1(k)$ with respect to time along the trajectory of (1) is

$$\Delta V_1(k) = \zeta^T(k)\Omega_1\zeta(k) \qquad (12)$$

where $\Omega_1 = [e_1 + e_4, \ e_3 + e_6]P[e_1 + e_4, \ e_3 + e_6]^T - [e_1, \ e_3]P[e_1, \ e_3]^T$.

Similarly, difference of $V_2(k)$ and $V_3(k)$ are

$$\Delta V_2 = \zeta^T(k)\Omega_2\zeta(k) \qquad (13)$$

where $\Omega_2 = [e_1, \ e_3](Q_1 + Q_2)[e_1, \ e_3]^T - [e_3, \ e_6]Q_1[e_3, \ e_6]^T - [e_2, \ e_5]Q_2[e_2, \ e_5]^T$

and

$$\Delta V_3(k) = h_M^2 y^T(k)R_1y(k) - h_M \sum_{s=k-h(k)}^{k-1} y^T(s)R_1y(s). \qquad (14)$$

The following inequality for the difference of $V_3(k)$ can be obtained by using Lemma 1 :

$$
\begin{aligned}
\Delta V_3(k) &\leq h_M^2 y^T(k)R_1y(k) \\
&\quad - (x(k) - x(k-h(k)))^T R_1(x(k) - x(k-h(k))) \\
&= \zeta^T(k)\Omega_3\zeta(k)
\end{aligned} \qquad (15)
$$

where $\Omega_3 = h_M^2 e_4 R_1 e_4^T - (e_1 - e_2)R_1(e_1 - e_2)^T$.

The difference of $V_4(k)$ is

$$
\begin{aligned}
\Delta V_4(k) &= h_M^2 \left[\begin{array}{c} x(k) \\ y(k) \end{array}\right]^T R_2 \left[\begin{array}{c} x(k) \\ y(k) \end{array}\right] \\
&\quad - h_M \sum_{s=k-h_M}^{k-1} \left[\begin{array}{c} x(s) \\ y(s) \end{array}\right]^T R_2 \left[\begin{array}{c} x(s) \\ y(s) \end{array}\right] \\
&= h_M^2 \left[\begin{array}{c} x(k) \\ y(k) \end{array}\right]^T R_2 \left[\begin{array}{c} x(k) \\ y(k) \end{array}\right] \\
&\quad - h_M \sum_{s=k-h(k)}^{k-1} \left[\begin{array}{c} x(s) \\ y(s) \end{array}\right]^T R_2 \left[\begin{array}{c} x(s) \\ y(s) \end{array}\right] \\
&\quad - h_M \sum_{s=k-h_M}^{k-h(k)-1} \left[\begin{array}{c} x(s) \\ y(s) \end{array}\right]^T R_2 \left[\begin{array}{c} x(s) \\ y(s) \end{array}\right].
\end{aligned} \qquad (16)
$$

Since the following equalities are always satisfied for any matrices M and N

$$0 = x^T(k)h_M M x(k) - x^T(k-h(k))h_M x(k-h(k))$$
$$- h_M \sum_{s=k-h(k)}^{k-1} \left(y^T(s)My(s) + 2x^T(s)My(s) \right), \qquad (17)$$

$$0 = e^T(k-h(k))h_M N x(k-h(k))$$
$$- x^T(k-h_M)h_M N x(k-h_M)$$
$$- h_M \sum_{s=k-h_M}^{k-h(k)-1} \left(y^T(s)Ny(s) + 2x^T(s)Ny(s) \right), \qquad (18)$$

we have

$$\Delta V_4(k) = h_M^2 \begin{bmatrix} x(k) \\ y(k) \end{bmatrix}^T R_2 \begin{bmatrix} x(k) \\ y(k) \end{bmatrix}$$
$$- h_M \sum_{s=k-h(k)}^{k-1} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}^T R_{2a} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} \qquad (19)$$
$$- h_M \sum_{s=k-h_M}^{k-h(k)-1} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}^T R_{2b} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}$$

$$- h_M \sum_{s=k-h(k)}^{k-1} y^T(s)My(s)$$
$$- h_M \sum_{s=k-h_M}^{k-h(k)-1} y^T(k)Ny(k) - e^T(k)h_M M e(k) \qquad (20)$$
$$- e^T(k-h_M)h_M N e(k-h_M)$$
$$- e^T(k-h(k))h_M(M-N)e(k-h(k))$$

By Schur complement and reciprocally convex approach in [6], the sum terms in (19) is bounded as

$$- h_M \sum_{s=k-h(k)}^{k-1} y^T(s)My(s) - h_M \sum_{s=k-h_M}^{k-h(k)-1} y^T(s)Ny(s)$$
$$\leq -\zeta^T(k) \begin{bmatrix} e_1^T - e_2^T \\ e_2^T - e_3^T \end{bmatrix}^T \begin{bmatrix} \frac{1}{\alpha(k)}M & 0 \\ \star & \frac{1}{1-\alpha(k)}N \end{bmatrix} \begin{bmatrix} e_1^T - e_2^T \\ e_2^T - e_3^T \end{bmatrix} \zeta(k)$$
$$\leq -\zeta^T(k) \begin{bmatrix} e_1^T - e_2^T \\ e_2^T - e_3^T \end{bmatrix}^T \begin{bmatrix} M & S \\ \star & N \end{bmatrix} \begin{bmatrix} e_1^T - e_2^T \\ e_2^T - e_3^T \end{bmatrix} \zeta(k)$$
$$(21)$$

where $\alpha(k) = (h_M - h(k))/h_M$.
For positive definite matrix $R_{2a} > 0$, $R_{2b} > 0$, $\Delta V_4(k)$ is bounded as

$$\Delta V_4(k) \leq \zeta(k)\Omega_4\zeta(k) \qquad (22)$$

where

$$\Omega_4 = h_M^2 [e_1, \ e_3] R_2 [e_1, \ e_3]^T$$
$$- h_M e_1 M e_1^T - h_M e_3 N e_3^T - h_M e_2(M-N)e_2^T \qquad (23)$$
$$- [e_1 - e_2, e_2 - e_3] \begin{bmatrix} M & S \\ S^T & N \end{bmatrix} [e_1 - e_2, \ e_2 - e_3]^T.$$

The following equality is obtained from the Eq. (6) with any matrices Λ^{-1}

$$2f^T(x(k))\Lambda^{-1}$$
$$\begin{bmatrix} \Delta(k) & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} \zeta(k) = 0. \qquad (24)$$

On the other hand, from the system (1) and the Leibniz-Newton formula, the following equation is satisfied with any matrix G^{-1} and a positive scalar δ:

$$2 \begin{bmatrix} x^T(k) & y^T(k) \end{bmatrix} \begin{bmatrix} G^{-1} \\ \delta G^{-1} \end{bmatrix}$$
$$\begin{bmatrix} A-I+BK & A_d & 0 & -I & 0 & 0 & F \end{bmatrix} \zeta(k)$$
$$= \zeta^T(k)(e_1 + \delta e_4)G^{-1}\Xi\zeta(k) = 0. \qquad (25)$$

An upper bound of the difference of $V(k)$ is derived by combining with (24) and (25)

$$\Delta V(k) \leq \zeta^T(k)(\sum_{i=1}^{4} \Omega_i - 2e_7\Lambda^{-1}e_7^T$$
$$+ e_7\Lambda^{-1}\Delta(k)e_1^T + e_1\Delta(k)\Lambda^{-1}e_7^T \qquad (26)$$
$$+ (e_1 + \delta e_4)G^{-1}\Xi G^{-1})\zeta(k), \qquad \forall \ \zeta(k) \neq 0.$$

Let us define

$$\hat{P} = diag\{G,G\}^T P \ diag\{G,G\},$$
$$\hat{Q}_1 = diag\{G,G\}^T Q_1 \ diag\{G,G\},$$
$$\hat{Q}_2 = diag\{G,G\}^T Q_2 \ diag\{G,G\},$$
$$\hat{R}_1 = G^T R_1 G,$$
$$\hat{R}_2 = diag\{G,G\}^T R_2 \ diag\{G,G\},$$
$$\hat{M} = G^T M G, \quad \hat{N} = G^T N G,$$
$$\hat{S} = G^T S G, H = KG,$$

then pre and post multiplying the matrix $diag\{G, \ G, \ G, \ G, \ G, \ G, \Lambda\}^T$ and $diag\{G, \ G, \ G, \ G, \ G, \ G, \Lambda\}$ in Eq. (26) leads to LMI (9). This completes the proof. ∎

Remark 1. In the stabilization problem for systems with time delay, one of the important index for checking the enhancement of the feasible region is to get maximum delay bounds which guarantees the stability of the closed-loop system. In following numerical example, we will verify the usefulness of proposed stabilization criterion by finding a solution for the feasibility problem of Theorem 1.

IV. NUMERICAL EXAMPLE

In this section, to illustrate the effectiveness of the proposed criterion given in Theorem 1, consider the following a discrete-time sector bounded system (1) with the following matrices.

$$A = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$F = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix}. \qquad (27)$$

Let the parameters be set as

$$\Delta^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ h_M = 26, \ \delta = 11.9, \qquad (28)$$

then by Theorem 1, we obtain

$$P = \begin{bmatrix} 7.4437 & 1.7704 & 0.1413 & -0.0412 \\ 1.7704 & 7.4437 & -0.0412 & 0.1413 \\ 0.1413 & -0.0412 & 0.5078 & 0.0893 \\ -0.0412 & 0.1413 & 0.0893 & 0.5078 \end{bmatrix},$$

$$Q1 = \begin{bmatrix} 0.4416 & 0.1847 & 0.4417 & 0.1024 \\ 0.1847 & 0.4416 & 0.1024 & 0.4417 \\ 0.4417 & 0.1024 & 2.0229 & -0.1302 \\ 0.1024 & 0.4417 & -0.1302 & 2.0229 \end{bmatrix},$$

$$Q2 = \begin{bmatrix} 0.4610 & 0.0590 & 0.0334 & 0.0220 \\ 0.0590 & 0.4610 & 0.0220 & 0.0334 \\ 0.0334 & 0.0220 & 0.8621 & -0.0247 \\ 0.0220 & 0.0334 & -0.0247 & 0.8621 \end{bmatrix},$$

$$R1 = \begin{bmatrix} 0.0177 & -0.0069 \\ -0.0069 & 0.0177 \end{bmatrix},$$

$$R2 = \begin{bmatrix} 0.2853 & 0.0972 & 0.0171 & -0.0058 \\ 0.0972 & 0.2853 & -0.0058 & 0.0171 \\ 0.0171 & -0.0058 & 0.5110 & 0.2042 \\ -0.0058 & 0.0171 & 0.2042 & 0.5110 \end{bmatrix},$$

$$K = \begin{bmatrix} 2.3555 & 2.3555 \end{bmatrix}$$

Through the example, it is easy to verify that the proposed method provide a feasible solution for the system (27). This means that the state feedback controller makes the discrete-time sector bounded systems with time-varying delay stable.

V. CONCLUSION

In this paper, we consider the problem of robust controller design for a discrete-time delay systems with sector bounded nonlinearity. A delay-dependent stabilization criterion has been proposed in terms of LMIs by using new Lyapunov-Krasovskii function. We have demonstrated the effectiveness of the proposed method via a numerical example.

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