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MHD Falkner-Skan Boundary Layer Flow with Internal Heat Generation or Absorption

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Abstract**—**This paper examines the forced convection flow of incompressible, electrically conducting viscous fluid past a sharp wedge in the presence of heat generation or absorption with an applied magnetic field. The system of partial differential equations governing Falkner - Skan wedge flow and heat transfer is first transformed into a system of ordinary differential equations using similarity transformations which is later solved using an implicit finite - difference scheme, along with quasilinearization technique. Numerical computations are performed for air (Pr = 0.7) and displayed graphically to illustrate the influence of pertinent physical parameters on local skin friction and heat transfer coefficients and, also on, velocity and temperature fields. It is observed that the magnetic field increases both the coefficients of skin friction and heat transfer. The effect of heat generation or absorption is found to be very significant on heat transfer, but its effect on the skin friction is negligible. Indeed, the occurrence of overshoot is noticed in the temperature profiles during heat generation process, causing the reversal in the direction of heat transfer.

Keywords**—**Heat generation / absorption, MHD Falkner- Skan flow, skin friction and heat transfer

I. Introduction

HISTORICALLY, the steady laminar flow over a wedge was first analyzed by Falkner and Skan [1] to illustrate the application of Prandtl's boundary layer theory. Hartree [2] later investigated the same problem with similarity transformation and gave numerical results for wall shear stress for different values of the wedge angle. Eckert [3] solved Falkner-Skan flow along an isothermal wedge and gave the first wall heat transfer values. Stewartson [4] made an attempt to establish further solutions of Falkner-Skan equation. Thereafter, many solutions have been obtained for different aspects of this class of boundary layer problems [5-8]. In recent years, magneto hydrodynamic (MHD) flows as boundary layer problems have great importance from a mathematical as well as from a physical stand point. Vajravelu and Nayfeh [9] examined the hydromagnetic convection at a cone and a wedge. Kafoussias and Nanousis [10] have considered MHD flow over a wedge with suction and injection. Recently, Abbasbandy and Hayat [11] have obtained analytic solutions for the magnetohydrodynamic Falkner-Skan flow. The objective of this paper is to investigate the steady, laminar boundary layer flow of an electrically conducting viscous fluid over a wedge in the presence of a transverse magnetic field, with internal heat generation / absorption.

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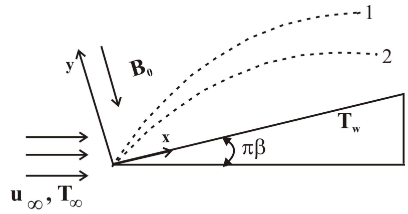

Fig. 1 Physical model and co-ordinate system for MHD Falkner-Skan wedge flow, where 1 and 2 represent edge of thermal and momentum boundary layers, respectively

II. Mathematical Analysis

The physical configuration of present investigation (See Fig.1) consists of a cartesian coordinate system where is measured along the surface of the wedge and is normal to it. We consider the steady, two-dimensional laminar boundary layer flow of an incompressible, electrically conducting fluid flowing towards the wedge with velocity. The temperature of the wall is uniform and constant and is greater than the free stream temperature. A transverse magnetic field of strength is applied in direction normal to the wedge surface and it is assumed that the magnetic Reynolds number is small, so that the induced magnetic field can be neglected. The fluid is assumed to have constant physical properties. Under the above assumptions, the boundary layer equations governing the forced convection steady, MHD Falkner-Skan wedge flow are given by: by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0{}^2}{\rho}(u-U) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty) \tag{3}$$

where u and v are, respectively, velocity components along x and y - directions; U is the inviscid flow velocity at the edge of the boundary layer and, is a function of x; α is the thermal diffusivity; T is the fluid temperature; μ, ν, σ, ρ are, respectively, dynamic viscosity, kinematic viscosity, electrical conductivity and density of the fluid; C_p is the specific heat and Q_0 is the heat generation/absorption coefficient.

The boundary conditions are given by

$$\begin{aligned}
&at\;\; y=0: && u=v=0 \;\; and \;\; T=T_w\\
&as\; y\to\infty: && u\to U(x)=u_\infty(x/L)^m \;\; and \;\; T=T_\infty\\
&at\;\; x=0: && u=u_\infty \;\; and \;\; T=T_\infty
\end{aligned} \tag{4}$$

where L is the length of the wedge, m is the Falkner – Skan parameter and x is measured from the tip of the wedge. The subscripts w and ∞ denotes conditions at wall and infinity respectively.

Introducing the following transformations

$$u=\frac{\partial\psi}{\partial y};\ v=-\frac{\partial\psi}{\partial x};\ f(\eta)=\sqrt{\frac{1+m}{2}\frac{L^m}{\upsilon u_\infty}}\ \left(\frac{\psi}{x^{(1+m)/2}}\right)$$
$$\eta=\sqrt{\frac{1+m}{2}\frac{u_\infty}{\nu L^m}}\ \left(\frac{y}{x^{(1-m)/2}}\right);\ G(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

to (1) - (3), we find that continuity (1) is identically satisfied and (2) and (3) are, respectively transformed to:

$$F''+fF'+\left(\frac{2m}{1+m}\right)\left(1-F^2\right)+M\left(1-F\right)=0 \tag{6}$$

$$\Pr^{-1}G''+f\,G'+\left(\frac{2}{1+m}\right)Q\,G=0 \tag{7}$$

where

$$v=-\sqrt{\frac{2}{1+m}\frac{\nu u_\infty}{L^m}}\,x^{(m-1)/2}\left\{\left(\frac{1+m}{2}\right)f+\eta\left(\frac{m-1}{2}\right)f'\right\};$$
$$\frac{u}{U}=f'=F;\ \ \nu=\frac{\mu}{\mu_\infty};\qquad \Pr=\frac{\nu}{\alpha};\ \mathrm{Re}_L=\frac{u_\infty L}{\nu};$$
$$M=\frac{2L\sigma B_0^{\,2}}{\rho u_\infty(m+1)};\ \ Q=\frac{Q_0 L}{\rho C_p u_\infty} \tag{8}$$

The transformed boundary conditions are:

$$F=0;\quad G=1\quad at\quad \eta=0$$
$$F=1;\quad G=0\quad as\quad \eta\to\infty \tag{9}$$

Here, ψ and f are dimensional and dimensionless stream functions, respectively; F and G are, respectively, dimensionless velocity and temperature of the fluid; M is dimensionless magnetic parameter; Re_L is the local Reynolds number, $\Pr$ is the Prandtl number; η is the transformed coordinate; Q is the heat generation or absorption parameter. Here prime $(')$ denotes derivative with respect to η.

We note that in (6) and (7), the parameter m is connected with the apex angle $\pi\beta$ by the relation $m=\beta/(2-\beta)$ or $\beta=2m/(m+1)$. Further, the numerical computations have been carried out for entire range of realistic flow i.e., for the range $0\le m\le 0.2$ (corresponding to wedge angle ranging from 0^o to 60^o). The skin friction and heat transfer coefficient in the form of Nusselt number, can be expressed, respectively as

$$C_f(\mathrm{Re}_L)^{1/2}=\frac{\tau_w}{\frac{1}{2}\rho U^2}=2\sqrt{\frac{1+m}{2}}\ (f'')_{\eta=0}$$
$$Nu(\mathrm{Re}_L)^{-1/2}=-k\frac{\left(\partial T/\partial y\right)_{y=0}}{(T_w-T_\infty)}=-\sqrt{\frac{1+m}{2}}\,(G')_{\eta=0} \tag{10}$$

where the wall shear stress τ_w is given by $\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ with μ and k are, respectively, dynamic viscosity and thermal conductivity and $\mathrm{Re}_L=\frac{u_\infty L}{\nu}$, the local Reynolds number.

III. Method of Solution

The system of nonlinear ordinary differential equations (6) and (7) under the boundary conditions (9) have been solved numerically by using a highly efficient implicit finite-difference scheme in conjunction with quasilinearization technique. Quasilinearisation technique can be viewed as a generalization of the Newton-Raphson approximation technique in functional space. An iterative sequence of linear equations is carefully constructed to approximate the non-linear (6) and (7) under boundary conditions (9) for achieving quadratic-convergence and monotonicity. This method, developed originally for ordinary differential equations by Inouye and Tate [12], has been successfully applied in a wide variety of thermo-fluid dynamic problems by many researchers. Following [12], we replace the non-linear ordinary differential equations (6) and (7) to the following sequence of linear ordinary differential equations:

$$F''^{(k+1)}+X_1^{(k)}F'^{(k+1)}+X_2^{(k)}F^{(k+1)}=U_1^{(k)} \tag{11}$$

$$G''^{(k+1)}+Y_1^{(k)}G'^{(k+1)}+Y_2^{(k)}G^{(k+1)}=U_2^{(k)} \tag{12}$$

where the coefficient functions with iterative index k are known and functions with iterative index $k+1$ are to be determined. The boundary conditions become

$$F^{(k+1)}=0,\quad G^{(k+1)}=1\quad \text{at}\quad \eta=0$$
$$F^{(k+1)}=1,\quad G^{(k+1)}=0\quad \text{at}\quad \eta=\eta_\infty \tag{13}$$

The coefficients in (11) and (12) are given by

$$X_1^{(k)}=f$$

$$X_2^{(k)}=-2\left(\frac{2m}{1+m}\right)F-M$$

$$U_1^{(k)}=-\left(\frac{2m}{1+m}\right)(1+F^2)-M$$

$$Y_1^{(k)}=\Pr f$$

$$Y_2^{(k)}=\left(\frac{2}{1+m}\right)Q\Pr$$

The equations (11) and (12) along with boundary conditions (13) were expressed in difference form, considering central difference scheme in $\eta-$direction. In each iteration step, equations were then reduced to a system of linear algebraic equations with a block tri-diagonal structure which is later solved using [13]. To ensure the convergence of the numerical solution to the exact solution, step size $\Delta\eta$ is optimized and taken as 0.01. The results presented here are independent of the step size in $\eta-$direction atleast up to the four decimal place. The value of η_∞ (i.e., the edge of the boundary layer) has been taken as 5.0 throughout the computation. Iteration is employed to deal with the nonlinear nature of the governing equations to become linear, locally. A convergence criterion based on the relative difference between the current and the previous iteration values of the velocity and temperature gradients at wall are employed. The solution is assumed to have converged and the iterative process is terminated when $Max\left[|(F'_w)^{(k+1)}-(F'_w)^{(k)}|,\ \ |(G'_w)^{(k+1)}-(G'_w)^{(k)}|\right]<10^{-4}$

IV. Results And Discussion

The computations have been carried out for various values of governing parameters viz., magnetic parameter (M) and heat generation or absorption parameter (Q), for air with $\Pr = 0.7$, that is appropriate for Helium (400^o F), Hydrogen (near about 370^o F) and Oxygen (near about 10^o F). With a view to validate the accuracy of the numerical method used, we have compared our stream function $f(\eta)$ and velocity field $F(\eta)$, with those of White [14] when $m = 0.0$ [See TABLE I]. Further, the skin friction (F'_w) and heat transfer (G'_w) parameters have also been compared with those of Watanabe [8] for the range of m, $0 \le m \le 1.0$. [See TABLE II]. Our results in the absence of magnetic parameter (i.e., $M = 0.0$) and heat generation or absorption parameter ($Q = 0.0$), are found to be in excellent agreement with these studies [8, 14].

Fig. 2 shows the effect of magnetic field (M) on skin friction [$C_f(\text{Re}_L)^{1/2}$] and heat transfer [$Nu(\text{Re}_L)^{-1/2}$] coefficients without heat generation or absorption parameter $(Q = 0)$. It is found that magnetic field (M) increases both $C_f(\text{Re}_L)^{1/2}$ and $Nu(\text{Re}_L)^{-1/2}$. This is because when M increases, Lorentz force produces more resistance to the transport phenomena which leads to the deceleration of the flow, enhancing the surface shear stress and rate of heat transfer at the wall. However, it is observed both $C_f(\text{Re}_L)^{1/2}$ and $Nu(\text{Re}_L)^{-1/2}$ monotonically increases with the increase of wedge angle parameter (m).

TABLE I
Comparison Of Numerical Results For The Case Of $m = 0.0$ When $M = 0.0$ And $Q = 0.0$ With Those Of White [14]

	$f(\eta)$		$F(\eta)$	
	Present	White[14]	Present	White[14]
0.0	0.0000	0.00000	0.0000	0.00000
0.2	0.0094	0.00939	0.0940	0.09391
0.4	0.0376	0.03755	0.1876	0.18761
0.6	0.0846	0.08439	0.2806	0.28058
0.8	0.1500	0.14967	0.3720	0.37196
1.0	0.2335	0.23299	0.4606	0.46063
2.0	0.8882	0.88680	0.8170	0.81669
3.0	1.7978	1.79557	0.9690	0.96905
4.0	2.7864	2.78388	0.9978	0.99777
5.0	3.7833	3.78323	1.0000	0.99994

TABLE II
Comparison Of Numerical Results For The Range Of m ($0 \le m \le 1.0$) When $M = 0.0$ And $Q = 0.0$ With Those of Watanabe [8]

	F'_w		G'_w	
m	Present	Watanabe[8]	Present	Watanabe[8]
0.0	0.4696	0.46960	0.4151	0.41512
0.0141	0.5046	0.50461	0.4205	0.42051
0.0425	0.5690	0.56898	0.4299	0.42984
0.0909	0.6550	0.65498	0.4413	0.44125
0.1429	0.7320	0.73200	0.4504	0.45042
0.2	0.8021	0.80213	0.4583	0.45826
0.3333	0.9277	0.92765	0.4708	0.47083
1.0	1.2326	1.23258	0.4957	0.49571

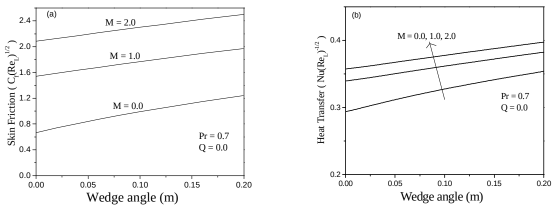

Fig. 2 Effect of magnetic field on (a) skin friction and (b) heat transfer coefficients

The influence of heat generation or absorption parameter ($Q > 0 / Q < 0$) on heat transfer coefficient [$Nu(\text{Re}_L)^{-1/2}$] in the presence of the magnetic field ($M = 2.0$) is displayed in Fig.3. It is observed that $Nu(\text{Re}_L)^{-1/2}$ decreases with the increase of Q $(0 \le Q \le 0.5)$ at $m = 0.0$ irrespective of heat generation or absorption. On the other hand, there is a mild increase in $Nu(\text{Re}_L)^{-1/2}$ during both heat generation $(Q > 0)$ and heat absorption $(Q < 0)$ as wedge angle parameter (m) increases from $m = 0.0$ to $m = 0.2$. Indeed, the percentage of decrease of $Nu(\text{Re}_L)^{-1/2}$ when Q increases from $Q = 0.0$ to $Q = 0.5$ at $m = 0.1$ is 57.38% while the percentage of increase of $Nu(\text{Re}_L)^{-1/2}$ when Q decreases from $Q = 0.0$ to $Q = -0.5$ at $m = 0.1$ is 29.23% [Fig.3 (a) & 3 (b)]. Further, it is found that the direction of heat transfer gets reversed when $Q = 0.5$ [Fig.3 (a)]. This is attributed to the fact that heat generation mechanism creates a layer of hot fluid near the surface and finally resultant temperature of the fluid exceeds the surface temperature resulting in the decrease of rate of heat transfer from the surface. The heat generation or absorption parameter doesn't cause any significant effect on skin friction coefficient [$C_f(\text{Re}_L)^{1/2}$] and hence it is not shown here.

Fig. 4 depicts the effect of heat generation or absorption parameter on temperature profile in the presence of magnetic field ($M = 2.0$) when $m = 1/7$, corresponding to the wedge angle 45^o. It is clearly observed that heat generation $(Q > 0)$ causes over shoot in the temperature profiles exposing the reversal in the direction of heat transfer [Fig.4 (a)] due to increase in the temperature of the fluid. Also, the thermal boundary layer thickness is increased in the presence of both heat generation and absorption [Fig.4 (b)]. Further, it is evident from these figures that the present numerical results

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confirm to satisfy the thermal boundary layer conditions. The velocity profiles are unaffected by heat generation or absorption parameter and hence they are not shown here, for the sake of brevity

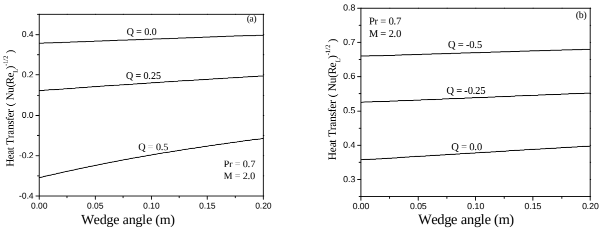

Fig. 3 Effect of (a) heat generation and (b) heat absorption parameter (Q) on heat transfer coefficients

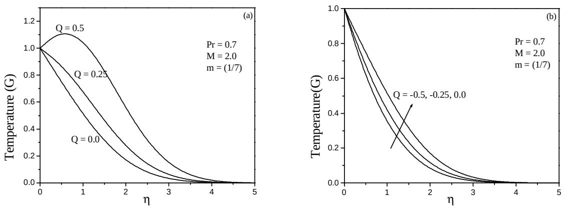

Fig. 4 Temperature profile for (a) heat generation $(Q > 0)$ and (b) heat absorption $(Q < 0)$ parameter at the wedge angle $m = 1/7$

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