Modeling of Heat and Mass Transfer in Soil Plant-Atmosphere. Influence of the Spatial Variability of Soil Hydrodynamic

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Abstract—The modeling of water transfer in the unsaturated zone uses techniques and methods of the soil physics to solve the Richards's equation. However, there is a disaccord between the size of the measurements provided by the soil physics and the size of the fields of hydrological modeling problem, to which is added the strong spatial variability of soil hydraulic properties. The objective of this work was to develop a methodology to estimate the hydrodynamic parameters for modeling water transfers at different hydrological scales in the soil-plant atmosphere systems.

Keywords—Hydraulic properties, Modeling, Unsaturated zone, Transfer, Water

I. Introduction

THE basic equation in the theory of groundwater flow through unsaturated porous media is Richards' equation which was suggested in 1931 [1]. This nonlinear convection-diffusion equation can be written as a conservation law for the water content, the quantity of water contained in a given soil volume. The convection term is due to gravity while the diffusive term comes from Darcy's law. In numerical modeling of water flow and solute transport in unsaturated porous media a simple analytical function is desirable and often necessary for representing the water retention curve, the relation between water content and matrix suction. Usually, a mathematical function is chosen and its parameter values are determined by a regression Analysis on the available data [Bruce and Luxmoore, 1986]. Various functions that describe the water retention curve are in use [e.g., Brooks and Core), 1966; van Genuchten, 1980].Generally, they are successful at high and medium water contents but often give poor results at water contents [Nimmo, 1991; Ross et al., 1991]. This may pose little difficulty for some applications, such as wetlands studies or humid region agriculture, but others, including water flow in arid region, require a more accurate representation of the hydraulic characteristics over the whole range of saturation. The study report the evaluation in a one-dimensional hydrodynamic model of the effect of spatially varied hydraulic properties (water content, capillary capacity and hydraulic conductivity) for modeling water transfer at different hydrological scales in the soil-plant atmosphere systems.

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The model of Van Genuchten (van Genuchten, 1980) was chosen to describe the soil hydrodynamic properties. The model is written in a Fortran program.

II. Theory And Model For Water Transfert In Soil

A. Basic Principles for Soil Water Transfer

Darcy's law describes the flow of an incompressible fluid in a homogeneous, rigid and isotropic porous medium. So, combining Darcy's law and the conservation "(1)" leads to soil water distribution in space and time given by the solution of Richard's equation [1]:

$$\frac{\partial \theta}{\partial t} = \nabla q \qquad (1)$$

where t is time (s), θ the volumetric water content ($m^3 \cdot m^{-3}$), q the water flux density ($m \cdot s^{-1}$) and the vector differential operator in space. Richard's equation can be written in general terms as:

$$\frac{\partial \theta}{\partial t} = \nabla . [K.(\nabla H)] \qquad (2)$$

H(m) is the total soil-water pressure head equal to (h - z), h(m) being the matrix soil-water pressure head and z (m) the depth. K is hydraulic water conductivity ($m \cdot s^{-1}$), which depends greatly on the matrix soil-water potential.

For reasons of simplification, contributions of heads to q, such as those due to solute concentration and coupling with temperature, are assumed to be negligible for this work. So, in one dimension, along a vertical axis z, Richard's equation expressed in a "-based form" [2] makes it possible to examine water potential dynamic:

$$C(\theta)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left(K(\theta)\left(\frac{\partial h}{\partial z} - 1\right)\right) \qquad (3)$$

Where $C(\theta) = \dfrac{\partial \theta}{\partial h}$ is soil-water capacity (m^{-1})

III. Heat Transfert In The Soil

The soil is prone to variations in temperature due to modifications of intensity short wave of the solar radiation and the long wave of atmospheric radiation.The ground temperature present continual variations, under the influence of the climatic conditions, which determine the intensity of the energy exchanges between the ground and the atmosphere. In the absence of internal phase changes, the soil heat transfer is assumed to obey the following Fourier law of diffusion:

$$q = -\lambda \frac{\partial T}{\partial z} \qquad (4)$$

And equation of continuity;

$$\frac{\partial q}{\partial z} = \rho c \frac{\partial T}{\partial z} \qquad (5)$$

Combining of the two preceding equations; "(4)" and "(5)" if thermal conductivity is constant, gives the equation of heat;

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c} \frac{\partial^2 T}{\partial z^2} = a \frac{\partial^2 T}{\partial z^2} \qquad (6)$$

Heat transfer in soils can be calculated if it is considered as a periodic phenomenon in an isotropic soil. A quantitative description of the heat flow and temperature is based on solutions of the Fourier equation of heat conduction with appropriate initial and boundary conditions. In this equation two independent thermal properties appear: the thermal conductivity (λ) and the volumetric heat capacity (C) of the soil. The ratio of these quantities ($\frac{\lambda}{\rho.c}$) is called the thermal diffusivity (a).

$$T(z,t) = \overline{T} + A_0 e^{-\frac{z}{d}} \sin\left(\omega t - \frac{z}{d}\right) \qquad (7)$$

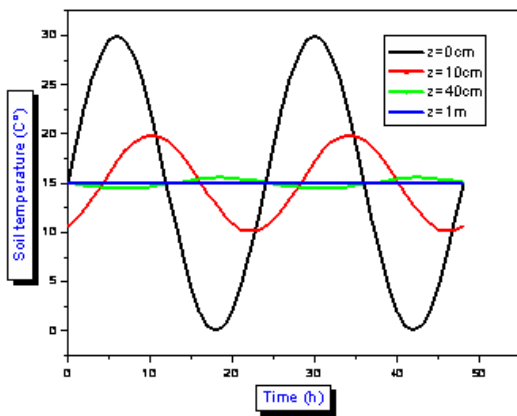With: $d = \sqrt{\frac{2a}{\omega}}$; $\omega = \frac{2\pi}{T_P}$

Fig. 1 Variation of the soil temperature to the surface of soil and to different depths in the soil

The Tp period of a wave is connected to its frequency ω by Tp = $2\pi/\omega$. In our case the thermal wave of surface posed two periods and amplitude of 15 C°. The phase shift between the temperature on the surface and that in-depth is easily remarkable by observing the time for which the temperature is maximal. In the same way, it is obvious that the waves in-depth are subjected to less variation in temperature than on the surface. The waves in-depth are thus attenuated compared to that on the surface from where the effect of the temperature on the surface of the ground does not have any more an influence beyond 40cm. It is it should be noted that with the time the damping and the phase shift of the thermal waves depend the depth and on the thermal diffusivity. In the same way, it is obvious that the waves in depth are submitted to least variations in temperature that to the surface.

IV. SOIL WATER RETENTION MODEL

Water retention curve is the relationship between the water content, θ, and the soil water potential, h. Several functions have been proposed to empirically describe the soil water retention curve. It is used to predict the soil water storage, water supply to the plants (field capacity) and soil aggregate stability. Due to the hysteretic effect of water filling and draining the pores, different wetting and drying curves may be distinguished. One of the most popular functions has been the equation of *van-Genuchten* [1980], further referred to as the VG-equation:

$$\theta(h) = \theta_r - \frac{\theta_s - \theta_r}{\left[\left(1 + (\alpha.|h|)\right)^n\right]^m} \qquad (8)$$

where θ_r and θ_s, are the residual and saturated water contents, respectively; α is an empirical parameter (L^{-1}) whose inverse is often referred to as the air entry value or bubbling pressure; α is related to the inverse of the air entry suction, α ($[L^{-1}]$, or cm^{-1}) and n is a measure of the pore-size distribution, n>1 (dimensionless) and $m = 1 - \frac{1}{n}$. For notational convenience, **h** and a for the remainder of this report are taken positive for unsaturated soils (i.e.,**h** denotes suction). The residual water content, θ_r in (8) specifies the maximum amount of water in a soil that will not contribute to liquid flow because of blockage from the flow paths or strong adsorption onto the solid phase [Luckner et al., 1989]. Formally, θ_r, may be defined as the water content at which both ($\frac{d\theta}{dh}$) and K go to zero when **h** becomes large. The residual water content is an extrapolated parameter, and hence may not necessarily represent the smallest possible water content in a soil.

V. SOIL WATER CAPACITY FUNCTION

To solve the unsaturated water flow or Richards equation, a priori knowledge of the water capacity function, *C*, is required. For any soil water retention model, the *C* function can be computed from the slope of the water retention curve:

$$C(h) = \frac{d\theta}{dh} = \frac{d}{dh}\left[\theta_r - \frac{\theta_s - \theta_r}{\left[\left(1 + (\alpha.|h|)\right)^n\right]^m}\right] \qquad (9)$$

VI. HYDRAULIC CONDUCTIVITY

Hydraulic conductivity, symbolically represented as K, is a property of vascular plants, soil or rock, which describes the ease with which water can move through pore spaces or fractures. It depends on the intrinsic permeability of the material and on the degree of saturation. Saturated hydraulic conductivity, Ksat, describes water movement through saturated media.

There are two broad categories of determining hydraulic conductivity:
• Empirical approach by which the hydraulic conductivity is correlated to soil properties like pore size and particle size (grain size) distributions, and soil texture.
• Experimental approach by which the hydraulic conductivity is determined from hydraulic experiments using Darcy's law.

$$K(h) = K_S \frac{\left[1 - (\alpha h)^{n-1}\left[1 + (\alpha h)^n\right]^{-m}\right]^2}{\left[1 + (\alpha h)^n\right]^{\frac{m}{2}}} \quad (10)$$

Ks [L T^{-1}] is the saturated hydraulic conductivity, K [L T^{-1}] is the unsaturated hydraulic conductivity as a function of pressure head, and α [L^{-1}] is a constant.

VII. RESULTS AND DISCUSSION OF THE CURVES

The relationship between water content and soil water potential (capillary tension) is described in the water retention curve. The curve is characteristic for two types of soils and is also called soil moisture characteristic. It also depends on the geometry and network of the pores. Fine-grained soils show high residual water contents and high changes in capillary tension are necessary that they release the water. The retention curve shows a hysteresis which means that depending on the history of the soil with regard to watering and drainage, the shape of the curve is different. The general features of a water retention curve can be seen in the fig 2, in which the volume water content, θ, is plotted against the matrix potential, h. At potentials close to zero, a soil is close to saturation, and water is held in the soil primarily by capillary forces. As θ decreases, binding of the water becomes stronger, and at small potentials (more negative, approaching wilting) water is strongly bound in the smallest of pores, at contact points between grains and as films bound by adsorptive forces around particles. The graph in fig 3 shows the soil water retention curve for two typical soil types: sand and clay. The graph indicates that as the soil water content decreases, the amount of pressure needed to extract remaining water from the soil matrix increases. We observe that sandy soils will involve mainly capillary binding, and will therefore release most of the water at higher potentials, while clayey soil, with adhesive and osmotic binding, will release water at lower (more negative) potentials. The water holding capacity of any soil is due to the porosity and the nature of the bonding in the soil. The fig 4 shows variation of soil water capacity for the sandy soil.
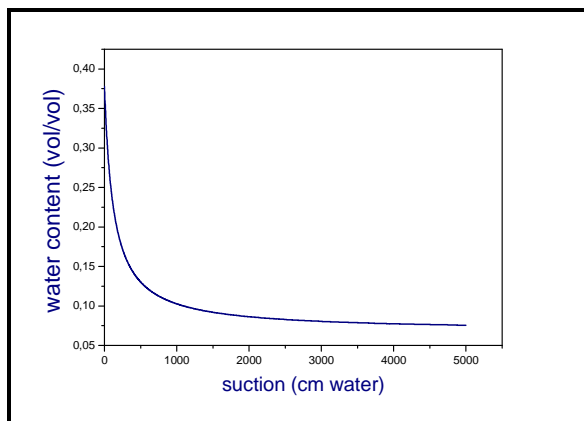

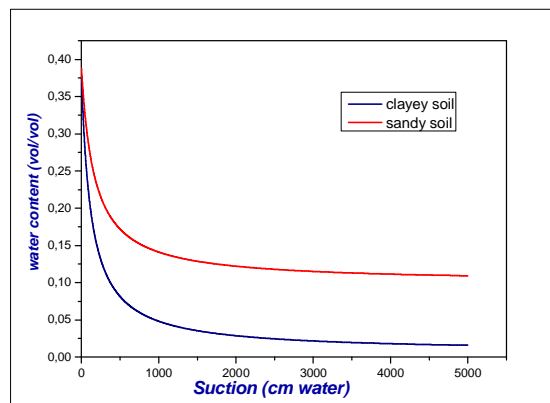Fig. 2 water content as a function soil water tension for sandy soil

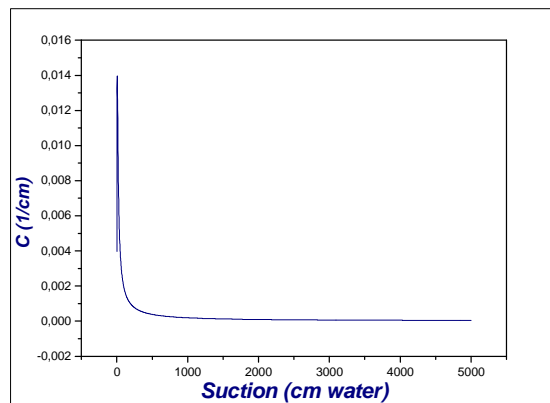Fig. 3 Water content as a function soil water tension for two soils

Fig. 4 soil water capacity as a function soil water tension for the sandy soil

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