Time-Derivative Estimation of Noisy Movie Data using Adaptive Control Theory

Soon-Hyun Park, and Takami Matsuo

Abstract— This paper presents an adaptive differentiator of sequential data based on the adaptive control theory. The algorithm is applied to detect moving objects by estimating a temporal gradient of sequential data at a specified pixel. We adopt two nonlinear intensity functions to reduce the influence of noises. The derivatives of the nonlinear intensity functions are estimated by an adaptive observer with σ-modification update law.

Keywords— Adaptive estimation, parameter adjustment law, motion detection, temporal gradient, differential filter.

I. INTRODUCTION

THE necessity to evaluate the time-derivative of signals arises frequently in many areas of research. In tracking or detecting moving objects in an image sequence, the need of velocity estimation from measured position data is still a difficult task and a challenging problem. The motion detection can be performed by calculating spatio-temporal gradient. A large set of motion detection algorithms has already been proposed in the literature. The first one is based on temporal gradient, *i.e.* the time-derivative: a motion likelihood index is measured by the instantaneous change in the image intensity computed by differentiation of consecutive frames. The second one is the background subtraction techniques. In the absence of any *a priori* knowledge about target and environment, the most widely adopted approach for moving object detection with fixed camera is based on the background subtraction. The principle of these methods is to build a model of the static scene (*i.e.* without moving objects) called background, and then to compare every frame of the sequence to this background in order to discriminate the regions of unusual motion, called foreground (the moving objects). Many algorithms

Manuscript received August 29, 2005.

S.-H. Park and T. Matsuo are with the Department of Human Wel. Engineering, Oita University, Dannoharu, Oita 870-1192, Japan (phone: +81-97-554-7804; fax: +81-97-554-7507; e-mail: matsuo@cc.oita-u.ac.jp).

have been developed for the background subtraction. The most important steps in the background subtraction algorithm are the background modeling and the foreground detection. In particular, background modeling is at the heart of any background subtraction algorithm. Much research has been devoted to developing a background model that is an estimate of background. The background model is computed by the difference between the current frame and the current background model[1]. The background model should be robust against environmental changes in the background, but sensitive enough to identify all moving objects of interest. A significant number of the described algorithms use a simple IIR filter applied to each pixel independently to update the background and use thresholding to classify pixels into foreground/background[4], [7]. Kalman filter is a widely used recursive technique for tracking linear dynamical systems under Gaussian noise. The dynamical systems such as the motion vector or intensity are assumed to be Markov models[8], [9]. These Markov models do not have physical meanings. Therefore, the estimator of the time-derivative for a signal, *i.e.* the motion vector, is required to be robust against disturbances without any knowledge of its dynamics. The third type of approach is based on the computation of the local apparent velocity (optical flow) that is used as input of a spatial segmentation. This method is in general more computationally complex and it is sensitive to the reliability of the optical flow. The fourth method is based on morphological filters. By using spatio-temporal structuring elements, local amplitude of variation can be computed as motion likelihood index. Such measure can be useful to detect small amplitude motion, but it is sensitive to outliers[11]. In particular, Richefeu *et al.*[11] presented a new differential operator based on a hybrid filter, combining morphological and linear operations. It computes a pixel-wise amplitude of time-variation over a recursively defined "temporal

window". It is particularly suited to small and low amplitude motion.

Recently, Ibrir have presented a time-varying linear system to estimate the first $(n-1)$-th derivative of any bounded signal[5], [6]. The time-derivative observer was formulated as a high-gain observer where the observer gain was calculated through a Lyapunov-like dynamical equation[6]. This paper presents a new differential filter of a nonlinear function based on the adaptive control theory[12], [13]. The proposed differential filter is applied to estimate the time-derivative of a nonlinear function of the intensity (nonlinear intensity) for each pixel to detect moving objects within a scene acquired by a stationary camera. The estimated variety of the nonlinear intensity is obtained by using the adaptive parameter adjustment law[12], [13]. We select two nonlinear functions, a quadratic function and logarithmic function, in order to attenuate the variety of disturbances with small amplitudes, because the filter should have certain noise immunity such as measurement noises, waving bushes or flowing water. As our differential filter recursively updates the estimate of variety of nonlinear intensity, its computational cost is very low. The proposed method allows us to attenuate the influence of noises in estimating the time-derivative of sequential data. The MATLAB simulations are performed to demonstrate its performance.

II. DIFFERENTIAL FILTER OF NONLINEAR INTENSITY FUNCTIONS BASED ON ADAPTIVE PARAMETER UPDATE LAW

Let $x(t)$ be an intensity of a specified pixel at a time t and $f(x(t))$ be its nonlinear function. We define $l(t)$ as the derivative with respect to time of the nonlinear function $f(x(t))$, *i.e.*

$$l(t) = \frac{df(x(t))}{dt} = \frac{df(x)}{dx}\frac{dx(t)}{dt}. \tag{1}$$

If $\frac{df(x)}{dx}$ has the inverse $\left(\frac{df(x)}{dx}\right)^{-1}$, we have

$$\frac{dx(t)}{dt} = \xi(t)l(t) \tag{2}$$

where

$$\xi(t) = \left(\frac{df(x)}{dx}\right)^{-1}. \tag{3}$$

We assume that the function $l(t)$ can be expressed in a polynomial form:

$$l(t) = \sum_{k=0}^{N} l_k t^k \tag{4}$$

where $l_k, k = 0, \cdots, N$ are constant. Equation (2) is rewritten by

$$\frac{dx(t)}{dt} = l_v^T \zeta(t) \tag{5}$$

where

$$l_v = \begin{bmatrix} l_0 \\ l_1 \\ \vdots \\ l_N \end{bmatrix}, \quad \zeta = \begin{bmatrix} \xi(t) \\ t\xi(t) \\ \vdots \\ t^N \xi(t) \end{bmatrix} \tag{6}$$

The problem is defined as follows:

Problem 1: Design a differential filter to estimate the derivative, $l(t)$, of the signal $x(t)$ without any knowledge of its dynamics.

If the signal $\zeta(t)$ is available, Problem 1 is equivalent to the estimation problem of the constant vector l_v in Equation (5) with the available signals $x(t)$ and ζ. This problem is well known as the adaptive identification problem[12], [13]. We design an adaptive observer for the system (5) as

$$\frac{d\hat{x}(t)}{dt} = \hat{l}_v^T(t)\zeta(t) - k(\hat{x}(t) - x(t)) \tag{7}$$

where k is a positive constant and $\hat{l}_v(t) = \begin{bmatrix} \hat{l}_0(t), \cdots, \hat{l}_N(t) \end{bmatrix}^T$ is an estimate of l_v, which is recursively updated by a parameter adjustment law. The estimate of the derivative of the nonlinear intensity function $f(x(t))$ is given by

$$\hat{l}(t) = \sum_{k=0}^{N} \hat{l}_k(t)t^k. \tag{8}$$

Defining two estimation errors as

state estimation error: $e(t) = \hat{x}(t) - x(t)$

parameter estimation error: $\tilde{l}_v(t) = \hat{l}_v(t) - l_v$,

we obtain the following error equation:

$$\frac{de(t)}{dt} = -ke(t) + \tilde{l}_v^T(t)\zeta(t). \tag{9}$$

The parameter update law is given by

$$\dot{\hat{l}}_v(t) = \dot{\tilde{l}}_v = -\gamma\zeta(t)e(t). \tag{10}$$

We get the following lemma using the adaptive control theory[12], [13].

Lemma 1: Consider the system

$$\frac{de(t)}{dt} = -ke(t) + \tilde{l}_v^T(t)\zeta(t)$$
$$\dot{\tilde{l}}_v = -\gamma\zeta(t)e(t)$$

where γ is any positive number. If $k > 0$ and $\zeta(t)$ is bounded, then $\tilde{l}_v(t)$ is bounded and

$$\lim_{t\to\infty} e(t) = 0.$$

Moreover, if we assume the following persistency of excitation condition[12], [13]:

$$\int_t^{t+T} \zeta(\tau)\zeta(\tau)^T d\tau \geq cI, \quad \forall t \geq 0$$

where c and T are positive, then the estimate $\hat{l}_v(t)$ converges to the true value l_v, *i.e.*,

$$\lim_{t\to\infty} \tilde{l}_v(t) = 0$$

Proof: Defining a Lyapunov-like function as

$$V(t) = \frac{1}{2}\left(e^2(t) + \frac{1}{\gamma}\tilde{l}_v^T(t)\tilde{l}_v(t)\right),$$

its derivative with respect to time is given by

$$\dot{V} = -ke^2(t) \leq 0.$$

Therefore, $e \in \mathcal{L}_2$ and $\tilde{l}_v$ is bounded. Since ζ is bounded, $\dot{e}$ is bounded. From Barbalat's lemma[13], we have $\lim_{t\to\infty} e(t) = 0$. From the persistently exciting condition, the element of ζ is independent each other. Thus, we have

$$lim_{t\to\infty}\tilde{l}_v(t) = 0,$$

because $\lim_{t\to\infty}\tilde{l}_v^T(t)\zeta(t) = 0$. Remark that without any persistently exciting condition, one can only conclude that V has a limit and $\lim_{t\to\infty}\dot{\tilde{l}}_v = 0$. ∎

The estimate of the derivative of the intensity is obtained by

$$\dot{\hat{x}}(t) = \hat{l}_v^T(t)\zeta(t). \tag{11}$$

The estimate of the derivative of the nonlinear intensity function is given by

$$\hat{l}(t) = \sum_{k=0}^{N} \hat{l}_k(t)t^k. \tag{12}$$

III. SELECTION OF NONLINEAR INTENSITY FUNCTION

Since we use the normalized intensity $x(t)$, the range of $x(t)$ is $[0, 1]$. To lessen the effect of noises, we adopt two nonlinear intensity functions as follows:

- the quadratic function: $f_1(x(t)) = x(t)^2$
- the logistic function:
  $f_2(x(t)) = \log(\cosh(x(t)/0.605))$.

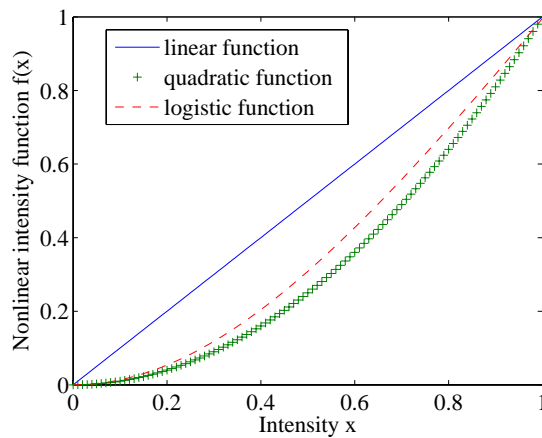Figure 1 shows the function shapes of the nonlinear functions.

Fig. 1 The nonlinear intensity functions

The derivatives of these functions are

$$\frac{df_1(x)}{dx} = 2x(t)$$
$$\frac{df_2(x)}{dx} = \frac{\tanh(x(t))}{0.605}.$$

To avoid zero-division, for a small positive number, $\xi_1(t)$ and $\xi_2(t)$ are defined as

$$\xi_1(t) = \begin{cases} \frac{1}{2x(t)} & (|x(t)| > \epsilon) \\ \frac{1}{2\epsilon} & (0 \leq x(t) \leq \epsilon) \\ -\frac{1}{2\epsilon} & (-\epsilon \leq x(t) < 0) \end{cases} \tag{13}$$

$$\xi_1(t) = \begin{cases} \frac{0.605}{\tanh(x(t))} & (|\tanh(x(t))| > \epsilon) \\ \frac{1}{\epsilon} & (0 \leq \tanh(x(t)) \leq \epsilon) \\ -\frac{1}{\epsilon} & (-\epsilon \leq \tanh(x(t)) < 0). \end{cases} \tag{14}$$

IV. DIGITIZATION OF DIFFERENTIAL FILTER

For the implementation, the derivative can be approximated by the difference as follows:

$$\dot{x}(t_k) = \frac{\hat{x}[k+1] - \hat{x}[k]}{h}$$

$$\dot{\hat{l}}_v(t_k) = \frac{\hat{l}_v[k+1] - \hat{l}_v[k]}{h}$$

where k is the frame number, h is the video-rate, and t_k is the time of k-th frame. The digital form of the differential filter is given by

$$\hat{x}[k+1] = \hat{x}[k] + h\left\{\hat{l}_v^T[k]\zeta[k]\right\} - ke[k] \quad (15)$$

$$\hat{l}_v[k+1] = \hat{l}_v[k] - \gamma\zeta[k]e[k] \quad (16)$$

where $e[k] = \hat{x}[k] - x[k]$. To guarantee the stability in the presence of measurement noise, we adopt the discrete-time version of the σ-modification update law[12], [13] as follows:

$$\hat{l}_v[k+1] = (1 - \sigma)\hat{l}_v[k] - \gamma\zeta[k]e[k] \quad (17)$$

where $0 < \sigma < 1$. The estimate of the derivative of the intensity, $\hat{x}[k]$, is obtained by

$$\hat{x}[k] = \hat{l}_v^T[k]\zeta[k]. \quad (18)$$

The estimate of the derivative of the nonlinear intensity function is given by

$$\hat{l}[k] = \sum_{j=0}^{N} \hat{l}_j[k]t_k^j. \quad (19)$$

V. SIMULATION RESULTS

A. Adaptive Differential Filter for Sinusoidal Signal with Noise

To demonstrate the performance of the proposed method, we use the sinusoidal signal with the Gaussian noise as follows:

$$x[k] = 2 + \sin t_k + 0.01w(t_k); \quad (20)$$
$$t_k = k/30, k = 1, 2, \cdots, 500$$

where $w(t_k)$ is the Gaussian noise $N(0, 1)$. We select the nonlinear function $f(x(t)$ as the linear function $f(x(t)) = x(t)$ to obtain the derivative of the sinusoidal signal (20) directly. The design parameters of the differential filter are selected as

$$\gamma = 0.3, \quad k = 0.195, \quad \sigma = 0.1$$

We adopt the simplest differential filter that is obtained by assuming that the function $l(t)$ is constant,

i.e. $l(t) = l_0$. In this case, the differential filter is given by

$$\hat{x}[k+1] = \hat{x}[k] + h\hat{l}[k]\xi[k] - ke[k] \quad (21)$$

$$\hat{l}[k+1] = (1 - \sigma)\hat{l}[k] - \gamma\xi[k]e[k]. \quad (22)$$

where $h = 1/30$ and $\xi[k] = 1$. Figure 2 shows the sinusoidal signal with the Gaussian noise $x[k]$. Figure 3 shows the estimate of its derivative using the frame difference $\frac{x[k]-x[k-1]}{h}, k = 1, 2, \cdots, 500$. Figure 4 shows the true value(dotted line) of the derivative of the noise-free sinusoidal signal and the estimate(solid line) of its derivative by the proposed adaptive differential filter. The frame difference amplifies the influence of the noises. However, the adaptive differential filter attenuate its influence and the estimate of the derivative of $x[k]$ approaches to the neiborhood of the real value as t increases.
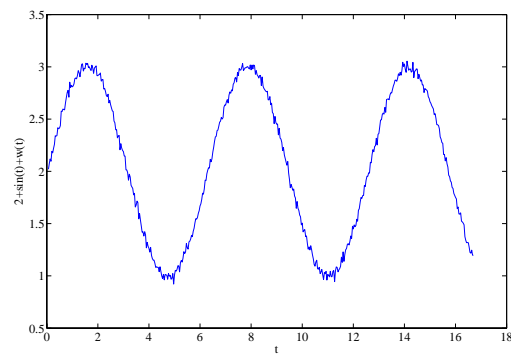

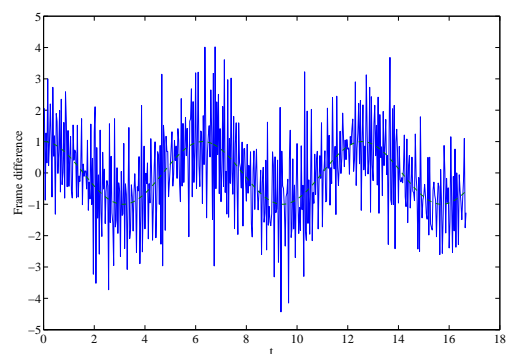Fig. 2 The data sequence with additive noise

Fig. 3 The frame difference (solid line (blue)) and its true value (dotted line (green))
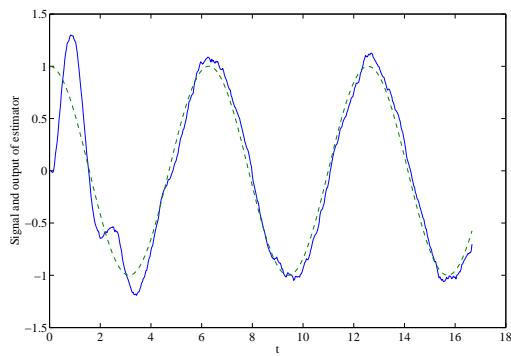
Fig. 4 The output of the proposed estimator (solid line(blue)) and
its true value (dotted line(green))

Fig. 6 The image of frame #83

B. Adaptive Differential Filter for Movie Data

Figure 5 shows the first frame of the movie file with 201 frames and 512×384 pixels, that is the Polyhedral scene with two moving marbled blocks and is downloaded from the famous image server[14]. Figures 6,7,8, and 9 show the 83rd frame, the 95th frame, the 122nd frame, and the 176th frame, respectively. We use the intensity

Fig. 7 The image of frame #95

Fig. 5 The image of frame #1

sequence $x_o[k]; k = 1, \cdots, 201$ at $(153, 225)$ pixel after converting the color image to the grayscale image using MATLAB function `rgb2gray.m` and normalizing the `uint8`-class to the `double`-class. The value 0 corresponds to black and 1 to white. This pixel is on the point marked with '+'. The dark side of the smallest polyhedron passes through the pixel from the frame #83 to the frame #94
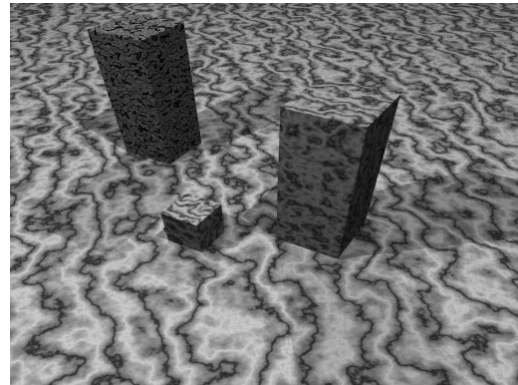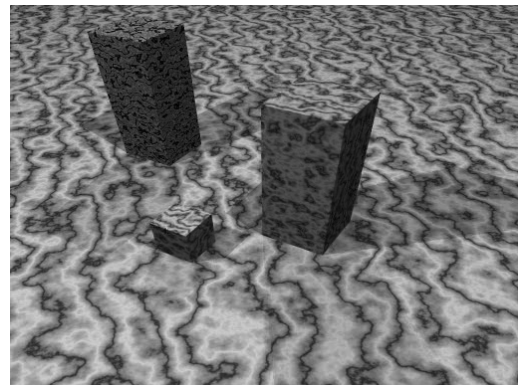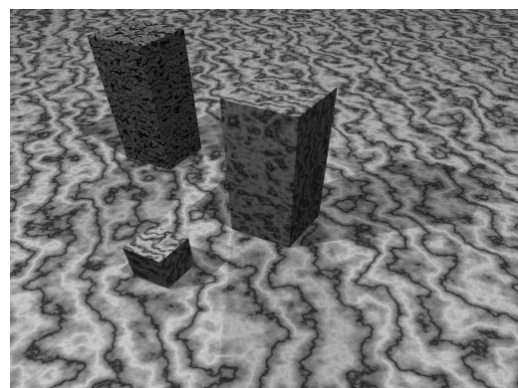

Fig. 8 The image of frame #122
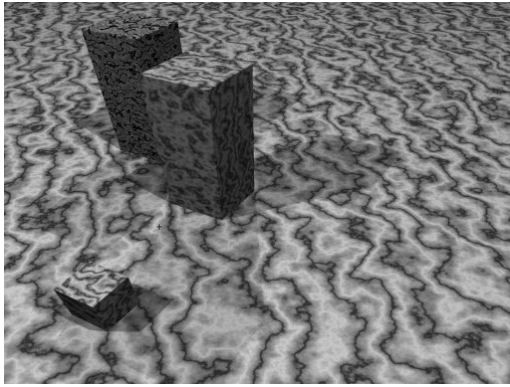
Fig. 9 The image of frame #176

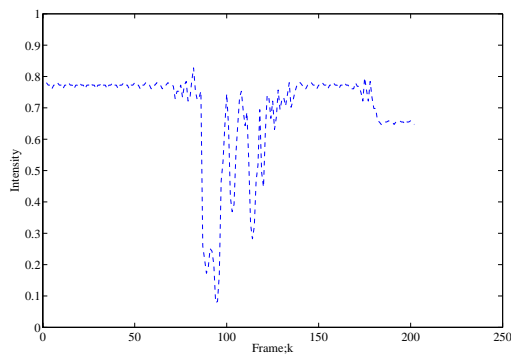Fig. 11   The temporal sequence $x[k]; k = 1, \cdots, 50$ with the additive Gaussian noise

Fig. 10   The temporal sequence $x_o[k]; k = 1, \cdots, 50$
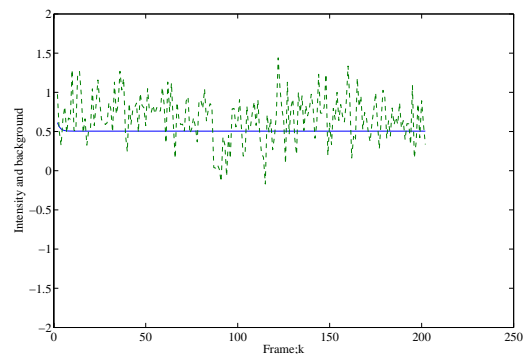
Fig. 12   The temporal sequence $x[k]; k = 1, \cdots, 50$(dotted line) and the background estimate(solid line)

and its bright side passes through the pixel from the frame #95 to the frame #122. Moreover, the shadow of the right-side polyhedron passes through the pixel from the frame #176 to the frame #201. Let $x[k]; k = 1, \cdots, 201$, be the temporal intensity sequence in the additive Gaussian noises with standard deviation $0.3 \times \max_k(x_o[k])$ in order to validate the performance of the noise attenuation. Figures 10 and 11 show the temporal sequences without the Gaussian noise and with the Gaussian noise at $(153, 225)$ pixel, respectively.

Figures 12 and 13 show the estimate of background with the M estimator[15], [16] and the estimate of the temporal-derivative with the frame difference. Both the estimators are influenced considerably by the Gaussian noise.

We also adopt the simplest differential filter given by (21) and (22) that is obtained by assuming

Fig. 13   The temporal sequence $x[k]; k = 1, \cdots, 50$(dotted line) and the estimate of the temporal-derivative using the frame difference(solid line)
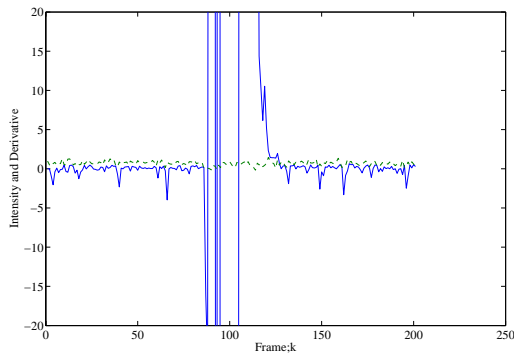
Fig. 14 The intensity (dotted line) and the estimate of the derivative(solid line) of the quadratic intensity function
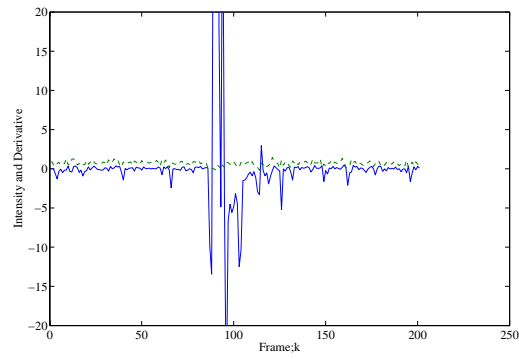

Fig. 15 The intensity (dotted line) and the estimate of the derivative(solid line) of the logistic intensity function

that the function $l(t)$ is constant. The following simulation results illustrate that its simplest differential filter can estimate the variety of time-varying functions.

1) Case of quadratic intensity function: The nonlinear intensity function $f(x[k])$ is selected as $f(x[k]) = x^2[k]$. The output of the differential filter $\hat{l}[k]$ is the estimate of the derivative of the quadratic intensity function $x^2[k]$ using the temporal intensity sequence $x[k]$ in the additive Gaussian noises with standard deviation $0.3 \times \max_k(x_o[k])$. The design parameters of the differential filter are selected as

$$\gamma = 1, \quad k = 0.895, \quad \sigma = 0.95.$$

Figure 14 shows the intensity sequence (dotted line) and the estimate (solid line) of the derivative of the quadratic intensity function using the adaptive observer. In this case, the estimator can detect the largest variety of the intensity, which is the motion of the smallest polyhedron from the frame #83 to the frame #122.

2) Case of logistic intensity function: The non-linear intensity function $f(x[k])$ is selected as the logistic function $f(x[k]) = \log(\cosh(x[k]))$. The output of the differential filter $\hat{l}[k]$ is the estimate of the derivative of the logistic intensity function. The design parameters of the differential filter are selected as

$$\gamma = 1, \quad k = 0.895, \quad \sigma = 0.95.$$

Figure 15 shows the intensity sequence (dotted line) and the estimate (solid line) of the derivative of the logistic intensity function using the adaptive observer. The selection of the logistic intensity

function also allows to detect the motion of the smallest polyhedron.

VI. CONCLUSION

This paper has presented an estimator to detect moving objects using the adaptive identification method. The estimator is constructed by an adaptive differential filter of an intensity nonlinear function. The nonlinear intensity function is introduced to reduce the effect of noises. The derivative of the nonlinear intensity function is estimated by an adaptive observer with σ-modification update law. The main subject in this paper is a new differentiator using the adaptive update law. It is the next issue to get an adaptive decision rule using a temporally varying threshold. In addition, we can get the relative differential filter. If the intensity function is represented as

$$x(t) = cf(t), \quad c : \text{constant},$$

we have the following differential equation:

$$\frac{dx(t)}{dt} = l(t)x(t)$$

where $l(t) = \frac{f'(t)}{f(t)}$. The signal $l(t)$ is the relative derivative of the intensity. We can derive the estimate of $l(t)$ by using the adaptive identification method in the same manner as in Chapter III.

REFERENCES

[1] Cucchiara,R., Grana,C., Piccardi,M., and Prati,A. : "Detecting Moving Objects, Ghosts, and Shadows in Video Streams", IEEE Trans. on PAMI, Vol.25, No.10, pp.1337–1342 (2003).

[2] Cheung,S.-C. and Kamath,C. : "Robust techniques for background subtraction in urban traffic video", Video Communications and Image Processing, SPIE Electronic Imaging, San Jose, January (2004), UCRL-CONF-200706.

[3] Manzanera,A. and Richefeu,J.C. : "A robust and computationally efficient motion detection algorithm based on $\Sigma - \Delta$ background estimation", ICVGIP, Kolkata, India (2004). http://www.ensta.fr/~manzaner

[4] Halevy,G. and Weinshall,D. : "Motion of disturbances: detection and tracking of multibody non-rigid motion",Machine Vision and Applications, Vol.11, pp.122–137 (1999).

[5] Ibrir,S.: "New differentiators for control and observation applications", Proc. of American Control Conference, pp.2522–2527 (2001).

[6] Ibrir,S.: "Linear time-derivative trackers", Automatica, Vol.40, pp.397–405 (2004).

[7] Wren,C., Azabayejani,A., Darrell,T. and Pentland,A.: "Pfinder: Real-time tracking of the human body", IEEE Trans. on PAMI, Vol.19, No.7, pp.780–785 (1997).

[8] Kuo,C.M., Hsieh,C.-H., Lin,H.-C., and Lu, P.-C.: "Motion estimation algorithm with Kalman filter", Electronics Letters, Vol.30, No.15, pp.1204–1206 (1994).

[9] Karmann,K.-P. and Brandt A.: "Moving object recognition using an adaptive background memory", Time-Varying Image Processing and Moving Object Recognition, V.Cappellini ed., 2, pp.289–307, Elsevier Sicence Publishers B.V. (1990).

[10] Köker,R., Cakar,S., and Öz,C.: "Moving object detection and target prediction in video image", IJCI Proceedings of International Conference on Signal Processing, Vol.1, No.2, pp.149–152 (2003).

[11] Richefeu,J. and Manzanera,A. : "A new hybrid differential filter for motion detection", ICCVG'04, Warsaw, Poland, 22-24, Sept. (2004). http://www.ensta.fr/~richefeu/Publications/iccvg82.pdf

[12] Narendra,K.S. and Annaswamy,A.M. : Stable Adaptive Systems, Prentice-Hall, Inc. (1989).

[13] Ioannou,P.A. and Sun,J. : Robust Adaptive Control, Prentice-Hall, Inc. (1996).

[14] Marbled-Block (new version) marmor_stat, Institut für Algorithmen und Kognitive Systeme (Group Prof. Dr. H.-H. Nagel), Universität Karlsruhe, http://i21www.ira.uka.de/image_sequences/.

[15] Shimai,H. , Kurita,T. , and Umeyama,S. : "Adaptive Background Estimation by Robust Statistics", IEICE Transaction, D-II, Vol.J86-D-II, No.6, pp.796–806 (2003) (in Japanese).

[16] Shimai,H., Mishima,T., Kurita,T., and Umeyama,S. : "Adaptive background estimation from image sequence by on-line M-estimation and its application to detection of moving objects", Proc. of Infotech Oulu Workshop on Real-Time Image Sequence Analysis, pp.99-108 (2000).