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Reliability Evaluation using Triangular Intuitionistic Fuzzy Numbers Arithmetic Operations

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***Abstract*—**In general fuzzy sets are used to analyze the fuzzy system reliability. Here intuitionistic fuzzy set theory for analyzing the fuzzy system reliability has been used. To analyze the fuzzy system reliability, the reliability of each component of the system as a triangular intuitionistic fuzzy number is considered. Triangular intuitionistic fuzzy number and their arithmetic operations are introduced. Expressions for computing the fuzzy reliability of a series system and a parallel system following triangular intuitionistic fuzzy numbers have been described. Here an imprecise reliability model of an electric network model of dark room is taken. To compute the imprecise reliability of the above said system, reliability of each component of the systems is represented by triangular intuitionistic fuzzy numbers. Respective numerical example is presented.

***Keywords*—**Fuzzy set, Intuitionistic fuzzy number, System reliability, Triangular intuitionistic fuzzy number.

I. Introduction

IT is well known that the conventional reliability analysis using probabilities has been found to be inadequate to handle uncertainty of failure data and modeling. To overcome this problem, the concept of fuzzy [1] approach has been used in the evaluation of the reliability of a system. In [2] Kaufmann et al. pointed out that the discipline of the reliability engineering encompasses a number of different activities, out of which the reliability modeling is the most important activity. For a long period of time efforts have been made in the design and development of reliable large-scale systems. In that period of time considerable work has been done by researchers to build a systematic theory of reliability based on the probability theory.

In [3] Cai et al. pointed out that there are two fundamental assumptions in the conventional reliability theory, i.e. (a) Binary state assumptions: the system is precisely defined as functioning or failing. (b) Probability assumptions: the system behaviour is fuzzy characterized in the context of probability measures. Because of the inaccuracy and uncertainties of data, the estimation of precise values of probability becomes very difficult in many systems. In [4], Cai et al introduced system failure engineering and its use of fuzzy methodology. In [5], Chen presented a method for analyzing the fuzzy system reliability using fuzzy number arithmetic opaerations. In [6], Cheng et al. used interval of confidence for analyzing the fuzzy system reliability. In [7], Singer presented a fuzzy set approach for fault tree and the reliability analysis. Verma [8] presented the dynamic reliability evaluation of the deteriorating system using the concept of probist reliability as a triangular fuzzy number.

Intuitionistic fuzzy set (IFS) is one of the generalizations of fuzzy sets theory [9]. Out of several higher-order fuzzy sets, IFS first introduced by Atanassov [10] have been found to be compatible to deal with vagueness. The concept of IFS can be viewed as an appropriate/alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In fuzzy sets the degree of acceptance is considered only but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [11]. Presently intuitionistic fuzzy sets are being studied and used in different fields of science. Among the works on these sets, Atanassov [12-14], Atanassov and Gargov [15], Szmidt and Kacrzyk [16], Buhaescu [17], Gerstenkorn and Manko [18], Stojona and Atanassov [19], Stoyanova [20], Deschrijver and Kerre [21] can be mentioned. With best of our knowledge, Burillo [22] proposed definition of intuitionistic fuzzy number and studied perturbations of intuitionistic fuzzy number and the first properties of the correlation between these numbers. Mitchell [23] considered the problem of ranking a set of intuitionistic fuzzy numbers to define a fuzzy rank and a characteristic vagueness factor for each intuitionistic fuzzy number. Here intuitionistic fuzzy number (IFN) is presented according to the approach of presentation of fuzzy number. Arithmetic operations of proposed IFN are evaluated.

This paper is organized as follows: Section 2 presents basic concept of intuitionistic fuzzy sets and intuitionistic fuzzy number. Section 3 presents arithmetic operations between two triangular intuitionistic fuzzy numbers. Section 4 presents expressions for finding the fuzzy reliability of a series and a parallel system using arithmetic operations on triangular intuitionistic fuzzy numbers. Section 5 presents the intuitionitic fuzzy success tree of imprecise reliability of a

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dark room. The conclusions are discussed in Section 6.

II. Basic Concept of Intuitionistic Fuzzy Sets

Fuzzy set theory was first introduced by Zadeh [9] in 1965. Let X be universe of discourse defined by $X = \{x_1, x_2, ..., x_n\}$. The grade of membership of an element $x_i \in X$ in a fuzzy set is represented by real value between 0 and 1. It does indicate the evidence for $x_i \in X$, but does not indicate the evidence against $x_i \in X$. Atanassov [10] presented the concept of IFS, and pointed out that this single value combines the evidence for $x_i \in X$ and the evidence against $x_i \in X$. An IFS $\overset{\sim i}{A}$ in X is characterized by a membership function $\mu_{\overset{\sim i}{A}}(x)$ and a non-membership function $\nu_{\overset{\sim i}{A}}(x)$. Here $\mu_{\overset{\sim i}{A}}(x)$ and $\nu_{\overset{\sim i}{A}}(x)$ are associated with each point in X, a real number in [0,1] with the values of $\mu_{\overset{\sim i}{A}}(x)$ and $\nu_{\overset{\sim i}{A}}(x)$ at X representing the grade of membership and non-membership of x in $\overset{\sim i}{A}$. Thus closure the value of $\mu_{\overset{\sim i}{A}}(x)$ to unity and the value of $\nu_{\overset{\sim i}{A}}(x)$ to zero; higher the grade of membership and lower the grade of non-membership of x. When $\overset{\sim i}{A}$ is an ordinary set its membership function (non-membership function) can take on only two values zero and one. If $\mu_{\overset{\sim i}{A}}(x) = 1$ and $\nu_{\overset{\sim i}{A}}(x) = 0$ the element x does not belongs to $\overset{\sim i}{A}$, similarly if $\mu_{\overset{\sim i}{A}}(x) = 0$ and $\nu_{\overset{\sim i}{A}}(x) = 1$ the element x does not belong to $\overset{\sim i}{A}$. An IFS becomes a fuzzy set $\tilde{A}$ when $\nu_{\overset{\sim i}{A}}(x) = 0$ but $\mu_{\overset{\sim i}{A}}(x) \in [0,1] \forall x \in \overset{\sim i}{A}$.

Definition: Intuitionistic Fuzzy Set

Let a set X be fixed. An intuitionistic fuzzy set $\overset{\sim i}{A}$ in X is an object having the form $\overset{\sim i}{A} = \left\{ < x, \mu_{\overset{\sim i}{A}}(x), \nu_{\overset{\sim i}{A}}(x) > : x \in X \right\}$, where the $\mu_{\overset{\sim i}{A}}(x)$:X → [0,1] and $\nu_{\overset{\sim i}{A}}(x)$:X→[0,1] define the degree of membership and degree of non-membership respectively, of the element x∈X to the set $\overset{\sim i}{A}$, which is a subset of X, for every element of x∈X, $0 \le \mu_{\overset{\sim i}{A}}(x) + \nu_{\overset{\sim i}{A}}(x) \le 1$.

Definition: (α,β)-Level Intervals or (α,β)-Cuts

A set of (α,β)-cut, generated by IFS $\overset{\sim i}{A}$, where $\alpha,\beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$$\overset{\sim i}{A}_{\alpha,\beta} = \left\{ \begin{array}{l} \left(x, \mu_{\overset{\sim i}{A}}(x), \nu_{\overset{\sim i}{A}}(x) \right) : x \in X, \\ \mu_{\overset{\sim i}{A}}(x) \le \alpha, \nu_{\overset{\sim i}{A}}(x) \le \beta,\ \alpha,\beta \in [0,1] \end{array} \right\}$$

(α,β)-level interval or (α,β)-cut, denoted by $\overset{\sim i}{A}_{\alpha,\beta}$, is defined as the crisp set of elements x which belong to $\overset{\sim i}{A}$ at least to the degree α and which does belong to $\overset{\sim i}{A}$ at most to the degree β.

2.1 Presentation of Intuitionistic Fuzzy Numbers and Its Properties

Intuitionistic fuzzy number was introduced by Burillo et al. [22] in 1994. The definition of IFN is given below

Definition: Intuitionistic Fuzzy Number

An intuitionistic fuzzy number $\overset{\sim i}{A}$ is

i) an intuitionistic fuzzy sub set of the real line

ii) normal i.e. there is any $x_0 \in R$ such that $\mu_{\overset{\sim i}{A}}(x_0) = 1$ (so $\nu_{\overset{\sim i}{A}}(x_0) = 0$)

iii) convex for the membership function $\mu_{\overset{\sim i}{A}}(x)$ i.e.

$$\mu_{\overset{\sim i}{A}}(\lambda x_1 + (1-\lambda)x_2) \ge \min\left(\mu_{\overset{\sim i}{A}}(x_1), \mu_{\overset{\sim i}{A}}(x_2)\right) \forall x_1, x_2 \in R, \lambda \in [0,1]$$

iv) concave for non-membership function $\nu_{\overset{\sim i}{A}}(x)$ i.e.

$$\nu_{\overset{\sim i}{A}}(\lambda x_1 + (1-\lambda)x_2) \le \max\left(\nu_{\overset{\sim i}{A}}(x_1), \nu_{\overset{\sim i}{A}}(x_2)\right) \forall x_1, x_2 \in R, \lambda \in [0,1]$$

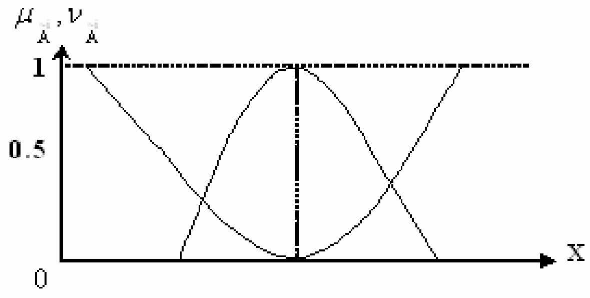

Fig. 1 Membership and non-membership functions of $\overset{\sim i}{A}$

Definition: Triangular Intuitionistic Fuzzy Number

A Triangular Intuitionistic Fuzzy Number (TIFN) $\overset{\sim i}{A}$ is an intuitionistic fuzzy set in R with following membership function $\left(\mu_{\overset{\sim i}{A}}(x)\right)$ and non-membership function $\left(\nu_{\overset{\sim i}{A}}(x)\right)$

$$\mu_{\tilde{A}^i}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & for\ a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2} & for\ a_2\le x\le a_3\\ 0 & otherwise\end{cases}\quad \text{and}\quad \nu_{\tilde{A}^i}(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'} & for\ a_1'\le x\le a_2\\ \dfrac{x-a_2}{a_3'-a_2} & for\ a_2\le x\le a_3'\\ 1 & otherwise\end{cases}$$

Where $a_1'\le a_1\le a_2\le a_3\le a_3'$ and $\mu_{\tilde{A}^i}(x),\nu_{\tilde{A}^i}(x)\le 0.5$ for $\mu_{\tilde{A}^i}(x)=\nu_{\tilde{A}^i}(x)\,\forall x\in R$. This TIFN is denoted by $\tilde{A}^i_{TIFN}=(a_1,a_2,a_3;a_1',a_2,a_3')$.

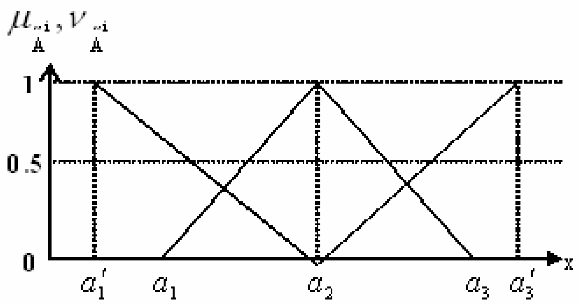

Fig. 2 Membership and non-membership functions of TIFN

Corollary 1. Transformation rule for the TIFN $\tilde{A}^i_{TIFN}=(a_1,a_2,a_3;a_1',a_2,a_3')$ *to triangular fuzzy number (TFN)* $\tilde{A}_{TFN}=(a_1,a_2,a_3)$ *is that* $a_1=a_1', a_3=a_3'$ *and* $\nu_{\tilde{A}^i}(x)=1-\mu_{\tilde{A}^i}(x),\forall x\in R.$

Proof: If $a_1=a_1'$, $a_3=a_3'$ are put and by considering $\nu_{\tilde{A}^i}(x)=1-\mu_{\tilde{A}^i}(x),\forall x\in R.$, membership and non-membership functions of TIFN are become as follows

$$\mu_{\tilde{A}^i}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & for\ a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2} & for\ a_2\le x\le a_3\\ 0 & otherwise\end{cases}\quad \text{and}\quad \nu_{\tilde{A}^i}(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1} & for\ a_1\le x\le a_2\\ \dfrac{x-a_2}{a_3-a_2} & for\ a_2\le x\le a_3\\ 1 & otherwise\end{cases}$$

It is clear from the above that $\nu_{\tilde{A}^i}(x)$ is the complement of $\mu_{\tilde{A}^i}(x)$, and hence the above transformation rules transform TIFN to TFN.

Corollary 2. Transformation rule from TIFN $\tilde{A}^i_{TIFN}=(a_1,a_2,a_3;a_1',a_2,a_3')$ *to crisp interval* $[a_1,a_3]$ *is* $a_1'=a_1$ *and* $a_3=a_3'.$

Proof: The proof is obvious.

Corollary 3. Transformation rule from TIFN $\tilde{A}^i_{TIFN}=(a_1,a_2,a_3;a_1',a_2,a_3')$ *to a real number* a *is* $a_1'=a_1=a_2=a_3=a_3'.$

Proof: The proof is obvious.

III. Arithmetic Operations on Intuitionistic Fuzzy Number

The arithmetic operation (*) of two intuitionistic fuzzy numbers is a mapping of an input vector $X=[x_1,x_2]^T$ which defined in the cartesian product space $R\times R$ onto an output y is defined in the real space R. If A_1 and A_2 are IFN then their resultant is also a IFN determined with the formula

$$(A_1*A_2)(y)=\left\{\left(y,\ \underset{y=x_1*x_2}{\vee}\left[A_1(x_1)\wedge A_2(x_2)\right],\ \underset{y=x_1*x_2}{\wedge}\left[A_1(x_1)\vee A_2(x_2)\right]\right)\forall x_1,x_2,y\in R\right\}$$

to calculate the arithmetic operation of IFNs it is sufficient to determine the membership function and non-membership function as follows

$$\mu_{(A_1*A_2)}(y)=\underset{y=x_1*x_2}{\vee}\left[A_1(x_1)\wedge A_2(x_2)\right] and\ \nu_{(A_1*A_2)}(y)=\underset{y=x_1*x_2}{\wedge}\left[A_1(x_1)\vee A_2(x_2)\right]$$

where $\vee$ denotes a set union operator and $\wedge$ denotes a set intersection operation.

A. Arithmetic Operation of Intuitionistic Fuzzy Number based on (α, β)-Cuts Method

If $\tilde{A}^i$ is an intuitionistic fuzzy number, (α,β)-level intervals or (α,β)-cut is given by

$$\tilde{A}^i_{\alpha,\beta}=\left\{\left[A_1(\alpha),A_2(\alpha)\right],\left[A_1'(\alpha),A_2'(\alpha)\right],\alpha+\beta\le 1,\alpha,\beta\in[0,1]\right\}$$

where $A_1(\alpha)(A_2(\alpha))$ and $A_2'(\beta)(A_1'(\beta))$ will be an increasing (decreasing) function of α with $A_1(1)=A_2(1);A_1'(0)=A_2'(0)$.

If $\tilde{A}^i$ is a TIFN then (i) $A_1(\alpha)\,and\,A_2'(\beta)$ will be continuous, monotonic increasing function of $\alpha,\beta\in[0,1]$; (ii) $A_2(\alpha)\,and\,A_1'(\beta)$ will be continuous, monotonic decreasing function of $\alpha,\beta\in[0,1]$ and (iii) $A_1(1)=A_2(1);A_1'(0)=A_2'(0)$.

In other form, if $\tilde{A}^i=(a_1,a_2,a_3;a_1',a_2,a_3')$ is a TIFN, then α-level intervals or α-cuts is

$$\tilde{A}^i_{\alpha,\beta}=\left\{\left[a_1+\alpha(a_2-a_1),a_3-\alpha(a_3-a_2)\right];\left[a_2-\beta(a_2-a_1'),a_2+\beta(a_3'-a_2)\right]\right\},$$

$\alpha,\beta\in[0,1]$ where $\alpha+\beta\le 1$.

Property 3.1 (a) If TIFN $\overset{\sim i}{A}=(a_1,a_2,a_3;a_1',a_2,a_3')$ *and* $y=ka$ *(with k>0) then* $\overset{\sim i}{y}=k\overset{\sim i}{A}$ *is a TIFN*$(ka_1,ka_2,ka_3;ka_1',ka_2,ka_3')$.

(b) If $y=ka$ *(with k<0) then* $\overset{\sim i}{y}=k\overset{\sim i}{a}$ *is a TIFN* $(ka_3,ka_2,ka_1;ka_3',ka_2,ka_1')$.

Proof. (a) When k>0, with the transformation $y=ka$, we can find the membership (acceptance) function and non-membership (rejection) function of IFN $\overset{\sim i}{y}=k\overset{\sim i}{a}$ by (α,β)-cut method.

α-cut of $\overset{\sim i}{A}$ is

$\mu_{\overset{\sim i}{A}}(x)\ge\alpha\Rightarrow[a_1+\alpha(a_2-a_1),a_3-\alpha(a_3-a_2)]$ for any $\alpha\in[0,1]$

i.e. $x\in[a_1+\alpha(a_2-a_1),a_3-\alpha(a_3-a_2)]$

So, $y(=ka)\in[ka_1+\alpha(ka_2-ka_1),ka_3-\alpha(ka_3-ka_2)]$.

Thus, the membership function of $\overset{\sim i}{y}=k\overset{\sim i}{A}$ is given by

$$\mu_{\overset{\sim i}{y}}(y)=\begin{cases}\dfrac{y-ka_1}{ka_2-ka_1} & for\ ka_1\le y\le ka_2\\ \dfrac{ka_3-y}{ka_3-ka_2} & for\ ka_2\le y\le ka_3\\ 0 & otherwise\end{cases}\qquad(3.1)$$

Hence the rule is proved for membership function.

For non-membership function, β-cut of $\overset{\sim i}{A}$ is

$\nu_{\overset{\sim i}{A}}(x)\le\beta\Rightarrow[a_2-\beta(a_2-a_1'),a_2+\beta(a_3'-a_2)]$ for any $\beta\in[0,1]$

i.e. $x\in[a_2-\beta(a_2-a_1'),a_2+\beta(a_3'-a_2)]$

So, $y(=ka)\in[ka_2-\beta(ka_2-ka_1'),ka_2+\beta(ka_3'-ka_2)]$.

Thus, the non-membership function of $\overset{\sim i}{y}=k\overset{\sim i}{A}$ shows as

$$\Rightarrow\nu_{\overset{\sim i}{A}}(x)=\begin{cases}\dfrac{ka_2-x}{ka_2-ka_1'} & for\ ka_1'\le x\le ka_2\\ \dfrac{x-ka_2}{ka_3'-ka_2} & for\ ka_2\le x\le ka_3'\\ 1 & otherwise\end{cases}\qquad(3.2)$$

Hence the rule is proved for non-membership function.

Thus $\overset{\sim i}{y}=k\overset{\sim i}{A}=(ka_1,ka_2,ka_3;ka_1',ka_2,ka_3')$ is a TIFN when k>0.

(b) Similarly it can be proved that $\overset{\sim i}{y}=k\overset{\sim i}{a}$, if $y=ka$,k<0 where

$$\mu_{\overset{\sim i}{y}}(y)=\begin{cases}\dfrac{y-ka_3}{ka_2-ka_3} & for\ ka_3\le y\le ka_2\\ \dfrac{ka_1-y}{ka_1-ka_2} & for\ ka_2\le y\le ka_1\\ 0 & otherwise\end{cases}$$

and

$$\nu_{\overset{\sim i}{y}}(y)=\begin{cases}\dfrac{ka_2-y}{ka_2-ka_3'} & for\ ka_3'\le y\le ka_2\\ \dfrac{y-ka_2}{ka_1'-ka_2} & for\ ka_2\le y\le ka_1'\\ 1 & otherwise\end{cases}\qquad(3.3)$$

Thus $\overset{\sim i}{y}=k\overset{\sim i}{A}=(ka_3,ka_2,ka_1;ka_3',ka_2,ka_1')$ is a TIFN when k<0.

Property 3.2 If $\overset{\sim i}{A}=(a_1,a_2,a_3;a_1',a_2,a_3')$ *and* $\overset{\sim i}{B}=(b_1,b_2,b_3;b_1',b_2,b_3')$ *are two TIFN, then* $\overset{\sim i}{A}\oplus\overset{\sim i}{B}$ *is also TIFN* $\overset{\sim i}{A}\oplus\overset{\sim i}{B}=(a_1+b_1,a_2+b_2,a_3+b_3;a_1'+b_1',a_2+b_2,a_3'+b_3')$.

Proof. With the transformation z=x+y, the membership (acceptance) function and non-membership (rejection) function of IFS $\overset{\sim i}{z}=\overset{\sim i}{A}\oplus\overset{\sim i}{B}$ can be found by (α, β)-cut method.

α-cut for membership function of $\overset{\sim i}{A}$ is $[a_1+\alpha(a_2-a_1),a_3-\alpha(a_3-a_2)]\forall\alpha\in[0,1]$

i.e. $x\in[a_1+\alpha(a_2-a_1),a_3-\alpha(a_3-a_2)]$

α-cut for membership function of $\overset{\sim i}{B}$ is $[b_1+\alpha(b_2-b_1),b_3-\alpha(b_3-b_2)]\forall\alpha\in[0,1]$

i.e. $y\in[b_1+\alpha(b_2-b_1),b_3-\alpha(b_3-b_2)]$

So, z (=x+y)

$\in[a_1+b_1+\alpha((a_2-a_1)+(b_2-b_1)),a_3+b_3-\alpha((a_3-a_2)+(b_3-b_2))]$

So, the membership function of acceptance (membership) of $\overset{\sim i}{z}=\overset{\sim i}{A}\oplus\overset{\sim i}{B}$ is given by

$$\mu_{\overset{\sim i}{z}}(z)=\begin{cases}\dfrac{z-a_1-b_1}{(a_2-a_1)+(b_2-b_1)} & for\ a_1+b_1\le z\le a_2+b_2\\ \dfrac{a_3+b_3-z}{(a_3-a_2)+(b_3-b_2)} & for\ a_2+b_2\le z\le a_3+b_3\\ 0 & otherwise\end{cases}\qquad(3.4)$$

Hence additions rule is proved for membership function.

For non-membership function, β-cut of $\overset{\sim i}{A}$ is $[a_2-\beta(a_2-a_1'),a_2+\beta(a_3'-a_2)]\forall\beta\in[0,1]$

i.e. $x \in \left[a_2 - \beta\left(a_2 - a_1'\right), a_2 + \beta\left(a_3' - a_2\right)\right]$

α-cut for membership function of $\overset{\sim i}{B}$ is $\left[b_2 - \beta\left(b_2 - b_1'\right), b_2 + \beta\left(b_3' - b_2\right)\right] \forall \beta \in [0,1]$

i.e. $y \in \left[b_2 - \beta\left(b_2 - b_1'\right), b_2 + \beta\left(b_3' - b_2\right)\right]$

So, z (=x+y)

$\in \left[a_2 + b_2 - \beta\left(\left(a_2 - a_1'\right) + \left(b_2 - b_1'\right)\right), a_2 + b_2 - \beta\left(\left(a_3' - a_2\right) + \left(b_3' - b_2\right)\right)\right]$

So, the non-membership (acceptance) function of $\overset{\sim i}{z} = \overset{\sim i}{A} \oplus \overset{\sim i}{B}$ is

$$\nu_{\overset{\sim i}{z}}(z) = \begin{cases} \dfrac{a_2 + b_2 - z}{\left(a_2 - a_1'\right) + \left(b_2 - b_1'\right)} & for\, a_1' + b_1' \le z \le a_2 + b_2 \\ \dfrac{z - a_2 - b_2}{\left(a_3' - a_2\right) + \left(b_3' - b_2\right)} & for\, a_2 + b_2 \le z \le a_3' + b_3' \\ 1 & otherwise \end{cases} \tag{3.5}$$

Hence the addition rule is proved for non-membership function.

Thus $\overset{\sim i}{A} \oplus \overset{\sim i}{B} = \left(a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1' + b_1', a_2 + b_2, a_3' + b_3'\right)$ is a TIFN.

Example 1: Let $\overset{\sim i}{A} = (1, 2, 3; 0.5, 2, 3.5)$ and $\overset{\sim i}{B} = (2, 3, 4; 1, 3, 4.5)$ are two TIFN then their sum $\overset{\sim i}{A} \oplus \overset{\sim i}{B}$ is define by $\overset{\sim i}{A} \oplus \overset{\sim i}{B} = (3, 5, 7; 1.5, 5, 8)$ with membership and non-membership function as follows

$$\mu_{\overset{\sim i}{A} \oplus \overset{\sim i}{B}}(x) = \begin{cases} \dfrac{x-3}{2} & if\ 3 \le x \le 5 \\ \dfrac{7-x}{2} & if\ 5 \le x \le 7 \\ 0 & otherwise \end{cases} \quad \text{and} \quad \nu_{\overset{\sim i}{A} \oplus \overset{\sim i}{B}}(x) = \begin{cases} \dfrac{5-x}{3.5} & if\ 1.5 \le x \le 5 \\ \dfrac{x-5}{3} & if\ 5 \le x \le 8 \\ 1 & otherwise \end{cases}$$

Note:- When the transformation $\overset{\sim i}{y} = k_1 \overset{\sim i}{A} + k_2 \overset{\sim i}{B}$ (k_1, k_2 are (not all zero) real numbers) is taken then the intuitionistic fuzzy set $\overset{\sim i}{y} = k_1 \overset{\sim i}{A} + k_2 \overset{\sim i}{B}$ is the following TIFN

(i) $(k_1a_1 + k_2b_1, k_1a_2 + k_2b_2, k_1a_3 + k_2a_3; k_1a_1' + k_2b_1', k_1a_2 + k_2b_2, k_1a_3' + k_2a_3')$ if $k_1 > 0, k_2 \ge 0$ or $k_1 \ge 0, k_2 > 0$,

(ii) $\left(k_1a_1 + k_2b_3, k_1a_2 + k_2b_2, k_1a_3 + k_2b_1; k_1a_1' + k_2b_3', k_1a_2 + k_2b_2, k_1a_3' + k_2b_1'\right)$ if $k_1 > 0, k_2 \le 0$ or $k_1 \ge 0, k_2 < 0$,

(iii) $\left(k_1a_3 + k_2b_1, k_1a_2 + k_2b_2, k_1a_1 + k_2b_3; k_1a_3' + k_2b_1', k_1a_2 + k_2b_2, k_1a_1' + k_2b_3'\right)$ if $k_1 < 0, k_2 \ge 0$ or $k_1 \le 0, k_2 > 0$,

(iv) $\left(k_1a_3 + k_2b_3, k_1a_2 + k_2b_2, k_1a_1 + k_2b_1; k_1a_3' + k_2b_3', k_1a_2 + k_2b_2, k_1a_1' + k_2b_1'\right)$ if $k_1 < 0, k_2 \le 0$ or $k_1 \le 0, k_2 < 0$.

Property 3.3 If $\overset{\sim i}{A} = \left(a_1, a_2, a_3; a_1', a_2, a_3'\right)$ *and* $\overset{\sim i}{B} = \left(b_1, b_2, b_3; b_1', b_2, b_3'\right)$ *are two TIFN, their product* $\overset{\sim i}{A} \otimes \overset{\sim i}{B}$ *is also TIFN* $\overset{\sim i}{A} \otimes \overset{\sim i}{B} = \left(a_1b_1, a_2b_2, a_3b_3; a_1'b_1', a_2b_2, a_3'b_3'\right)$.

Proof. With the transformation z=x×y, the membership (acceptance) function and non-membership (rejection) function of IFS $\overset{\sim i}{z} = \overset{\sim i}{A} \otimes \overset{\sim i}{B}$ can be establish by (α, β)-cut method.

α-cut for membership function of $\overset{\sim i}{A}$ is $\left[a_1 + \alpha\left(a_2 - a_1\right), a_3 - \alpha\left(a_3 - a_2\right)\right] \forall \alpha \in [0,1]$

i.e. $x \in \left[a_1 + \alpha\left(a_2 - a_1\right), a_3 - \alpha\left(a_3 - a_2\right)\right]$

α-cut for membership function of $\overset{\sim i}{B}$ is $\left[b_1 + \alpha\left(b_2 - b_1\right), b_3 - \alpha\left(b_3 - b_2\right)\right] \forall \alpha \in [0,1]$

i.e. $y \in \left[b_1 + \alpha\left(b_2 - b_1\right), b_3 - \alpha\left(b_3 - b_2\right)\right]$

So, z (=x×y)

$\in \left[\left(a_1 + \alpha\left(a_2 - a_1\right)\right)\left(b_1 + \alpha\left(b_2 - b_1\right)\right), \left(a_3 - \alpha\left(a_3 - a_2\right)\right)\left(b_3 - \alpha\left(b_3 - b_2\right)\right)\right]$

So, the membership function of acceptance (membership) of $\overset{\sim i}{z} = \overset{\sim i}{A} \otimes \overset{\sim i}{B}$ is

$$\mu_{\overset{\sim i}{z}}(z) = \begin{cases} \dfrac{-B_1 + \sqrt{B_1^2 - 4A_1\left(a_1b_1 - z\right)}}{2A_1} & for\, a_1b_1 \le z \le a_2b_2 \\ \dfrac{B_2 - \sqrt{B_2^2 - 4A_2\left(a_3b_3 - z\right)}}{2A_2} & for\, a_2b_2 \le z \le a_3b_3 \\ 0 & otherwise \end{cases} \tag{3.6}$$

where $A_1 = \left(a_2 - a_1\right)\left(b_2 - b_1\right)$,

$B_1 = b_1\left(a_2 - a_1\right) + a_1\left(b_2 - b_1\right)$, $A_2 = \left(a_3 - a_2\right)\left(b_3 - b_2\right)$ and $B_2 = -\left(b_3\left(a_3 - a_2\right) + a_3\left(b_3 - b_2\right)\right)$.

For non-membership function, β-cut of $\overset{\sim i}{A}$ is $\left[a_2 - \beta\left(a_2 - a_1'\right), a_2 + \beta\left(a_3' - a_2\right)\right] \forall \beta \in [0,1]$

i.e. $x \in \left[a_2 - \beta\left(a_2 - a_1'\right), a_2 + \beta\left(a_3' - a_2\right)\right]$

α-cut for membership function of $\overset{\sim i}{B}$ is $\left[b_2 - \beta\left(b_2 - b_1'\right), b_2 + \beta\left(b_3' - b_2\right)\right] \forall \beta \in [0,1]$

i.e. $y \in \left[b_2 - \beta\left(b_2 - b_1'\right), b_2 + \beta\left(b_3' - b_2\right)\right]$

So, z (=x×y)

$\in \left[\left(a_2 - \beta\left(a_2 - a_1'\right)\right)\left(b_2 - \beta\left(b_2 - b_1'\right)\right), \left(a_2 + \beta\left(a_3' - a_2\right)\right)\left(b_2 + \beta\left(b_3' - b_2\right)\right)\right]$

So, the non-membership (rejection) function of $\overset{\sim i}{z} = \overset{\sim i}{A} \otimes \overset{\sim i}{B}$ is defined as

$$\Rightarrow \nu_{\overset{\sim i}{z}}(z)=\begin{cases}1-\dfrac{-B_1'+\sqrt{B_1'^2-4A_1'\left(a_1'b_1'-z\right)}}{2A_1'} & for\ a_1'b_1'\le z\le a_2b_2\\ 1-\dfrac{B_2'-\sqrt{B_2'^2-4A_2'\left(a_3'b_3'-z\right)}}{2A_2'} & for\ a_2b_2\le z\le a_3'b_3'\\ 1 & otherwise\end{cases} \quad (3.7)$$

where $A_1'=\left(a_2-a_1'\right)\left(b_2-b_1'\right)$,

$B_1'=b_1'\left(a_2-a_1'\right)+a_1'\left(b_2-b_1'\right)$, $\quad A_2'=\left(a_3'-a_2\right)\left(b_3'-b_2\right)$

and $B_2'=-\left(b_3'\left(a_3'-a_2\right)+a_3'\left(b_3'-b_2\right)\right)$.

So $\overset{\sim i}{A}\otimes\overset{\sim i}{B}$ represented by (3.6) and (3.7) is a triangular shaped intuitionistic fuzzy number. It can be approximated to a TIFN $\overset{\sim i}{A}\otimes\overset{\sim i}{B}=\left(a_1b_1,a_2b_2,a_3b_3;a_1'b_1',a_2b_2,a_3'b_3'\right)$ (as shown in Fig. 3 with doted line).

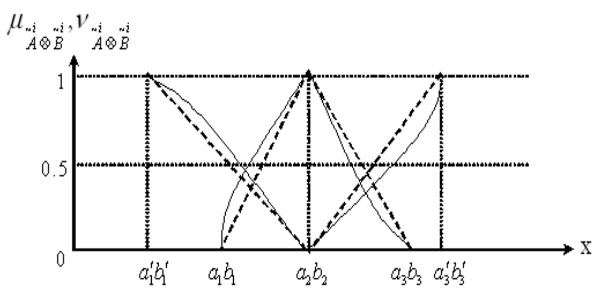

Fig. 3 Membership function of Intuitionistic fuzzy number $\overset{\sim i}{A}\otimes\overset{\sim i}{B}$

Example 2: Let $\overset{\sim i}{A}=\left(1,2,3;0.5,2,3.5\right)$ and

$\overset{\sim i}{B}=\left(2,3,4;1,3,4.5\right)$ are two TIFN then their product $\overset{\sim i}{A}\otimes\overset{\sim i}{B}$ is define by $\overset{\sim i}{A}\otimes\overset{\sim i}{B}=\left(2,6,12;0.5,6,15.75\right)$ with membership and non-membership function as follows

$$\mu_{\overset{\sim i}{A}\otimes\overset{\sim i}{B}}(x)=\begin{cases}\dfrac{-3+\sqrt{9-4(2-x)}}{2} & if\ 2\le x\le 6\\ \dfrac{7-\sqrt{49-4(12-x)}}{2} & if\ 6\le x\le 12\\ 0 & otherwise\end{cases} \quad \text{and} \quad \nu_{\overset{\sim i}{A}\otimes\overset{\sim i}{B}}(x)=\begin{cases}\dfrac{8.5-\sqrt{6.25-12(0.5-x)}}{6} & if\ 0.5\le x\le 6\\ \dfrac{-7.5+\sqrt{144-9(15.75-x)}}{4.5} & if\ 6\le x\le 15.75\\ 1 & otherwise\end{cases}$$

Above $\overset{\sim i}{A}\otimes\overset{\sim i}{B}$ is a triangular shaped intuitionistic fuzzy number. It can be approximated to TIFN as $\overset{\sim i}{A}\otimes\overset{\sim i}{B}=\left(2,6,12;0.5,6,15.75\right)$ with membership and non-membership function as follows:

$$\mu_{\overset{\sim i}{A}\otimes\overset{\sim i}{B}}(x)=\begin{cases}(x-2)/4 & if\ 2\le x\le 6\\ (12-x)/6 & if\ 6\le x\le 12\\ 0 & otherwise\end{cases} \quad \text{and} \quad \nu_{\overset{\sim i}{A}\otimes\overset{\sim i}{B}}(x)=\begin{cases}(6-x)/5.5 & if\ 0.5\le x\le 6\\ (x-6)/9.75 & if\ 6\le x\le 15.75\\ 1 & otherwise\end{cases}$$

IV. Expression of Imprecise Reliability of Series and Parallel System Using Arithmetic Operations over Triangular Intuitionistic Fuzzy Numbers

This section presents the expressions for evaluation the imprecise reliability of a series and a parallel system where the reliability of each component of the system is represented by a triangular intuitionistic fuzzy number.

A. Series System

Let a series system consisting of n components is considered as shown in Fig. 4. The intuitionistic fuzzy reliability $\overset{\sim i}{R_s}$ of the series system shown below can be evaluated by using the expression as follows:

$$\overset{\sim i}{R_s}=\overset{\sim i}{R_1}\otimes\overset{\sim i}{R_2}\otimes\ldots\otimes\overset{\sim i}{R_n}$$

$$=\left\{\left(r_{11},r_{12},r_{13};r_{11}',r_{12},r_{13}'\right)\otimes\left(r_{21},r_{22},r_{23};r_{21}',r_{22},r_{23}'\right)\otimes\ldots\otimes\left(r_{n1},r_{n2},r_{n3};r_{n1}',r_{n2},r_{n3}'\right)\right\}$$

It can be approximated to a TIFN as

$$\cong\left(\prod_{j=1}^{n}r_{j1},\prod_{j=1}^{n}r_{j2},\prod_{j=1}^{n}r_{j3}\ ;\ \prod_{j=1}^{n}r_{j1}',\prod_{j=1}^{n}r_{j2},\prod_{j=1}^{n}r_{j3}'\right)$$

where $\overset{\sim i}{R_j}=\left(r_{j1},r_{j2},r_{j3};r_{j1}',r_{j2},r_{j3}'\right)$ is intuitionistic fuzzy reliability of the j[th] component for j=1,2,…,n.

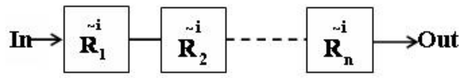

Fig. 4 A Schematic diagram of Series system

B. Parallel System

Let a parallel system consisting of n components is considered as shown in Fig. 5. The fuzzy reliability $\overset{\sim i}{R_s}$ of the parallel system shown below can be evaluated by using the expression as follows:

$$\overset{\sim i}{R_s}=1\Theta\prod_{j=1}^{n}(1\Theta\overset{\sim i}{R_j})$$

$$=1\Theta[(1\Theta\left(r_{11},r_{12},r_{13};r_{11}',r_{12},r_{13}'\right))\otimes\ldots..\otimes(1\Theta\left(r_{n1},r_{n2},r_{n3};r_{n1}',r_{n2},r_{n3}'\right))]$$

$$\cong\left[1-\prod_{j=1}^{n}(1-r_{j1}),1-\prod_{j=1}^{n}(1-r_{j2}),1-\prod_{j=1}^{n}(1-r_{j3});1-\prod_{j=1}^{n}(1-r_{j1}'),1-\prod_{j=1}^{n}(1-r_{j2}),1-\prod_{j=1}^{n}(1-r_{j3}')\right]$$

It is an approximated TIFN, where $\overset{\sim i}{R_j}=\left(r_{j1},r_{j2},r_{j3};r_{j1}',r_{j2},r_{j3}'\right)$ is intuitionistic fuzzy reliability of the j[th] component for j=1,…,n.

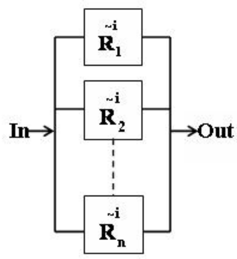

Fig. 5 Diagram of Parallel system

V. Fuzzy Success Tree Analysis of Reliability of Dark Room Using Intuitionistic Fuzzy Number

An electric network model [24] of a dark room is considered. A windowless room has a switch and two light bulbs. Develop a success tree for the desired event (*i.e.* top event): a dark room having light. Thus, in this case the room can only be brightened if there is availability of electricity, the switch is OK, and both the light bulbs are not burnt out. By using the success tree shown in Fig. 6 for this example is developed. Each success event in the success tree diagram is considered as TIFN.

$\overset{\sim i}{R}_1$ represents the reliability no fuse failure of electric supply of dark room,

$\overset{\sim i}{R}_2$ represents the reliability no power failure of electric supply of dark room,

$\overset{\sim i}{R}_3$ represents the reliability of switch of dark room,

$\overset{\sim i}{R}_4$ represents the reliability of bulb no. 1 of dark room,

$\overset{\sim i}{R}_5$ represents the reliability of bulb no. 2 of dark room,

$\overset{\sim i}{R}_6$ represents the reliability of electric supply of dark room,

$\overset{\sim i}{R}_7$ represents the resultant reliability of bulbs connecting in parallel,

$\overset{\sim i}{R}_8$ represents the reliability of desired event of dark room.

All imprecise components reliability $\overset{\sim i}{R}_j$ are represented by TIFN $\left(r_{j1}, r_{j2}, r_{j3}; r'_{j1}, r_{j2}, r'_{j3}\right)$ for j=1,…,5. Let us calculate the reliability of occurrence of the desired event (dark room with illumination).

The reliability value, for the occurrence of event electricity supply, $\overset{\sim i}{R}_6$:

$$\overset{\sim i}{R}_6 = \overset{\sim i}{R}_1 \otimes \overset{\sim i}{R}_2 \cong \left(r_{11}r_{21}, r_{12}r_{22}, r_{13}r_{23}; r'_{11}r'_{21}, r_{12}r_{22}, r'_{13}r'_{23}\right) \quad (5.1)$$

It is an approximated TIFN.

Similarly, the reliability value, for the occurrence of event both bulbs not burnt out, $\overset{\sim i}{R}_7$:

$$\overset{\sim i}{R}_7 = 1\,\Theta(1\,\Theta\,\overset{\sim i}{R}_4)(1\,\Theta\,\overset{\sim i}{R}_5)$$

It is approximated to a TIFN as follows

$$\cong \left(1-\prod_{j=4}^{5}(1-r_{j1}), 1-\prod_{j=4}^{5}(1-r_{j2}), 1-\prod_{j=4}^{5}(1-r_{j3}); 1-\prod_{j=4}^{5}(1-r'_{j1}), 1-\prod_{j=4}^{5}(1-r_{j2}), 1-\prod_{j=4}^{5}(1-r'_{13})\right) \quad (5.2)$$

By substituting the above two calculated values and the given data value, we get the reliability value for the occurrence of the top event, dark room, $\overset{\sim i}{R}_8$:

$$\overset{\sim i}{R}_8 = \overset{\sim i}{R}_1 \otimes \overset{\sim i}{R}_2 \otimes \overset{\sim i}{R}_3 \otimes (1\Theta(1\Theta\,\overset{\sim i}{R}_4)(1\Theta\,\overset{\sim i}{R}_5))$$

It is approximated to a TIFN as follows

$$\cong \left(r_{11}r_{21}r_{31}\left(1-\prod_{j=4}^{5}(1-r_{j1})\right), r_{12}r_{22}r_{32}\left(1-\prod_{j=4}^{5}(1-r_{j2})\right), r_{13}r_{23}r_{33}\left(1-\prod_{j=4}^{5}(1-r_{j3})\right);\right.$$

$$\left. r'_{11}r'_{21}r'_{31}\left(1-\prod_{j=4}^{5}(1-r'_{j1})\right), r_{12}r_{22}r_{32}\left(1-\prod_{j=4}^{5}(1-r_{j2})\right), r'_{13}r'_{23}r'_{33}\left(1-\prod_{j=4}^{5}(1-r'_{13})\right)\right) \quad (5.3)$$

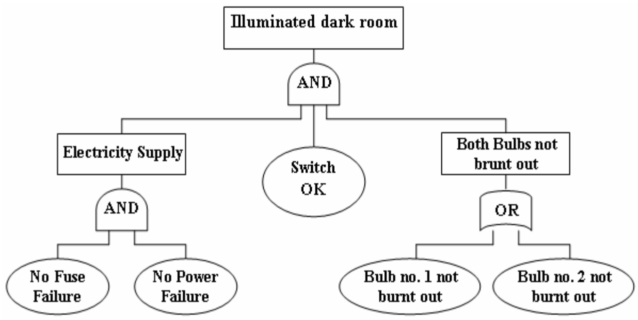

Fig. 6 Success tree for the top or desired event: illuminated dark room

Let the reliability of events are

$\overset{\sim i}{R}_1 = (0.75, 0.8, 0.85; 0.65, 0.8, 0.9)$,

$\overset{\sim i}{R}_2 = (0.8, 0.85, 0.9; 0.75, 0.85, 0.95)$,

$\overset{\sim i}{R}_3 = (0.8, 0.88, 0.92; 0.75, 0.88, 0.96)$,

$\overset{\sim i}{R}_4 = (0.75, 0.85, 0.9; 0.7, 0.85, 0.95)$, and

$\overset{\sim i}{R}_5 = (0.65, 0.75, 0.85; 0.6, 0.75, 0.9)$.

So results of equation (5.1) and (5.2) are as follows

$\overset{\sim i}{R}_6 = (0.6, 0.68, 0.765; 0.4875, 0.68, 0.855)$,

$\overset{\sim i}{R}_7 = (0.9125, 0.9625, 0.985; 0.88, 0.9625, 0.995)$,

By substituting the above two calculated values and the given data value in (5.3), the reliability value for the occurrence of the top event, illuminated dark room, $\overset{\sim i}{R}_8$ is

$$\overset{\sim i}{R}_8 = (0.438, 0.57596, 0.693243; 0.32175, 0.57596, 0.816696)$$

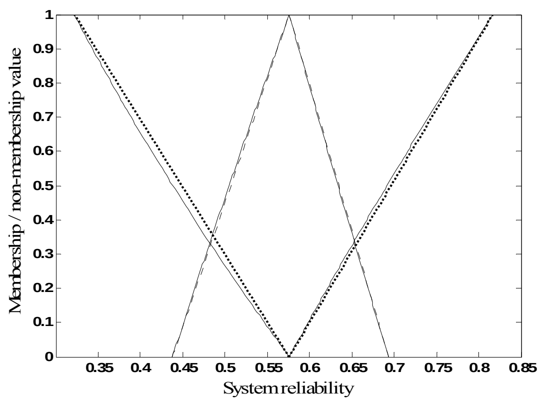

Fig. 7 System reliability of dark room having light

VI. Conclusion

In this paper, a definition of IFN according to the approach of fuzzy number presentation is proposed. Also arithmetic operation of proposed TIFN is evaluated based on intuitionistic fuzzy (α, β)-cut method. Here, a method to analyze system reliability which is based on intuitionistic fuzzy set theory has been presented, where the components of the system are represented by triangular intuitionistic fuzzy number. Arithmetic operations over triangular intuitionistic fuzzy number are used to analyze the fuzzy reliability of the series system, parallel system. Intuitionistic fuzzy success tree are used to analyze the imprecise reliability of a dark room with desired event. The major advantage of using intuitionistic fuzzy sets over fuzzy sets is that intuitionistic fuzzy sets separate the positive and the negative evidence for the membership of an element in a set.

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