A Study of Thermal Convection in Two Porous Layers Governed by Brinkman's Model in Upper Layer and Darcy's Model in Lower Layer

M. S. Al-Qurashi

***Abstract*—**This work examines thermal convection in two porous layers. Flow in the upper layer is governed by Brinkman's equations model and in the lower layer is governed by Darcy's model. Legendre polynomials are used to obtain numerical solution when the lower layer is heated from below.

***Keywords*—**Brinkman's law, Darcy's law, porous layers, Legendre polynomials, the Oberbeck-Boussineq approximation.

I. Introduction

THERMAL instability theory has attracted considerable interest and has been recognized as a problem of fundamental importance in many fields of fluid dynamics. The earliest experiments to demonstrate the onset of thermal instability in fluids are those of Bernard's [1, 2]. Benard worked with very thin layers of an incompressible viscous fluid standing on a levelled metallic plate maintained at a constant temperature. The upper surface was usually free and, being in contact with the air, was at a lower temperature. In his experiments, Benard deduced that a certain critical adverse temperature gradient must be exceeded before instability can set in. The instability of a layer of fluid heated from below and subjected to Coriolis forces has been studied by Chandrasekhar [3, 4] for a stationary and overstability case. He showed that the presence of these forces usually has the effect of inhibiting the onset of thermal convection. Nield [5] considered the onset of salt-finger convection in a porous layer .Taunton et al. [6] considered the thermohaline instability and salt-finger in a porous medium and solved the boundary value problem. Sun [7] was the first to consider such a problem, and he used a shooting method to solve the linear stability equations. Sun [7] and Nield [8] used Darcy's law in formulating the equations of porous layer. In Darcy's law of motion in porous mediums, the Darcy resistance term took the place of the Navier-stokes viscosity term, while in the modified Darcy's law (Brinkman model), suggested by Brinkman [9], the Navier- stokes viscosity term still exists. Chen & Chen [10] considered the multi-layer problem when the above layer is heated and salted from above, and the solution of the problem is obtained using a shooting method. Lindsay & Ogden [11] worked in the implementation of spectral methods resistant to the generation of spurious eigenvalues. Lamb [12] used expansion of Chebyshev polynomials to investigate an eigenvalue problem arising from a model discussing a finitely conducting inner core of the earth on magnetically driven instability. Bukhari [13] studied the effects of surface-tension in a layer of conducting fluid with an imposed magnetic field and obtained results when the free surface is deformable and non-deformable. He solved that by using Chebyshev spectral method, and thus obtained some different results from that of Chen & Chen [10]. Straughan [14] studied the thermal convection in fluid layer overlying a porous layer and considered the problem of lower layer heated from below and surface tension driven on the free top boundary of upper layer. In [15], he also dealt with the same problem considering the ratio depth of the relative layer and investigated the effect of the variation of relevant fluid and porous material properties. Allehiany [16] studied Benard convection in a horizontal porous layer permeated by a conducting fluid in the presence of magnetic field and coriolis forces. Al-Qurashi & Bukhari [17] studied the salt finger convection in a horizontal porous layer superposed by a fluid layer affected by rotation and vertical linear magnetic field on both layers. The solution is obtained using Legendre polynomials when the heat and the salt concentration affected from above.

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II. Math

Let L_1 and L_2 be two horizontal porous layers such that the top of the layer L_2 touches the bottom of the layer L_1. The plane interface between the two layers is $x_3 = 0$, the upper boundary of L_1 is $x_3 = d_B$ and the lower boundary of L_2 is $x_3 = -d_D$. We suppose that the two layers occupied by a porous medium permeated by an incompressible thermally and electrically conducting viscous fluid. The fluid flow in the porous layer L_2 is governed by Darcy's law, whereas the fluid flow in the porous layer L_1 is governed by Brinkman's law. Gravity **g** acts in the negative direction of x_3 (Fig. 1).

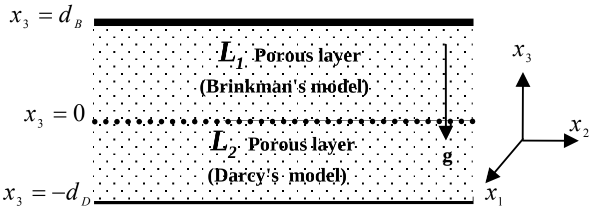

Fig. 1 Schematic representation of the physical configuration

Convection is driven by temperature dependence of the fluid density and damped by viscosity. The Oberbeck-Boussineq approximation is used as the density of fluid is constant everywhere except in the body force term where the density is linearly proportional to temperature, i.e

$$\rho = \rho_0 \left[1 - \alpha \left(T - T_0\right)\right]. \tag{1}$$

the governing equations of the porous layer L_1 are

$$\frac{1}{\varphi_B}\frac{\partial V_B}{\partial t} = -\nabla \frac{P_B}{\rho_0} - \frac{\mu}{K_B} V_B + \nu \nabla^2 V_B - g\left[1 - \alpha\left(T_B - T_0\right)\right]$$
$$\frac{\partial T_B}{\partial t} + V_B . \nabla T_B = k_B \nabla^2 T_B , \tag{2}$$

and the governing equations of the porous layer L_2 are

$$\frac{1}{\varphi_D}\frac{\partial V_D}{\partial t} = -\nabla \frac{P_D}{\rho_0} - \frac{\mu}{K_D} V_D - g\left[1 - \alpha\left(T_D - T_0\right)\right]$$
$$\frac{\partial T_D}{\partial t} + V_D . \nabla T_D = k_D \nabla^2 T_D , \tag{3}$$

where p_D, p_m are the pressure of the porous layers L_1 and L_2 respectively, V_D, V_B are seepage velocity of the porous layers L_1 and L_2 respectively, T_D, T_B are the Kelvin temperature of the porous layers L_1 and L_2 respectively, k_D, k_B are the thermal and overall thermal conductivity of the porous layers L_1 and L_2 respectively, μ is the viscosity, K_D, K_B is the permeability of the porous layers L_1 and L_2 respectively, ϕ_D, ϕ_B is its porosity of the porous layers L_1 and L_2 respectively.

A. The boundary Conditions

Suppose that $x_3 = d_B$ is rigid and maintained at constant temperature T_u, and $x_3 = -d_D$ is rigid and maintained at constant temperature T_l, then the boundary conditions can be written as:

$$w_B\left(d_B\right) = 0, \quad \frac{\partial w_B}{\partial x_3}\left(d_B\right) = 0, \quad T_B\left(d_B\right) = T_l, \tag{4}$$

on the upper boundary, and

$$w_D\left(-d_D\right) = 0, \quad \frac{\partial w_D}{\partial x_3}\left(-d_D\right) = 0, \quad T_D\left(-d_D\right) = T_u, \tag{5}$$

on the lower boundary, where w_B and w_D are the normal axial velocity components of the porous layers L_1 and L_2 respectively. The boundary conditions on the interface plane $x_3 = 0$ are based on the assumption that temperature, heat flux, normal fluid velocity and normal stress tensor are continuous so that

$$T_B(0) = T_D(0), \qquad k_B \frac{\partial T_B}{\partial x_3}(0) = k_D \frac{\partial T_D}{\partial x_3}(0),$$
$$w_B(0) = w_D(0), \qquad -p_B(0) + 2\mu \frac{\partial w_B}{\partial x_3}(0) = -p_D(0), \tag{6}$$

Equations (2) and (3) have an equilibriam solution satisfying the boundary conditions (4)-(6) on the form

$$V_B = 0, \qquad V_D = 0,$$
$$-\nabla P_B + \rho_f g = 0, \qquad -\nabla P_D + \rho_f g = 0, \tag{7}$$
$$\nabla^2 T_D = \nabla^2 T_B = 0,$$

and with the boundary conditions

$$T_B\left(d_B\right) = T_u, \quad T_D\left(-d_D\right) = T_l, \tag{8}$$

and the interface conditions

$$T_B(0) = T_D(0), \quad k_B \frac{\partial T_B}{\partial x_3}(0) = k_D \frac{\partial T_D}{\partial x_3}(0), \quad P_B(0) = P_D(0), \tag{9}$$

the equilibrium temperature field, hydrostatic pressure and salt concentration in the fluid layer and porous medium layer are respectively:

$$T_B = T_0 - \left(T_0 - T_u\right)\frac{x_3}{d_B}, \quad P_B = P_B\left(x_3\right), \quad 0 \le x_3 \le d_B,$$
$$T_D = T_0 - \left(T_l - T_0\right)\frac{x_3}{d_D}, \quad P_D = P_B\left(x_3\right), \quad -d_D \le x_3 \le 0, \tag{10}$$

Where $T_0 = \dfrac{k_B d_B T_u + k_B d_D T_l}{k_D d_B + k_B d_D}$,

B. The Perturbation Equations

Suppose that the equilibrium solution be perturbed by following linear perturbation quantities:

$$\begin{aligned} V_B &= 0 + \varepsilon v_B, \quad P_B = P_B(x_3) + \varepsilon p_B, \\ T_B &= T_0 - (T_0 - T_u)\frac{x_3}{d_B} + \varepsilon\theta_B, \\ V_D &= 0 + \varepsilon v_D, \quad P_D = P_D(x_3) + \varepsilon p_D, \\ T_D &= T_0 - (T_l - T_0)\frac{x_3}{d_D} + \varepsilon\theta_D, \end{aligned} \tag{11}$$

then we may verify that the linearised version of equations (2) are

$$\begin{aligned} \frac{\rho_0}{\varphi_B}\frac{\partial v_B}{\partial t} &= -\nabla p_B - \frac{\mu}{K_B} v_B + \rho_0 \alpha g \theta_B, \\ \frac{\partial \theta_B}{\partial t} - v_B \frac{(T_u - T_0)}{d_B} &= k_B \nabla^2 \theta_B, \end{aligned} \tag{12}$$

and equations (3) are

$$\begin{aligned} \frac{\rho_0}{\varphi_D}\frac{\partial v_D}{\partial t} &= -\nabla p_D - \frac{\mu}{K_D} v_D + \rho_0 \alpha g \theta_D, \\ \frac{\partial \theta_D}{\partial t} - v_D \frac{(T_l - T_0)}{d_D} &= k_D \nabla^2 \theta_D, \end{aligned} \tag{13}$$

The boundary conditions (4)-(6) become respectively

$$\begin{aligned} & w_B(d_B) = 0, \quad \frac{\partial w_D}{\partial x_3}(d_B) = 0, \quad \theta_B(d_B) = 0, \\ & \theta_B(0) = \theta_D(0), \quad k_B \frac{\partial \theta_B}{\partial x_3}(0) = k_D \frac{\partial \theta_D}{\partial x_3}(0), \\ & w_B(0) = w_D(0), \quad -p_B(0) + 2\mu \frac{\partial w_B}{\partial x_3}(0) = -p_D(0), \\ & w_D(\text{-}d_D) = 0, \quad \frac{\partial w_D}{\partial x_3}(\text{-}d_D) = 0, \quad \theta_D(\text{-}d_D) = 0. \end{aligned} \tag{14}$$

C. Non-Dimensionalisation

We now non-dimensionalize the equations (12) and (13) by using the transformation

$$\begin{aligned} & x = d_B x^{\bullet}{}_B, \quad v_B = \frac{\lambda_B}{d_B} v^{\bullet}{}_B, \quad \theta_B = |T_0 - T_u| \theta^{\bullet}{}_B, \\ & t = \frac{d_B^2}{\lambda_B} t^{\bullet}{}_B, \quad p_B = \frac{\mu \lambda_B}{K_B} p^{\bullet}{}_B, \end{aligned} \tag{15}$$

for the fluid layer, and using the transformation

$$\begin{aligned} & x = d_D x^{\bullet}{}_D, \quad v_D = \frac{\lambda_D}{d_D} v^{\bullet}{}_D, \quad \theta_D = |T_l - T_0| \theta^{\bullet}{}_D, \\ & p_D = \frac{\mu \lambda_D}{K_D} p_D{}^{\bullet}, \quad t = \frac{d_D^2}{\lambda_D} t^{\bullet}{}_D, \end{aligned} \tag{16}$$

Thus equations (12) can be written in the form

$$\begin{aligned} \frac{Da_B}{\varphi_B P_{r_B}} \frac{\partial v_B}{\partial t_B} &= -\nabla p_B - v_B + Da_B \nabla^2 v_B + Rt_B \theta_B \\ \frac{\partial \theta_B}{\partial t_B} + F v_B &= \nabla^2 \theta_B, \end{aligned} \tag{17}$$

where P_{r_B}, Da_B and Rt_B are non-dimensional numbers denote the viscous Prandtl number, Darcy number and thermal Rayleigh number of the porous layer L_1 and given by

$$P_{rB} = \frac{\mu}{\rho_0 \lambda_B}, \quad Da = \frac{K_B}{d_B^2}, \quad Rt_B = \frac{\rho_0 g \alpha |T_0 - T_u| K_B d_B}{\mu \lambda_B},$$

and the equations (13) can be written in the form

$$\begin{aligned} \frac{Da_D}{\varphi P_{r_D}} \frac{\partial v_D}{\partial t_D} &= -\nabla p_D - v_D + Rt_D \theta_D, \\ \frac{\partial \theta_D}{\partial t_D} + F v_D &= \nabla^2 \theta_D, \end{aligned} \tag{18}$$

where P_{r_m}, Da_D and Rt_D are non-dimensional numbers denote viscous Prandtl number, Darcy number and thermal Rayleigh number of the porous medium layer L_2 and given by:

$$P_{rD} = \frac{\mu}{\rho_0 \lambda_D}, \quad Da_D = \frac{K_D}{d_D^2}, \quad Rt_D = \frac{g\alpha |T_l - T_0| K_D d_D}{\mu \lambda_D},$$

and where

$$F=\frac{-(T_0-T_u)}{|T_0-T_u|}=\frac{-(T_l-T_0)}{|T_l-T_0|}=\begin{cases}-1, \text{ when heating from below,}\\ 1, \text{ when heating from above.}\end{cases}$$

We will discuss the problem in case of heating from below, so we take $F=-1$. Using (15) and (16) in the boundary conditions (14) we obtain

$$\begin{aligned}
&w_B(1)=0, \quad \frac{\partial w_B}{\partial x_3}(1)=0, \quad \theta_B(1)=0,\\
&\gamma_T\theta_B(0)=\theta_D(0), \quad \frac{\partial \theta_B}{\partial x_3}(0)=\frac{\partial \theta_D}{\partial x_3}(0),\\
&w_B(0)=\gamma_T w_D(0), \quad \frac{1}{\gamma_T\hat{d}}Da_D\left[\frac{1}{Da_B}p_B(0)-2\frac{\partial w_B}{\partial x_3}(0)\right]=p_D(0),\\
&w_D(-1)=0, \quad \frac{\partial w_D}{\partial x_3}(-1)=0, \quad \theta_D(-1)=0,
\end{aligned} \tag{19}$$

where $\hat{d},\varepsilon_T,\gamma_T$ and $\hat{K}$ are given by

$$\hat{d}=\frac{d_B}{d_D}, \quad \varepsilon_T=\frac{k_B}{k_D}, \quad \gamma_T=\frac{|T_u-T_0|}{|T_0-T_l|}=\frac{\hat{d}}{\varepsilon_T}, \quad \hat{K}=\frac{K_B}{K_D},$$

And $\quad P_{r_B}=\frac{1}{\varepsilon_T}P_{r_D}, \quad Da_B=\frac{\hat{K}}{\hat{d}}Da_D \quad Rt_B=\frac{\hat{d}^4 Da_B}{\varepsilon_T{}^2 Da_D}Rt_D.$

Note: the (•) superscript has been dropped from equations (17)-(19) for simplify.

D. Linearisation of Equations

We take the curl curl of the equations $(17)_1$ and $(18)_1$ to eliminate p_B and p_D respectively and considering the third component of the result equations and the equations $(17)_2$ and $(18)_2$, we get

$$\begin{aligned}
&\frac{Da_B}{\varphi_B P_{r_B}}\frac{\partial}{\partial t_B}\nabla^2 w_B=-\nabla^2 w_B+Da_B\nabla^4 w_f+Rt_B\nabla_2^2\theta_B,\\
&\frac{\partial\theta_B}{\partial t_B}\text{-}w_B=\nabla^2\theta_B,
\end{aligned} \tag{20}$$

and

$$\begin{aligned}
&\frac{Da_D}{\varphi_D P_{r_D}}\frac{\partial}{\partial t_D}\nabla^2 w_D=-\nabla^2 w_D+Rt_D\nabla_2^2\theta_D,\\
&\frac{\partial\theta_D}{\partial t_D}\text{-}w_D=\nabla^2\theta_D.
\end{aligned} \tag{21}$$

where $\nabla_2^2=\nabla^2-\frac{\partial^2}{\partial x_3^2}$ is tow-dimensional Laplacian operator and $\nabla^4=(\nabla^2)^2$. To simple the normal stress boundary condition on the interface plane $(19)_7$ by eliminate hydrostatic pressure term so taking tow-dimensional Laplacian of $(19)_7$ we obtain:

$$\frac{1}{\gamma_T\hat{d}^3}Da_D\left[\frac{1}{Da_B}\nabla_2^2 p_B(0)-2\frac{\partial}{\partial x_3}\nabla_2^2 w_B(0)\right]=\nabla_2^2 p_m(0). \tag{22}$$

Since

$$\begin{aligned}
\nabla\cdot v_B=0 &\Rightarrow \frac{\partial u_B}{\partial x_1}+\frac{\partial v_B}{\partial x_2}=-\frac{\partial w_B}{\partial x_3},\\
\nabla\cdot v_D=0 &\Rightarrow \frac{\partial u_D}{\partial x_1}+\frac{\partial v_D}{\partial x_2}=-\frac{\partial w_D}{\partial x_3}.
\end{aligned} \tag{23}$$

then we take the divergence of equations $(17)_1$ and $(18)_1$ we get respectively

$$\nabla_2^2 p_B=\frac{Da_B}{\phi_B P_{r_B}}\frac{\partial}{\partial t}\frac{\partial w_B}{\partial x_3}+\frac{\partial w_B}{\partial x_3}-Da_B\nabla^2\frac{\partial w_B}{\partial x_3}, \tag{24}$$

$$\nabla_2^2 p_D=\frac{Da_D}{\phi_D P_{r_D}}\frac{\partial}{\partial t}\frac{\partial w_D}{\partial x_3}+\frac{\partial w_D}{\partial x_3}. \tag{25}$$

Substitute (24) and (25) in (22) we have

$$\begin{aligned}
&\frac{1}{\gamma_T\hat{d}^3}Da_D\frac{\partial}{\partial x_3}\left(\nabla^2 w_B(0)-\frac{1}{Da_B}w_B-\frac{1}{\phi_B P_{r_B}}\frac{\partial w_B}{\partial t}(0)+\right.\\
&\left.2\nabla_2^2 w_B(0)\right)=-\left(\frac{Da_D}{\phi_D P_{r_D}}\frac{\partial}{\partial t}+1\right)\frac{\partial w_D}{\partial x_3}(0).
\end{aligned} \tag{26}$$

Now we look for solution of the form

$$\Phi(x,t)=\Phi(x_3)\exp[i(nx_1+mx_2)+\sigma t],$$

for the functions w_B,θ_B,w_D and θ_D. Thus the governing equation are:

$$\begin{aligned}
&\frac{Da_B}{\phi_B P_{r_B}}\sigma_B L_B w_B=-L_B w_B+Da_B L_B^2 w_B-a_B^2 Rt_B\theta_B,\\
&\sigma_B\theta_B=w_B+L_B\theta_B,\\
&-\frac{Da_D}{\phi_D P_{r_D}}\sigma_D L_D w_D=L_D w_D+a_D^2 Rt_D\theta_D,\\
&\sigma_D\theta_D=w_D=L_D\theta_D,
\end{aligned} \tag{27}$$

where $a_B=\sqrt{n_B^2+m_B^2}$ and $a_D=\sqrt{n_D^2+m_D^2}$ are non-dimensional wave numbers in the fluid layer and porous medium layer respectively, σ is the grouth rate and

$$a_B = \hat{d} a_D, \quad \sigma_B = \frac{\hat{d}^2}{\varepsilon_T}\sigma_D, \quad D_D = \frac{\partial}{\partial x_3}, \quad x_3 \in [-1,0],$$

$$D_B = \frac{\partial}{\partial x_3}, \quad x_3 \in [0,1],$$

$$L_B = \left(D_B^2 - a_B^2\right), \quad \text{and} \quad L_D = \left(D_D^2 - a_D^2\right).$$

The final boundary conditions are:

$$w_B = 0, \quad D_B w_B = 0, \quad \theta_B = 0, \quad on\ x_3 = 1, \tag{28}$$

$$\left.\begin{aligned} & w_B = \gamma_T w_D, \quad \gamma_T \theta_B = \theta_D, \\ & D_B \theta_B = D_D \theta_D, \\ & \frac{1}{\gamma_T \hat{d}^3} Da_D \left(D_B^3 w_B - \frac{1}{Da_B} D_B w_B - 3a_B^2 D_B w_B - \frac{\sigma_B}{\phi_B P_{r_B}} D_B w_B \right) = \\ & -\left(\frac{Da_D}{\phi_D P_{r_D}} \sigma_D + 1 \right) D_D w_D, \end{aligned}\right\} on\ x_3 = 0, \tag{29}$$

$$w_D = 0, \quad D_D w_D = 0, \quad \theta_D = 0, \quad on\ x_3 = -1. \tag{30}$$

III. Numerical Solution

A Legender polynomials is applied to solve the equations (27) with the relevant boundary conditions (28)-(30), and we map $x_3 \in [0,1]$ and $x_3 \in [-1,0]$ in to $z \in [-1,1]$ by the transformations $z = 2x_3 - 1$ and $z = 2x_3 + 1$ respectively, and get

$$\frac{\partial}{\partial x_3} = 2\frac{\partial}{\partial z}, \quad \text{thus} \quad D_B = D_D = 2\frac{\partial}{\partial z} = D, \quad z \in [-1,1].$$

then, suppose that

$$y_r(z) = \sum_{k=0}^{M-1} \alpha_{kr} P_k(z), \quad 1 \le r \le 10 \quad z \in [-1,1],$$

let the variables y_r where $1 \le r \le 10$ be defined by:

$$\begin{aligned} & y_1 = w_B, \quad y_2 = D_B w_B, \quad y_3 = D_B^2 w_B, \quad y_4 = D_B^3 w_B, \\ & y_5 = \theta_B, \quad y_6 = D_B \theta_B, \\ & y_7 = w_D, \quad y_8 = D_D w_D, \quad y_9 = \theta_D, \quad y_{10} = D_D \theta_D. \end{aligned} \tag{31}$$

Then the equations (27) can be rewritten in a system of eighteen ordinary differential equations of first order as follows

$$D_B y_1 = y_2,$$
$$D_B y_2 = y_3,$$
$$D_B y_3 = y_4,$$
$$D_B y_4 = -\left(\frac{a_B^2}{Da_B} + a_B^4\right) y_1 + \left(2a_B^2 + \frac{1}{Da_B}\right) y_3 + \frac{a_B^2}{Da_B} Rt_B y_5 + \frac{\sigma_B}{\varphi_B P_{r_B}}\left(y_3 - a_B^2 y_1\right),$$
$$D_B y_5 = y_6,$$
$$D_B y_6 = -y_1 + a_B^2 y_5 + \sigma_B y_5,$$
$$D_D y_7 = y_8,$$
$$D_D y_8 = a_D^2 y_7 - a_D^2 Rt_D y_9 + \sigma_D \frac{Da_D}{\varphi_D P_{r_D}}\left(a_D^2 y_7 - D y_8\right),$$
$$D_D y_9 = y_{10},$$
$$D_D y_{10} = -y_7 + a_D^2 y_9 + \sigma_D y_9,$$

and the boundary conditions are

$$y_1 = 0, \quad y_2 = 0, \quad y_5 = 0, \quad on\ z = 1,$$

$$\left.\begin{aligned} & y_1(-1) - \gamma_T y_7(1) = 0, \quad \gamma_T y_5(-1) - y_9(1) = 0, \\ & y_6(-1) - y_{10}(1) = 0, \\ & \frac{1}{\gamma_T \hat{d}^3} Da_D \left(y_4(-1) - \left(\frac{1}{Da_B} + 3a_B^2\right) y_2(-1) + y_8(1) \right) = \sigma \left(\frac{\hat{d}^2}{\varepsilon_T \phi_B P_{r_B}} y_2(-1) - \frac{Da_D}{\phi_D P_{r_D}} y_8(1) \right), \end{aligned}\right\} on\ z = 0$$

$$y_7 = 0, \quad y_8 = 0, \quad y_9 = 0, \quad on\ z = -1,$$

Since $D_B = D_D = D$ and if we put $\sigma_m = \sigma$ then $\sigma_B = \frac{\hat{d}^2}{\varepsilon_T}\sigma$ so the eigenvalue problem can be reformulated in the form

$$\frac{dY}{dz} = AY + \sigma BY, \quad z \in [-1,1],$$

where A and B are real 10×10 matrices.

IV. Results and Discussion

The eigen value problem (27) with boundary conditions (28)- (30) by using Legendre polynomials is transformed to a system of five ordinary differential equations of first order in the porous layer L_1 and a system of five ordinary differential equations of first order in the porous layer L_2 with ten boundary conditions. We will find the thermal Rayleigh numbers of the porous medium Rt_D corresponding to the wave numbers a_D for different values of depth ratio $\hat{d}$, permeability ratio $\hat{K}$ and thermal conductivity ratio ε_T as shown in the following Figs. 2-9. Therefore, we concluded that:

- The deeper the space between the two porous layers is the less value the thermal Rayleigh numbers will be, which leads to the instability of the fluid. This means that the less deep the Darcy, governed porous layer is the more the thermal convection, as shown in Fig. 2.
- The increases of the rate of permeability $\hat{K}$ helps suppress the thermal convection which leads to the stability of the fluid. This case becomes clearer when the space between two porous layers decreases, as shown in Figs. 3-5.
- As thermal conductivity ratio ε_T increases, the thermal Rayleigh number increases. This means that when the porous layer governed by Brinkman's model is more thermal conductive than the porous layer governed by Darcy's model it helps stabilize the fluid. This case becomes more clear when the space between two porous layers decreases and the rate of permeability increases, as shown in Figs. 6-9.

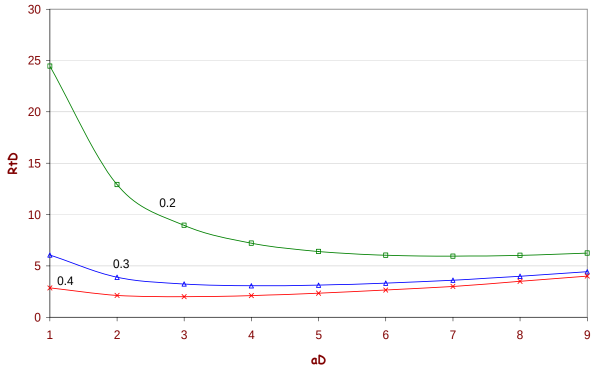

Fig. 2 The relation between a_D and Rt_D for different value of $\hat{d}$, $Da_D = 4\times10^{-6}$, $\varepsilon_T = 0.7$ and $\hat{K} = 0.01$

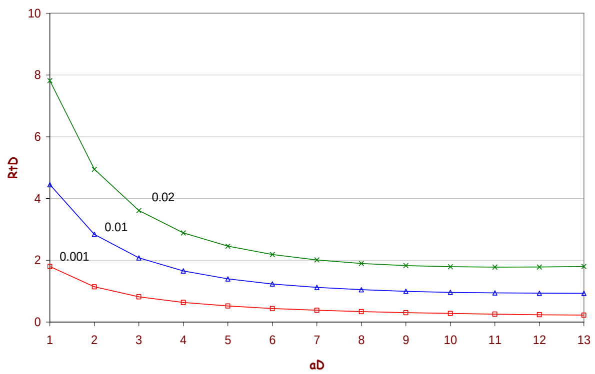

Fig. 3 The relation between a_D and Rt_D for different value of $\hat{K}$, $\hat{d} = 0.14$, $Da_D = 4\times10^{-6}$ and $\varepsilon_T = 0.7$

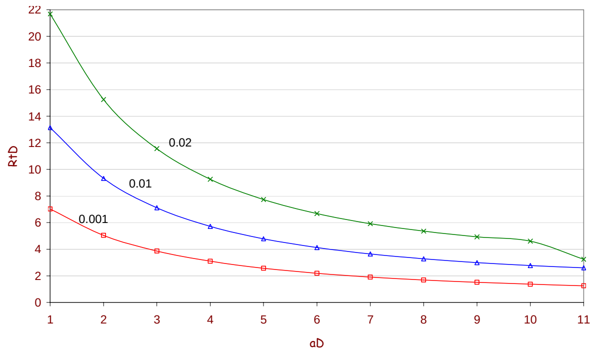

Fig. 4 The relation between a_D and Rt_D for different value of $\hat{K}$, $\hat{d} = 0.09$, $Da_D = 4\times10^{-6}$ and $\varepsilon_T = 0.7$

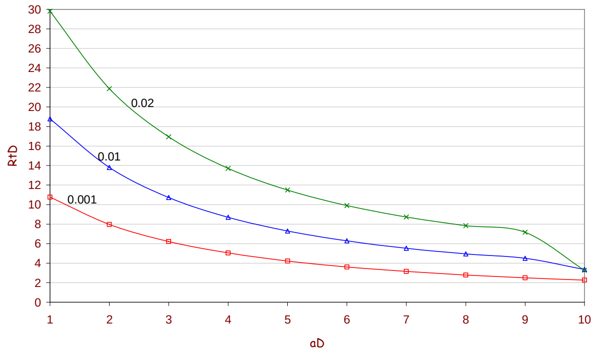

Fig. 5 The relation between a_D and Rt_D for different value of $\hat{K}$, $\hat{d} = 0.08$, $Da_D = 4\times10^{-6}$ and $\varepsilon_T = 0.7$

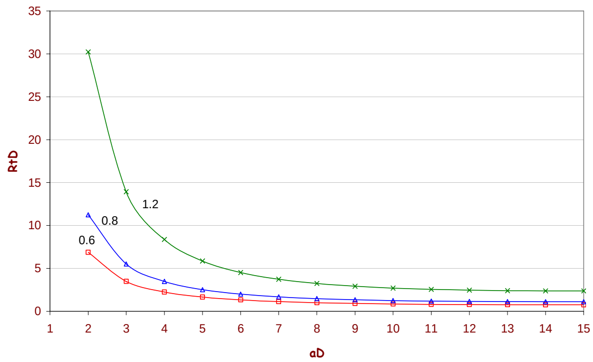

Fig. 6 The relation between a_D and Rt_D for different value of ε_T, $\hat{K} = 0.01$, $\hat{d} = 0.14$ and $Da_D = 4\times10^{-6}$

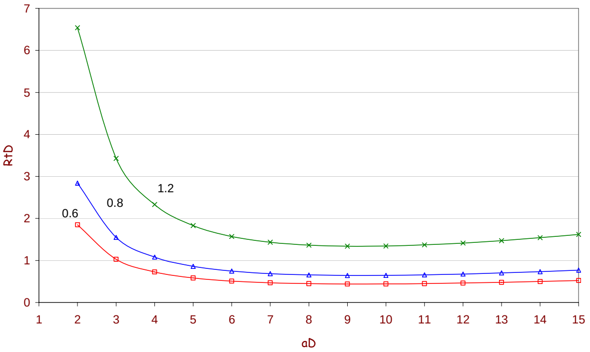

Fig. 7 The relation between a_D and Rt_D for different value of ε_T, $\hat{K} = 0.01$, $\hat{d} = 0.2$ and $Da_D = 4\times10^{-6}$

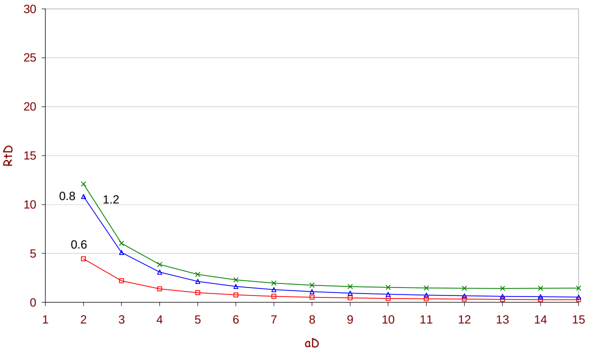

Fig. 8 The relation between a_D and Rt_D for different value of ε_T, $\hat{K} = 0.001$, $\hat{d} = 0.14$ and $Da_D = 4\times10^{-6}$

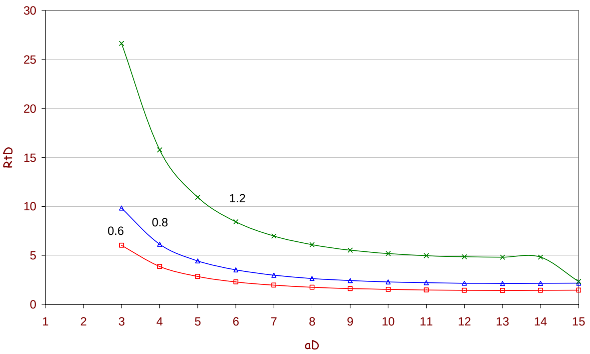

Fig. 9 The relation between a_D and Rt_D for different value of ε_T, $\hat{K} = 0.02$, $\hat{d} = 0.14$ and $Da_D = 4\times10^{-6}$.

References

[1] Benard, H., Les tourbillons cellulaires dans une nappe liquide., Revue genral des sciences pures et appliquess, 11, 1216-1271 and 1309-1328 (1900).
[2] Benard, H., Les tourbillons cellulaires dans une nappe liquide transportant de la chalaur par convection en regime permanent., Ann. Chim. Phys., 23, 62-114 (1901).
[3] Chandrasekhar, S., The instability of a layer of fluid heated below and subject to Coriolis forces., Proc. Roy. Soc.(London) Vol. 217, 306-327 (1953).
[4] Chandrasekhar, S. and Elbert, D. D., The instability of a layer of fluid heated below and subject to Coriolis forces.II., Proc. Roy. Soc.(London). No. 231, 198-210 (1955).
[5] Nield, D. A., Onset of themohaline convection in a porous medium, water resources Res., 4, 553-560 (1968).
[6] Taunton, J. & Lightfoot, E., Thermohaline instability and salt fingers in a porous medium, Phys. Of fluid, 15, No. 5, 784-753 (1972).
[7] Sun, W. J., Convection Instability in Superposed Porous and Free Layers, Ph.D. Dissertation, University of Minnesota, Minneapolis. MN. 1973.
[8] Nield, D. A., Onset of Convection in a fluid layer overlying a porous me dium, J. Fluid Mech., 81, 513-522 (1977).
[9] Brinkman, H. C., A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles,. Appl. Sci. Res. Al. 27-34 (1947).
[10] Chen, F. & Chen, C.F., Onset of finger convection in a horizontal porous layer underlying a fluid layer., J. of Heat Tran., 110, 403-409 (1988).
[11] Lindsay K. A. & Ogden R. R., A Practical Implementation of Spectral Methods Resistant To The Generation of Spurious Eigenvalues Paper Nn. 92/23 University of Glasgow, Department of Mathematics preprint series (1992).
[12] Lamb, C. J., Hydromagnetic Instability In the Earth's Core, Ph.D. thesis, Department of Mathematics, University of Glasgow 1994.
[13] Bukhari, A. f., A Spectral Eigenvalue Method for Multi-layered Continuua., Ph.D. thesis, University of Glasgow, Scot. 1996.
[14] Straughan, B., Surface-tension-driven convection in a fluid overlying a porous layer, J. Comput. Phys. 170, 320-337 (2001).
[15] Straughan, B., Effect of property variation and modelling on convection in a fluid overlying a porous layer, Int. J. Numer. Anal. Meth. Geomech. 26, 75-97 (2002).
[16] Allehiany, F. , Benard convection in a horizontal porous layer permeated by a conducting fluid in the presence of magnetic field and coriolis forces, An M. Sc. Thesis Umm Al- Qura University 2003.
[17] AL-Qurashi, M.S. & Bukhari, Abdul-Fattah K., Salt-finger convection in a horizontal porous layer superposes a fluid layer affected by rotation and vertical linear magnetic field on both layers., Advances and Applications in Fluid Mechanics, vol.11, 119-146,Apr.(2012).