Analytical Model for Brine Discharges from a Sea Outfall with Multiport Diffusers

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***Abstract*—**Multiport diffusers are the effective engineering devices installed at the modern marine outfalls for the steady discharge of effluent streams from the coastal plants, such as municipal sewage treatment, thermal power generation and seawater desalination. A mathematical model using a two-dimensional advection-diffusion equation based on a flat seabed and incorporating the effect of a coastal tidal current is developed to calculate the compounded concentration following discharges of desalination brine from a sea outfall with multiport diffusers. The analytical solutions are computed graphically to illustrate the merging of multiple brine plumes in shallow coastal waters, and further approximation will be made to the maximum shoreline's concentration to formulate dilution of a multiport diffuser discharge.

***Keywords*—**Desalination brine discharge, mathematical model, multiport diffuser, two sea outfalls.

I. Introduction

As desalinated water is indispensably required at all costs in hot and arid climate countries, there are intense seawater desalination activities in certain coastlines of the Arabian Gulf, Red Sea, Mediterranean Sea, and the Gulf of Oman. In particular, more than half of the world's desalination plants are constructed along the coasts of the Arabian Gulf, Gulf of Oman and Red Sea [1]. Thus, along such coastal areas, many seawater desalination plants are commonly found to be operated closely together. Furthermore, as the needs for desalinated water are steadily increasing, not only are the number of new large scale desalination plants growing, the existing plants are also gradually increasing their water production capacities. Like any large scale industrial process, seawater desalination unfortunately also has its potential environmental impacts and is a serious threat to marine ecosystems [2]. Desalination plants extract large volumes of seawater and discharge hypersaline brine, a reject concentrate stream, back into the marine environment [3]. Current technology limits the efficiency of producing desalinated water, and up to 60% is lost via concentrate stream that is more than double the typical seawater salinity.

Most large scale coastal desalination plants dispose of their concentrate via long outfall pipes that stretch far into the ocean [4]-[5], and as concentrate stream enters the receiving marine waters, it creates a high salinity plume. An engineering solution utilizing the best available technology is required where a multiport diffuser would be installed at the pipe-end to rapidly dilute the concentrate. Without proper dilution [5], the brine plume will tend to sink and propagate down the slope for hundreds of meters, harming the ecosystem along the way, and most at risk are the benthic marine organisms living at the sea bottom. The worst situation may occur along the highly populated coastal areas of the Arabian Gulf, Gulf of Oman and Red Sea, where multiple sea outfalls are discharging large volumes of desalination brine.

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Fig. 1 shows two marine outfall systems of the (up to 4 co-location) Barka power generation and seawater desalination plants in the Gulf of Oman [4]. Each outfall system is designed for a maximum capacity of 122,100 m^3/h to discharge the cooling water from the power generation plants and mix it with brine reject (and other effluents) from seawater desalination plants. The old (currently in use by the existing Barka I and II plants) outfall pipe length is about 650 m, while the new (not yet been used) outfall pipe length is about 1200 m, and the distance between the two discharge points is 1000 m. The old outfall system comprises of four parallel pipes angled at 62 degrees to the coastline, each with a diameter of 2.5 m, buried at 5 m below the seabed (not visible on the surface) and spaced equally at 4.8 m apart. Each pipe has a 62.4 m long multiport diffuser, consisting of nine ports equally spaced at 7.5 m apart, installed at the end of each outfall pipe. The multiport diffusers are arranged in two nested V shapes as illustrated in Fig. 1, and each pair diverges at an angle of 30 degrees on either side of the outfall pipelines. The two internal pipes of length 653 m have its end at a depth of 9 m below the mean sea level, while the other two shorter external pipes of length 582 m end at a depth of 8.4 m. The ports of each diffuser are oriented in an alternating way, each with an angle of 20 degrees to the diffuser pipe. The port diameter is 0.7 m and located at 1 m above the seabed, and the ports are oriented upwards with an angle of 10 degrees against the horizontal.

Owing to the highly variable nature of the sea, we do not yet have a full understanding of the mixing processes of desalination brine discharges, and the use of mathematical models has been a key strategy for assessing the potential marine environmental impacts [4],[6]. To demonstrate the effectiveness of a multiport diffuser in diluting the effluent stream, many laboratory and field experimental measurements have been carried out to form several empirical equations for the effluent plume formed from the merging of individual port discharges [7]-[8]. However, no analytical or numerical computations have been done to model and reproduce the interaction and overlapping of multiple effluent plumes. A clear understanding of these processes is needed so that predictive models can be developed which form the basis of sound engineering design. The analytical formulation for the dilution of a multiport diffuser discharge is derived here to measure its effectiveness over the single outfall discharge.

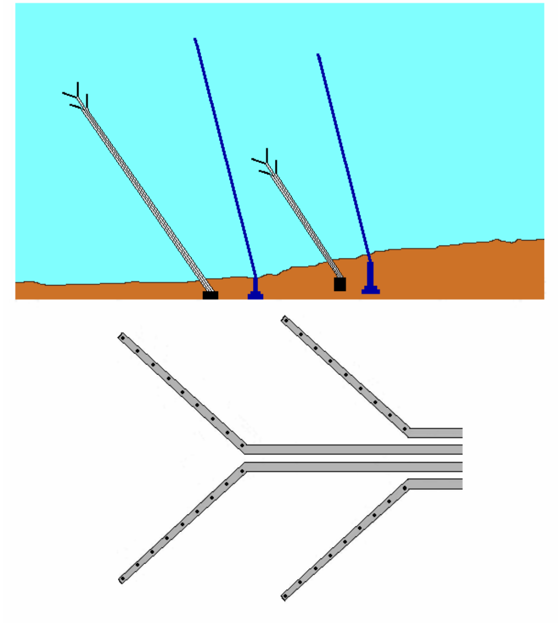

Fig. 1 Two marine outfall systems of Barka plants, Oman (top), and the multiport diffusers at the end of the outfall pipes (below)

II. Model Formulation

Immediately after steady release from the submerged multiport diffusers, vigorous and rapid dilution of desalination brine is governed by the effluent buoyancy, momentum of the discharge and its interaction with the sea currents [5],[7]. At the end of this mixing zone stage, adjacent brine plumes interact with each other and merge to form a rising curtain, then it continues to drift away with the currents. Because of relatively shallow water depth, it is observed that the elongated brine plumes are spreading towards the shoreline and may cause salinity build-up in the coastal waters [9]-[10].

As we are only concerned with the effect of longshore currents on the long-term (far field) desalination brine plume, a highly simplified semi-infinite flat seabed is considered, where the shoreline is straight and of a constant water depth [11]-[12]. The coastal current is assumed to be uniform over water depth and remains in the *x*-direction parallel to the shoreline. A simple model of a longshore current that consists of a steady (residual) drift and a periodic component with amplitude U_0 can be represented by [10]-[12]: $u(t) = v + U_0 \sin \omega t$. Using typical numerical values of the mean tidal amplitude $U_0 = 0.2$ m/s [4],[10] relevant to the Gulf of Oman and the period $2\pi/\omega = 4.5\times10^4$ s, we define a length scale U_0/ω of the order of 1400 m.

The dispersion processes are represented by the longitudinal diffusivity D_x and lateral diffusivity D_y. Note that for shallow coastal waters, the dispersion in the vertical direction occurs much faster than in the lateral direction. For simplicity, the other complexities such as density and temperature are ignored. We assume that the outfall's brine plume is vertically well mixed over the water depth, and we also consider the desalination brine stream to be steadily discharged, starting from the initial time t_i, at a rate Q_0 from the (original) single outfall at the position $(x_0 = 0, y_0 = \alpha)$, at a different rate Q_1 from the (new) first outfall at the position $(x_1 = -\ell, y_1 = \alpha + h)$, at a rate Q_2 from the second outfall at $(x_2 = -2\ell, y_2 = \alpha + 2h)$, and so on, where h is the outfall's offshore and ℓ along the shore separation distances. For a single outfall, the total effluent load is a function of Q_0. As illustrated in Fig. 2, these points $(x_k = -k\ell, y_k = \alpha + kh)$ represent a series of outfalls, each releasing an effluent stream with rate Q_k, and if both values of h and ℓ are too small compared to α , they are similar to the engineering design of a multiport line diffuser. For example, for the Barka plant's multiport diffusers [4], $\ell = 3.75$ m, $h = 6.5$ m and $\alpha = 576.5$ m. Furthermore, for a line diffuser with *n* ports, the total effluent load is distributed into *n* individual discharges, so that each port discharges equally at a rate $Q_n = Q_0/n$.

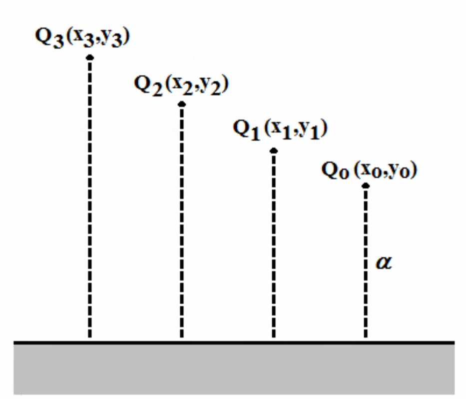

Fig. 2 Definition diagram of multiple sea outfalls

Following [10] and by applying a linear superposition, the conventional equation used to model the concentration *c* of desalination brine discharges from the *n*+1 outfalls is the two-dimensional advection-diffusion equation

$$\frac{\partial c}{\partial t} + u(t)\frac{\partial c}{\partial x} - D_x \frac{\partial^2 c}{\partial x^2} - D_y \frac{\partial^2 c}{\partial y^2} = \delta(t - t_i)\sum_{k=0}^{n} Q_k\ \delta(x + x_k)\left[\delta(y - y_k) + \delta(y + y_k)\right] \quad (1)$$

where c is assumed to be ultimately dissolved into the ocean, δ the Dirac delta function, and in order to satisfy the boundary condition at $y = 0$, for each point source (x_k, y_k), an imaginary source is added at $(x_k, -y_k)$. In terms of the dimensionless variables, the solution of (1) is given by

$$C = \int_0^{T_i} \frac{dT_0}{T_0} \sum_{k=0}^{n} q_{k*} \exp\left[-\frac{\lambda}{T_0}\{X + kL - X_0(T)\}^2\right] \times \left\{\exp\left[-\frac{\lambda\eta(Y - \Lambda - kH)^2}{T_0}\right] + \exp\left[-\frac{\lambda\eta(Y + \Lambda + kH)^2}{T_0}\right]\right\} \quad (2)$$

where $C = 4\pi c\sqrt{D_x D_y}/Q_0$, $T_i = \omega t_i$, $T_0 = T - \omega t_0$, $q_{k*} = Q_k/Q_0$, $\lambda = U_0{}^2/4\omega D_x$, $X = \omega x/U_0$, $L = \omega\ell/U_0$, $X_0(T) = VT_0 - \cos T + \cos(T - T_0)$, $\eta = D_x/D_y$, $Y = \omega y/U_0$, $\Lambda = \omega\alpha/U_0$ and $H = \omega h/U_0$. The interaction and merging process of the brine plumes' behaviour is controlled by the interplay of the model parameters [10]; V, the ratio of the drift current to tidal amplitude; λ, the distances by which the plume is transported and spread over by advection to that by longitudinal diffusion [12]; η, the ratio of longitudinal to lateral diffusivities, and other parameters related to the multiple outfalls, such as Λ the (original) single outfall (offshore) distance, H port's (offshore) separation distance, and L port's (along the shore) separation distance. Note that, for a multiport line diffuser with n ports, both values of H and L are much smaller than Λ, and $q_{k*} = 1/n$.

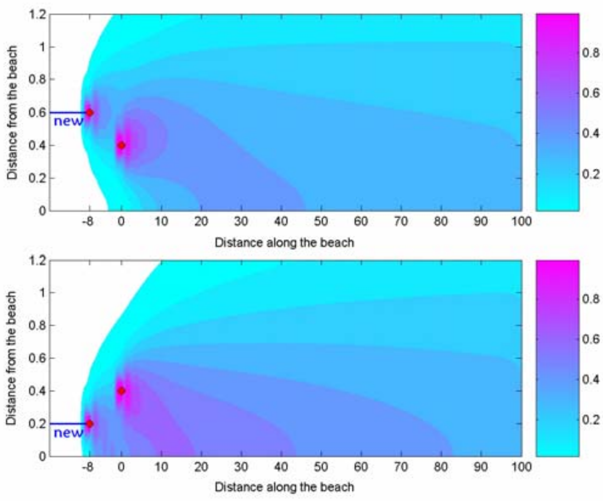

Fig. 3 Simulated merging of effluent plumes from two sea outfalls

Due to the unpredictable sea conditions, very little information is available on the model parameters [13]-[14], and for the quantitative illustration of the model applications, the values of $V = 0.3$, $\lambda = 10$, $\eta = 25$ will be used in all plots. By plotting the results of numerical integrations of (2), the merging processes of two brine plumes from coastal seawater desalination plants are reproduced graphically. Fig. 3 shows a dynamically equivalent representation of the overlapping of two plumes drifting along the coast at $T = 2\pi$ when $\Lambda = 0.4$, $q_{1*} = 1$, the separation distances (offshore) $H = \pm 0.2$ and along the shore distance $L = 8$. Taking a closer look at Fig. 3, the compounded contribution from the (new) first outfall can be minimized [9], provided $Y_1 = \Lambda + H > Y_0 = \Lambda$, i.e. by building a longer new outfall far away from the shoreline. Note that the actual plumes are very elongated in the x-direction, and the peakiness of the plume reflects the physical feature of flow oscillations.

III. Multiport Diffuser Discharges

For a large volume effluent discharge, the engineering practice is to distribute the effluent stream over a large expanse by installing a multiport diffuser at the end of a marine outfall to substantially improve the mixing and dilution of effluent plumes in the coastal waters [5],[7]-[8]. A multiport diffuser is a linear structure consisting of many closely spaced ports or nozzles which release a series of effluent plumes into the receiving coastal water. As shown in Fig. 4, we consider two major diffuser designs that take special account of coastal current direction: *perpendicular*, where the line diffuser with n ports is placed along the y-axis in the offshore direction; and *parallel*, where the line diffuser is constructed aligned to the x-axis parallel to the shoreline.

An appropriate measure for assessing the impact of desalination brine stream discharges into the sea would be the shoreline's concentration values [9]-[10]. On substituting $y = 0$ into (2), we obtain

$$C_* = \frac{2}{n+1}\int_0^{T_i} \frac{dT_0}{T_0} \times \sum_{k=0}^{n} \exp\left[-\frac{\lambda}{T_0}\left(\{X + kL - X_0(T)\}^2 + \eta\{\Lambda + kH\}^2\right)\right] \quad (3)$$

If we are only interested in the long-term impact, i.e. in the limit as $T_i \to \infty$, then the term $\cos(T - T_0)$ may be neglected from (3) as it has little contribution to the integral [11]. From the integral formula $\int_0^{\infty} \frac{dx}{x}\exp\left(-\frac{A}{x} - Bx\right) = 2K_0\left(2\sqrt{AB}\right)$, where K_0 is a modified Bessel function of the second kind

[15], the resulting closed form of the shoreline's concentration (3) simplifies to

$$C_{*\infty} = \frac{4}{n+1}\sum_{k=0}^{n}\exp(2\lambda V\{X+kL+\cos T\}) \times K_0\left(2\lambda V\sqrt{\{X+kL+\cos T\}^2+\eta\{\Lambda+kH\}^2}\right) \quad (4)$$

Next, using the asymptotic representation $K_0(x) \approx \sqrt{\pi/2x}\,\exp(-x)$, (4) can be written as

$$C_{*\infty} \approx \frac{1}{n+1}\sum_{k=0}^{n}\sqrt{\frac{4\pi}{\lambda V(X+kL+\cos T)}} \times \exp\left(-\frac{\lambda V\eta\{\Lambda+kH\}^2}{X+kL+\cos T}\right) \quad (5)$$

By substituting X in (5) with the (original) single outfall's maximum value $X_{\max} = 2\lambda V\eta\Lambda^2 - \cos T$ [10], the maximum shoreline's concentration is approximated by

$$C_{*\max} \approx \frac{1}{(n+1)\lambda V}\sqrt{\frac{4\pi}{\eta}} \times \sum_{k=0}^{n}\frac{1}{\sqrt{2\Lambda^2+kL/\lambda V\eta}}\exp\left(-\frac{\{\Lambda+kH\}^2}{2\Lambda^2+kL/\lambda V\eta}\right) \quad (6)$$

It is easy to see that for a single outfall when $L = 0 = H$, then (6) reduces to $C_{*0} = \frac{1}{\lambda V\Lambda}\sqrt{\frac{2\pi}{\eta e}}$, the maximum shoreline's concentration from the (original) single outfall [10].

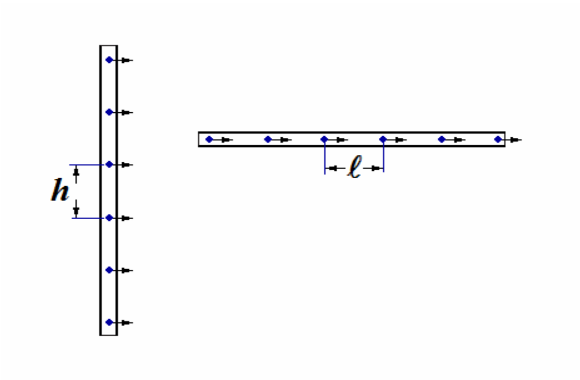

Fig. 4 Multiport line diffuser designs: Perpendicular (left) and parallel (right) to the direction of current

A. Perpendicular Multiport Diffuser

As shown in Fig. 4, geometrically the line diffuser with *n* ports is placed in the *y*-direction perpendicular to the current direction, and it consists of a series of ports equally spaced by the offshore separation distance *H*. Thus, when $L = 0$, and using the fact that H/Λ is small, we can linearize (6), and then after summing for *n* ports, obtain

$$C_{*\max} \approx C_{*0}\left(1-\frac{n}{2}\frac{H}{\Lambda}\right) \quad (7)$$

As the number of ports increases and the longer the offshore distance, the maximum shoreline's concentration (7) gets smaller than that of the (original) single outfall value. Fig. 5 shows the brine plume dilution, which is defined as the ratio of the initial concentration at the outfall discharge point to that at a given location, when $\Lambda = 0.5$ for three values of H/Λ. In particular for a 9-port line diffuser, a dilution of 5.1 is obtained for $H/\Lambda = 0.01$, and it increases to 6.2 as H/Λ increases to 0.03. Similarly, for $H/\Lambda = 0.01$, a dilution of 5.5 can be achieved by increasing the number of ports to 21. This finding is in agreement with the general fact that a multiport diffuser improves the mixing of effluent plumes substantially with dilutions between about 5 and 20, mainly because the individual plumes are collapsed and swept away rapidly by the current.

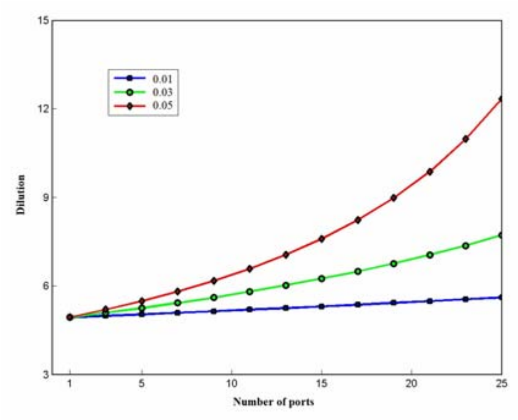

Fig. 5 Dilution for a perpendicular line diffuser discharge

B. Parallel Multiport Diffuser

Geometrically, this line diffuser is placed in the *x*-direction parallel to the current direction, and it consists of a series of ports equally spaced by along the shore separation distance *L*. Thus, when $H = 0$, and since the numerical values of $L/4\lambda V\eta\Lambda^2$ are much smaller than H/Λ, we can again linearize (6), and then after summing for *n* ports, obtain

$$C_{*\max} \approx C_{*0}\left[1-\frac{n(2n+1)}{6}\left(\frac{L}{4\lambda V\eta\Lambda^2}\right)^2\right] \quad (8)$$

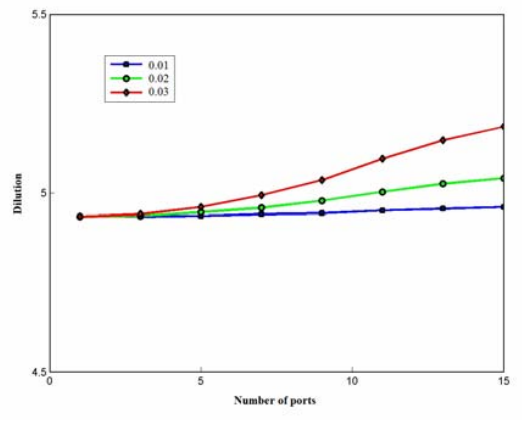

Fig. 6 Dilution for a parallel line diffuser discharge

Although the formula (8) also shows a reduction in the maximum concentration, it is much less than that of the perpendicular line diffuser (7). Fig. 6 shows the brine plume dilution when $\Lambda = 0.5$ for three values of $L/4\lambda V\eta\Lambda^2$. In the case of a 9-port line diffuser, only a dilution of 4.9 is obtained for $L/4\lambda V\eta\Lambda^2 = 0.01$, and it increases slightly to 5.0 as $L/4\lambda V\eta\Lambda^2$ increases to 0.03. Similarly, for $L/4\lambda V\eta\Lambda^2 = 0.01$, a dilution of slightly below 5 can be achieved by increasing the number of ports to 15. These are mainly due to the location of ports in respect to the current; unless the separation distance is large enough, the current repeatedly sweeps the individual plumes from the multiport diffuser discharges and cannot distinguish it from the plume steadily being discharged from a single outfall.

IV. Conclusion

The multiport diffusers are commonly designed in the form of a linear structure consisting of many closely spaced ports or nozzles which inject a series of effluent plumes into the receiving coastal waters. Such diffusers have increasingly been installed at the end of marine outfall pipelines from coastal seawater desalination plants as part of an engineering solution to minimize the potential environmental impacts of a large volume of brine discharges. A multiport diffuser is an efficient mixing device, capable of thoroughly mixing and diluting the effluent within a short distance with dilutions of about 5 to 20. A mathematical model is developed here to replicate and capture the process of merging discharge plumes from a multiport diffuser. The maximum diffuser-induced shoreline concentration is then formulated, and the results show that the perpendicular line diffuser (to the current direction) is better than that of the parallel line diffuser. The model could be extended to formulate dilution for other designs of multiport line diffusers.

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