

# Stabilization of Nonnecessarily Inversely Stable First-Order Adaptive Systems under Saturated Input

M. De la Sen, and O. Barambones

**Abstract**—This paper presents an indirect adaptive stabilization scheme for first-order continuous-time systems under saturated input which is described by a sigmoidal function. The singularities are avoided through a modification scheme for the estimated plant parameter vector so that its associated Sylvester matrix is guaranteed to be non-singular and then the estimated plant model is controllable. The modification mechanism involves the use of a hysteresis switching function. An alternative hybrid scheme, whose estimated parameters are updated at sampling instants is also given to solve a similar adaptive stabilization problem. Such a scheme also uses hysteresis switching for modification of the parameter estimates so as to ensure the controllability of the estimated plant model.

**Keywords**—Hybrid dynamic systems, discrete systems, saturated input, control, stabilization.

## I. INTRODUCTION

THE inputs to physical systems usually present saturation phenomena which limit the amplitudes which excite the linear dynamics, [1-2]. Also, the adaptive stabilization and control of linear continuous and discrete systems has been successfully investigated in the last years. Classically, the plant is assumed to be inversely stable and its relative degree and its high-frequency gain sign are assumed to be known together with an absolute upper-bound for that gain in the discrete case. Attempts of relaxing such assumptions have been made for continuous systems, [5-7]. The assumption on the knowledge of the order can be relaxed by assuming a known nominal order and considering the exceeding modes and unmodelled dynamics, [13-16], [19]. The assumption on the knowledge of the high frequency gain has been removed in [6] and [17] and the assumption of the plant being inversely stable has been successfully removed in the discrete case and more recently in the continuous one, [10-16]. The problem has been solved by using either excitation of the plant signals or by exploiting the properties of the standard least-squares covariance matrix combined with an estimation modification rule based upon the use of a hysteresis switching function, [12-16], [18].

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Such an estimates modification technique guarantees that the modified estimated plant model is controllable at all time provided that the plant is controllable. *This paper presents an adaptive stabilization algorithm for first - order continuous - time systems with a zero which can be either stable or unstable under saturated input. The saturating device is modelled by a sigmoidal function .* Such an approach is a very good approximation to the common saturations usually modelled as piecewise-continuous functions. Also, it is an exact model for saturations inherent to practical MOS-type amplifiers. The adaptive scheme uses a parameter modification rule which guarantees that the absolute value of the determinant of the Sylvester matrix associated with the modified parameter estimates is bounded from below by a positive threshold and, thus, the estimated model is guaranteed to be controllable. That feature is the main contribution of this manuscript. The results are then extended to the case when an adaptive stabilizer, which re-updates at sampling instants the plant estimates, modified estimates and controller parameters, is used for the above continuous - time plant. This strategy results in a hybrid closed-loop system because of the discrete nature of the updating procedure of the parametrical estimation / modification.

## II. ADAPTIVE STABILIZATION

### A. Plant, Estimation / Modification Scheme and Adaptive Stabilization Law

Consider the following continuous-time first-order controllable system under saturated input:

$$\dot{y} + a^* y = b_0^* \dot{u} + b_1^* u' \quad (1.a)$$

$$u' = \text{sat}_v^*(u) = \begin{cases} u & |u| \leq v \\ v \cdot \frac{u}{|u|} & |u| > v \end{cases} \quad (1.b)$$

where the saturated input  $u'$  to the plant (1.a) is modelled by a sigmoidal function (1.b), [2]. To simplify the writing, the argument (t) is omitted and all the constants are denoted by superscripts by '\*'. Eqn. 1.a can be rewritten as

$$\dot{y} = -a^* y + b_0^* \dot{u} + b_1^* u + b_0^* (\dot{u}' - \dot{u}) + b_1^* (u' - u) \quad (2)$$

Note that the equivalence between (1.a) and (2) is an identity where positive and negative terms concerned with the

unsaturated input and its time-derivative are cancelled in the right-hand-side of (2). Define filtered signals

$$\dot{u}_f = -d^* u_f + u ; \quad \dot{u}'_f = -d^* u'_f + u' ; \quad \dot{y}_f = -d^* y_f + y \quad (3)$$

for some scalar  $d^* > 0$  so that one gets from (2) for filtered signals

$$\dot{y}_f = \theta^{*T} \varphi = -a^* y_f + b_0^* \dot{u}_f + b_1^* u'_f + \varepsilon_0^* e^{-d^* t} \quad (4.a)$$

$$\dot{y}_f = -a^* y_f + b_0^* \dot{u}_f + b_1^* u'_f + b_0^* (\dot{u}'_f - \dot{u}_f) + b_1^* (u'_f - u_f) + \varepsilon_0^* e^{-d^* t} \quad (4.b)$$

where

$$\theta^* = [b_0^*, b_1^*, a^*, b_0^*, b_1^*, \varepsilon_0^*]^T \quad (5.a)$$

$$\varphi = [\dot{u}_f, u_f, -y_f, \dot{u}'_f - \dot{u}_f, u'_f - u_f, e^{-d^* t}]^T \quad (5.b)$$

where  $\varepsilon_0^* = y_f(0) - u'_f(0)$  has been included in  $\theta^{*T}$  to obtain (4) without neglecting the exponentially decaying term due to initial conditions of the filters  $1/(s + d^*)$  used in (4) as proposed in [13], [15] and [16]. Also, the over-parametrization of (5.a)-(5.b), in the sense that the coefficients of the numerator polynomial are estimated twice with different regressors, allows describing (4.a) as driven by  $u_f$  and  $u'_f - u_f$ . This idea will be then exploited for the stability analysis of the adaptive stabilizer. The parameter vector  $\theta^{*T}$  can now be estimated by using the least-squares algorithm

$$e = \dot{y}_f - \theta^T \varphi \quad (6)$$

$$\dot{\theta} = P \varphi e \quad (7)$$

$$\dot{P} = -P \varphi \varphi^T P ; P(0) = P^T(0) > 0 \quad (8)$$

where  $e$  is the prediction error,  $\theta = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)^T$  is the estimate of  $\theta^*$ , defined in (5.a), and  $P$  is the covariance matrix. The use of (4.b) into (6) yields

$$\dot{y}_f = \theta_1 \dot{u}_f + \theta_2 u_f - \theta_3 y_f + \theta_4 (\dot{u}'_f - \dot{u}_f) + \theta_5 (u'_f - u_f) + \theta_6 e^{-d^* t} + e \quad (9)$$

The following modification rule of the parameter estimates is used to guarantee the controllability of the estimated plant model

$$\dot{\bar{\theta}} = \theta + P \beta \quad (10)$$

with  $\beta$  being a vector which can be chosen to be equal to one of the following vectors:

$$\beta_1 = [0, 0, \dots, 0]^T ; \quad \beta_2 = v ; \quad \beta_3 = -\beta_2 \quad (11.a)$$

$$\beta_4 = p_1 - p_4 + p_3 ; \quad \beta_5 = -\beta_4 ; \quad \beta_6 = p_1 - p_4 - p_3 \quad (11.b)$$

$$\beta_7 = -(p_1 - p_4) + p_3 ; \quad v = (\theta_1 - \theta_4) p_3 + \theta_3 (p_1 - p_4) - (p_2 - p_5) \quad (11.c)$$

and whose current value is selected from a hysteresis switching function which is defined by the following rule. Define

$$c(\beta) = |(\bar{\theta}_1 - \bar{\theta}_4) \bar{\theta}_3 - (\bar{\theta}_2 - \bar{\theta}_5)|$$

$$= \left| \text{Det} \begin{bmatrix} 1 & 0 & 0 \\ \bar{\theta}_3 & 1 & \bar{\theta}_1 - \bar{\theta}_4 \\ 0 & \bar{\theta}_3 & \bar{\theta}_2 - \bar{\theta}_5 \end{bmatrix} \right|$$

which is the absolute value of the Sylvester matrix of the modified parameter estimates associated with the estimation of the plant numerator and denominator polynomials obtained from (8)-(9) and (10)-(12). Assume that  $\beta(t^-) = \beta_j(t^-)$  and  $c(\beta_j(t^+)) \geq c(\beta_m(t^+))$  for some  $j = 1, 2, \dots, 7$  with  $j \neq i$  and all  $m = 1, 2, \dots, 7$ . Thus, for some prefixed design scalar  $\alpha^* \in (0, 1]$ :

$$\beta(t^+) = \begin{cases} \beta_j(t^+) & \text{if } c(\beta_j(t^+)) \geq (1 + \alpha^*) c(\beta_i(t^+)) \\ \beta_i(t^+) & \text{otherwise} \end{cases} \quad (12)$$

where  $p_i$  denotes the  $i$ -th column of  $P$ . This modification strategy, first proposed in [13] for the linear continuous-time case and then extended in [15-16] to linear hybrid systems, guarantees that the parametrical error lies in the image of the of  $P$  (see [13]), while allowing that the diophantine equation, which will be then used for the synthesis of the adaptive stabilizer, will have no cancellations at any time. It will be then shown that the two following conditions are satisfied:

- C1)  $\beta$  converges
- C2)  $c(\beta) \geq \delta^* > 0$ .

which will be then required in the proofs of convergence and stability. Eqn. 9 can be rewritten as dependent of the modified estimates (10)-(12) as follows:

$$\dot{y}_f = \bar{\theta}_1 \dot{u}_f + \bar{\theta}_2 u_f - \bar{\theta}_3 y_f + \bar{\theta}_4 (\dot{u}'_f - \dot{u}_f) + \bar{\theta}_5 (u'_f - u_f) + \bar{\theta}_6 e^{-d^* t} + e - \beta^T P \varphi + \bar{\theta}_5 (u'_f - u_f) + \bar{\theta}_6 e^{-d^* t} + e - \beta^T P \varphi \quad (13)$$

The filtered control input  $u_f$  to the saturating device and its unfiltered version  $u$  are generated as follows:

$$\begin{aligned} \dot{u}_f &= -s_1 u_f - r_0 y_f ; \\ u &= d^* u_f + \dot{u}_f = (d^* - s_1) u_f - r_0 y_f \end{aligned} \quad (14)$$

with the parameters  $r_0$  and  $s_1$  of the adaptive stabilizer being calculated for all time from the diophantine polynomial equation

$$\begin{aligned} (D + \bar{\theta}_3)(D + s_1) + [(\bar{\theta}_1 - \bar{\theta}_4)D + (\bar{\theta}_2 - \bar{\theta}_5)]r_0 &= C^*(D) \\ =_{\text{def}} D^2 + c_1^* D + c_2^* \end{aligned} \quad (15)$$

with  $D = d / dt$  in (15.a) and  $C^*(D)$  being a strictly Hurwitz polynomial that defines the suited nominal closed-loop dynamics.

**B. Stability and Convergence Results**

They are summarized in the following main result:

*Theorem 1.* Consider the plant (1) subject to the estimation scheme (6)-(8), the modification scheme (10)-(12) and the control law (14)-(15). Assume that either  $a^* \geq 0$  (i.e., the

open-loop plant is stable) or  $|y(0)| \leq \left| \frac{b_1^* - a^* b_0^*}{a^*} \right|$  if  $a^* < 0$  (i.e., the initial condition is sufficiently small if the plant is unstable). □

Thus, the resulting closed-loop scheme has the following properties:

- (i) The modified estimated plant model is controllable for all time for the chosen  $\beta$  in such a way that  $c(\beta) \geq \delta^* > 0$ .
- (ii)  $\tilde{\theta} = \theta - \theta^* \in L_\infty$  and  $e$  and  $P\phi$  are in  $L_\infty \cap L_2$ .
- (iii)  $\theta, P, \beta, \bar{\theta}, s_1$  and  $r_0$  are uniformly bounded and converge asymptotically to finite limits. Also, the number of switches in  $\beta$  is finite. Also,  $\dot{\theta} \in L_2 \cap L_\infty$ .
- (iv) The signals  $u, u'$  and  $y$  and their corresponding filtered signals are in  $L_\infty \cap L_2$ . The signals  $u, u', u_f, u'_f, y$  and  $y_f$  converge to zero and their time-derivatives are in  $L_\infty \cap L_2$  so that they converge to zero asymptotically. □

An outline proof of Theorem 1 is given in Appendix A. Note that the requirement of the initial conditions being sufficiently small when the plant is unstable is a usual requirement for stabilization in the presence of input saturation since it is impossible to globally stabilize an open-loop unstable system with saturated input. This avoids the closed-loop system trajectory to explode. Such a phenomenon occurs when the initial time-derivative of the state vector is positive and continues to be positive for all time because its sign cannot be modified for any input value within the allowable input range. Note also that Theorem 1 (i)-(iii) imply that Conditions C1-C2 for the  $\beta(\cdot)$ -functions of the modification scheme are fulfilled. Finally, note that the controllability of the modified estimation scheme allows to keep coprime the modified estimates of the polynomials for

zeros and poles. Thus, the diophantine equation (15) associated with the controller synthesis is solvable for all time without any singularities.

The mechanism which is used to ensure local stability for unstable plants and global one for stable ones is to guarantee the boundedness of all the unsaturated filtered and unfiltered signals from the regressor boundedness while the saturated ones are bounded by construction. This also ensures the identification (or adaptation) error to be bounded for all sampling time since the unmodified and modified plant parameter estimates as well as those of the adaptive controller are all bounded. The fact that the control signal is bounded is ensured since it is saturated. In the unsaturated control case, the control boundedness has to be proven explicitly (see, for instance, [21-24]) irrespective of the particular theoretical design or application. On the other hand, it turns out the main future interest of applying saturating controls to otherwise positive systems in the presence of delays or under hybrid controls (see, [25-27]). Related research would be an interesting future investigation field.

**III. ADAPTIVE ESTIMATES AND CONTROL**

Now, the continuous-time plant (1) is subject to the control law (14)-(15) under the saturating sigmoidal function (1.b) but the estimation algorithm (6)-(8) only updates parameters at the sampling instants  $t_{k+1} = t_k + h = (k+1)h$  of the sampling period  $h$  while the regressor is evaluated at all time for re-updating the various estimates at sampling instants only. The estimation modification and calculation of the controller parameters is also updated at sampling instants. The discrete-time parameter estimation and inverse of the covariance matrix adaptation laws are:

$$\begin{aligned} \theta_k &= \theta_{k-1} + \Delta \theta_{k-1} = \theta_{k-1} - \\ &- P_k \frac{\int_0^h \|\phi[(k-1)h + \tau]\|^2 \phi[(k-1)h + \tau] \phi^T [(k-1)h + \tau] d\tau}{c_k (1 + \int_0^h \phi^T [(k-1)h + \tau] \phi[(k-1)h + \tau] d\tau)} \tilde{\theta}_{k-1} \end{aligned} \quad (16.a)$$

$$\begin{aligned} P_{k+1}^{-1} &= P_k^{-1} + \Delta P_k^{-1} = P_k^{-1} \\ &+ \frac{\int_0^h \|\phi[(k-1)h + \tau]\|^2 \phi[(k-1)h + \tau] \phi^T [(k-1)h + \tau] d\tau}{c_k (1 + \int_0^h \phi^T [(k-1)h + \tau] \phi[(k-1)h + \tau] d\tau)} \end{aligned} \quad (16.b)$$

$$c_k \geq c_{k0} =_{\text{def}} \lambda_{\max}^2 (P_k) \frac{\int_0^h \|\phi[(k-1)h + \tau]\|^4 d\tau}{1 + \int_0^h \|\phi[(k-1)h + \tau]\|^4 d\tau} \quad (16.c)$$

with  $P(0) = P^T(0) > 0$  and  $\tilde{\theta}_k = \theta_k - \theta^*$  for all integer  $k \geq 0$ . The main result of this section is announced as follows:

*Theorem 2.* Consider the plant (1) subject to the estimation scheme (6) and (16), i.e., the parameter estimates are only

updated at sampling instants, the modification scheme (10)-(12), with (12) being updated only at  $t = kh$ , and the stabilizing control law (14)-(15). Thus, the resulting closed-loop scheme fulfils the same properties of Theorem 1 under the same assumptions.  $\square$

The proof of Theorem 2 is outlined in Appendix B.

#### IV. CONCLUSION

This paper has developed a continuous-time adaptive stabilizer for a continuous-time first-order controllable plants which can have an unstable zero and is subject to an input saturation of sigmoidal function type. The mechanism used to guarantee the scheme's closed-loop stability is a modification scheme of the parameter estimates which is based on the use of a hysteresis switching function. The switches are built so that the modified plant estimated model is controllable and then it has no pole-zero cancellation. An alternative adaptive stabilizer which only modifies the parameter estimates at sampling instants, but which is based on continuous-time input / output measurements, is also addressed for the same kind of simple plant. The resulting closed-loop system is of a hybrid nature because of the discrete updating of the estimation scheme. A similar hysteresis switching function, which operates at sampling instants, is also used in that case so as to guarantee the controllability of the modified estimated plant model.

#### APPENDIX

##### A. Outline of proof of Theorem 1

Define the Lyapunov function candidate  $V = 1/2 \tilde{\theta}^T P^{-1} \tilde{\theta}$  by using the parametrical error  $\tilde{\theta} = \theta - \theta^*$  and the inverse of the covariance matrix. It follows that  $P^{-1} \tilde{\theta}$  is constant for all time so that  $\theta^* = \theta + P\beta$ . Thus,  $0 < \delta_0^* \leq c(\beta^*) = |f_1|$

$$\leq (|f + f_1| + \|v\| + \|p_3\| \|p_1 - p_2\|) \max(1, \|\beta^*\|^2)$$

where

$$f = \theta_1 \theta_4 - \theta_4^2 + \theta_5 - \theta_2$$

$$f_1 = b_0^* a^* - b_1^*$$

$$v^T = (p_5 - p_2)^T + (p_1 - p_4)^T \theta_3 + p_3^T (\theta_1 - \theta_4)$$

It follows directly that

$$c(\beta) = |(\bar{\theta}_1 - \bar{\theta}_4) \bar{\theta}_3 - (\bar{\theta}_2 - \bar{\theta}_5)| \\ = |f + \beta^T (v + (p_1 - p_4) \beta^T p_3)| > 0$$

since  $f + f_1$ ,  $v$ ,  $p_3$  and  $p_1 - p_4$  cannot be simultaneously zero since  $c(\beta^*) > 0$ .  $f_1 = -f \neq 0$  if  $f + f_1 = 0$  so that  $c(\beta) > 0$ . If  $\beta = \pm v \neq 0$  then  $c(\beta) > 0$ . If  $f = v = 0$  then  $\beta$  equalizes one of the combinations  $\pm(p_1 - p_4) \pm p_3$  and

$c(\beta) > 0$ . Property (i) has been proven. Property (ii) is proven as follows. First note that  $2\dot{V} = -e^2 \leq 0$  what implies that  $V \leq V(0) < \infty$ . Then,  $e(t)$  is bounded and square-integrable and the parametrical error is also bounded for all time. Finally,  $d(\text{tr } P)/dt = -\varphi^T P^2 \varphi \leq 0$  what implies that  $P\varphi$  is bounded and square-integrable. Properties (iii)-(iv) follow from the fact that  $P$  is non-increasing and positive semidefinite from its updating rule so that it converges. Also,

$$\theta(t) - \theta(0) \leq \int_0^t \|\dot{\theta}(\tau)\| d\tau = \int_0^t \|P(\tau)\varphi(\tau)e(\tau)\| d\tau \\ \leq \frac{1}{2} \int_0^t (\|P(\tau)\varphi(\tau)\|^2 + \|e(\tau)\|^2) d\tau < \infty$$

for all time. It follows that the parametrical error converges asymptotically to a finite limit. From this partly result, the remaining of the proof follows by calculating a bounded upper-bound of the norm-square integral of the time derivative of the estimate time-derivative. It follows that  $\dot{\theta}$  is bounded and square-integrable. Then, using the Diophantine equation for the controller synthesis, it follows that the modified estimated vector  $\bar{\theta}$  also converges asymptotically as well as they converge the various controller parameters.  $\square$

##### B. Outline of Proof of Theorem 2

One gets from (16) that  $\Delta \tilde{\theta}_{k-1} = -P_k \Delta \tilde{P}_k^{-1} \tilde{\theta}_{k-1}$  with the one-step incremental error being:

$$\Delta \tilde{\theta}_{k-1} = \tilde{\theta}_k - \tilde{\theta}_{k-1} \quad \text{and} \quad \Delta \tilde{P}_k^{-1} = P_{k+1}^{-1} - P_k^{-1}$$

Then, for a Lyapunov sequence candidate  $V_k = \tilde{\theta}_k^T P_k^{-1} \tilde{\theta}_k$ , one gets a one-step increment from 816):

$$\Delta V_{k-1} = V_k - V_{k-1} \\ = -\tilde{\theta}_{k-1}^T \Delta \tilde{P}_k^{-1} (I - P_k \Delta \tilde{P}_k^{-1} P_k) \Delta \tilde{P}_k^{-1} \tilde{\theta}_{k-1} \leq 0$$

if  $c_k \geq c_{k0}$ . Then, the candidate is a Lyapunov sequence with bounded eigenvalues of the covariance matrix implying strictly positive eigenvalues of its inverse, what leads to the results of Theorem 2.  $\square$

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