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Receding Horizon Filtering for Mobile Robot Systems with Cross-Correlated Sensor Noises

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Abstract— This paper reports on a receding horizon filtering for mobile robot systems with cross-correlated sensor noises and uncertainties. Also, the effect of uncertain parameters in the state of the tracking error model performance is considered. A distributed fusion receding horizon filter is proposed. The distributed fusion filtering algorithm represents the optimal linear combination of the local filters under the minimum mean square error criterion. The derivation of the error cross-covariances between the local receding horizon filters is the key of this paper. Simulation results of the tracking mobile robot's motion demonstrate high accuracy and computational efficiency of the distributed fusion receding horizon filter.

Keywords— Distributed fusion, fusion formula, Kalman filter, multisensor, receding horizon, wheeled mobile robot

I. INTRODUCTION

Nowadays, more and more autonomous mobile robots are being developed and deployed in many real-world applications. Predominantly tracking and motion generation topics have been treated [1]-[5]. Recently attention is given to active sensing, that incorporates in itself tracking and motion planning solutions in the presence of uncertainties. The mobile robots often work in unknown and inhospitable environments. To survive, these robots must be able to constantly monitor and appropriately react to variation and uncertainty in their environments. Model uncertainty appears due to un-modeled dynamics and inaccurate parameters of the robot system. One must admit that there is no mathematical model that can perfectly represent a real physical system. Thus, any discrepancy between the physical system and the mathematical model causes the model uncertainty. The error between the system parameters used in the mathematical model and the ones presented in the physical system also contributes to the model uncertainty since it is almost impossible to obtain all system parameters correctly.

In practical applications, there could be cross-correlations between the sensor noises. This is true in practical situations when the dynamic process is observed in a common noisy environment such as when a target is taking an electronic countermeasure, e.g. noise jamming, or when the sensor noises are coupled because of their dependence on the target state [6].

To get a more accurate and robust estimate of a state of system under potential uncertainty, various kinds of techniques have been introduced and discussed. Among them, receding horizon estimation is one of popular and successful strategies, which is robust against temporal uncertainty has been rigorously investigated [7]-[10]. The local receding horizon Kalman filters (LRHKFs), which we fuse, utilize finite measurements over the most recent time interval [7]-[9], [11]. It has been a general rule that the LRHKF is often robust against dynamic model uncertainties and numerical errors than the standard local Kalman filter, which utilize all measurements. Based on the LRHKFs [7], [9] and the optimal fusion formula with matrix weights [10], [12], [13], we propose a distributed receding horizon fusion filtering for the mobile robot systems with cross-correlated sensor noises and uncertainties which has a better accuracy than every LRHKF, and it has the reduced computational burden as compared to the centralized fusion receding horizon filter.

This paper is organized as follows. The problem is set up in Section II. The centralized fusion receding horizon filter is described in Section III. In Section IV, we present the main result regarding the distributed fusion receding horizon filter for multisensory environment. Here the key equations for cross-covariances between LRHKFs are derived. In Section V, an example of tracking error model with three sensors illustrates the proposed distributed filter. In Section VI, concluding remarks are given.

II. PROBLEM SETTING

Consider the continuous-time linear dynamic system with additive white Gaussian noise

$$\dot{x}_t = F_t x_t + G_t \xi_t, \quad t \geq t_0, \quad x_0 = x_{t_0}, \tag{1}$$

$$y_t^{(i)} = H_t^{(i)} x_t + w_t^{(i)}, \quad i = 1, \ldots, N, \tag{2}$$

where $x_t \in \mathbb{R}^n$ is the state, $y_t^{(i)} \in \mathbb{R}^{m_i}$ is the local sensor measurement, $\xi_t \in \mathbb{R}^r$ and $w_t^{(i)} \in \mathbb{R}^{m_i}$ are white Gaussian noises with zero mean and intensity matrices Q_t and $R_t^{(i)}$, respectively, and $F_t, G_t, H_t^{(i)}$ are matrices with compatible

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dimensions. Superscript (i) denotes the *ith* sensor, N is the number of sensors.

We assume that the initial state $x_0 \sim \mathbb{N}(m_0;P_0)$, $m_0 = E(x_0)$, $P_0 = \operatorname{cov}\{x_0,x_0\}$; the system noise ξ_t is uncorrelated with sensor noises $w_t^{(i)},...,w_t^{(N)}$, but all sensor noises $w_t^{(i)} \in \mathbb{R}^{m_i}$, $i=1,\dots,N$ are mutually correlated with intensity matrices $R_t^{(ij)}$, $i,j=1,\dots,N$, $i \neq j$.

Our aim is to find the distributed weighted fusion estimate of the state x_t based on the overall horizon cross-correlated sensor measurements

$$y_{t-\Delta}^{t} = \left\{ y_s^{(1)},\dots,y_s^{(N)},\ t-\Delta \le s \le t \right\}. \tag{3}$$

III. Centralized Fusion Receding Horizon Filter

Let consider the original dynamic system (1) and rewrite the measurement model (2) in equivalent form. We obtain

$$\begin{aligned} &\dot{x}_t = F_t x_t + G_t \xi_t,\ \ t \ge t_0,\ \ x_0 = x_{t_0}, \\ &Y_t = H_t x_t + w_t, \end{aligned} \tag{4}$$

where

$$Y_t = \begin{bmatrix} y_t^{(1)} \\ \vdots \\ y_t^{(N)} \end{bmatrix},\ H_t = \begin{bmatrix} H_t^{(1)} \\ \vdots \\ H_t^{(N)} \end{bmatrix},\ w_t = \begin{bmatrix} w_t^{(1)} \\ \vdots \\ w_t^{(N)} \end{bmatrix}. \tag{5}$$

Then the optimal *centralized fusion receding horizon filter* (CFRHF) $\hat{x}_t^{opt}$ of the state x_t based on the overall receding horizon measurements (3) is described by the following differential equations [7]-[9]:

$$\begin{aligned} &\dot{\hat{x}}_s^{opt} = F_s x_s^{opt} + K_s^{opt}\left[Y_s - H_s \hat{x}_s^{opt} \right], \\ &\dot{P}_s^{opt} = F_s P_s^{opt} + P_s^{opt} F_s^T - P_s^{opt} H_s^T R_s^{-1} H_s P_s^{opt} + \tilde{Q}_s, \\ &K_s^{opt} = P_s^{opt} H_s^T R_s^{-1},\ \ \tilde{Q}_s = G_s Q_s G_s^T, \\ &R_s = \begin{bmatrix} R_s^{(11)} & \cdots & R_s^{(1N)} \\ \vdots & \ddots & \vdots \\ R_s^{(N1)} & \cdots & R_s^{(NN)} \end{bmatrix},\ R_s^{(ii)} \equiv R_s^{(i)},\ i=1,...,N;\ t-\Delta \le s \le t, \end{aligned} \tag{6}$$

where the horizon initial conditions at time instant $s = t-\Delta$ represent the unconditional mean

$$\hat{x}_{t-\Delta}^{opt} \overset{def}{=} m_{t-\Delta} = E\left(x_{t-\Delta}\right)$$

and covariance

$$P_{t-\Delta}^{opt} \overset{def}{=} P_{t-\Delta} = E\left[\left(x_{t-\Delta} - m_{t-\Delta}\right)\left(x_{t-\Delta} - m_{t-\Delta}\right)^T \right]$$

of the horizon state $x_{t-\Delta}$ satisfying the Lyapunov equations

$$\begin{aligned} &\dot{m}_\tau = F_\tau m_\tau,\ \ t_0 \le \tau \le t-\Delta,\ \ \ m_{t_0} = m_0 = E\left(x_0\right), \\ &\dot{P}_\tau = F_\tau P_\tau + P_\tau F_\tau^T + \tilde{Q}_\tau,\ \ \ P_{t_0} = P_0 = \operatorname{cov}\left\{x_0,x_0\right\}, \\ &t_0 \le \tau \le t-\Delta. \end{aligned} \tag{7}$$

Let us now summarize the algorithmic procedure for the CFRHF. First, the horizon initial conditions $\hat{x}_{t-\Delta}^{opt}$ and $P_{t-\Delta}^{opt}$ are determined by the Lyapunov equations (7). Then, the CFRHF equations (6) are solved on the horizon interval $s \in [t-\Delta, t]$ using current horizon measurements (3).

The CFRHF represents a joint estimator. To compute the state estimate $\hat{x}_t^{opt}$, the implementation of the CFRHF requires all the horizon sensor measurements (3) jointly at each time instant t. Therefore, in the case of several limitations, such as computational cost, communication resources, the CFRHF cannot produce well-timed results, especially for the large number of sensors. So distributed receding horizon filter is preferable as there is no need to estimate state by using *overall* sensor measurements (3) simultaneously.

IV. Distributed Fusion Receding Horizon Filter

Now we show that the fusion formula [10], [12], [13] can serve as the basis for designing of a distributed fusion filter. A new suboptimal *distributed fusion receding horizon filter* (DFRHF) is described as follows: first, the local sensor measurements $y_t^{(1)},\dots,y_t^{(N)}$ are processed separately by using the optimal LRHKF [7]-[9] and, second, the obtained local estimates (filters) are fused in an optimal linear combination.

Let denote local receding horizon Kalman estimate of the state x_t based on the individual sensor $y_t^{(i)}$ by $\hat{x}_t^{(i)}$. To find $\hat{x}_t^{(i)}$ we can apply the LRHKF to system (1) with sensor $y_t^{(i)}$ [7]-[9]. We obtain the following differential equations:

$$\begin{aligned} &\dot{\hat{x}}_s^{(i)} = F_s \hat{x}_s^{(i)} + K_s^{(i)}\left[y_s^{(i)} - H_s^{(i)} \hat{x}_s^{(i)} \right], \\ &\dot{P}_s^{(ii)} = F_s P_s^{(ii)} + P_s^{(ii)} F_s^T - P_s^{(ii)} H_s^{(i)^T} R_s^{(i)^{-1}} H_s^{(i)} P_s^{(ii)} + \tilde{Q}_s, \\ &K_s^{(i)} = P_s^{(ii)} H_s^{(i)^T} R_s^{(i)^{-1}}, \\ &P_s^{(ii)} = \operatorname{cov}\left\{ e_s^{(i)}, e_s^{(i)} \right\},\ \ e_s^{(i)} = x_s - \hat{x}_s^{(i)},\ t-\Delta \le s \le t, \end{aligned} \tag{8}$$

with the horizon initial conditions $\hat{x}_{t-\Delta}^{(i)} = m_{t-\Delta}$, $P_{t-\Delta}^{(ii)} = P_{t-\Delta}$ determined by (7).

Thus the distributed fusion receding horizon suboptimal estimate $\hat{x}_t^{sub}$ based on the overall sensor measurements (3) is constructed by using the fusion formula, i.e.,

$$\hat{x}_t^{sub} = \sum_{i=1}^{N} c_t^{(i)} \hat{x}_t^{(i)},\ \ \ \sum_{i=1}^{N} c_t^{(i)} = I_n, \tag{9}$$

where I_n is the identity matrix, $c_t^{(1)},\dots,c_t^{(N)}$ are the time-varying weighted matrices determined by the mean square criterion.

Theorem 1 [10], [12]. *(a) The optimal weights $c_t^{(1)},\dots,c_t^{(N)}$ satisfy the linear algebraic equations*

$$\sum_{i=1}^{N} c_t^{(i)}\left[P_t^{(ij)} - P_t^{(iN)}\right] = 0,\ \ \sum_{i=1}^{N} c_t^{(i)} = I_n, \tag{10}$$

and they can be explicitly written in the following form

$$c_t^{(i)} = \sum_{j=1}^{N} W_t^{(ij)}\left(\sum_{l,h=1}^{N} W_t^{(lh)}\right)^{-1},\quad i=1,\dots N, \tag{11}$$

where $W_t^{(ij)}$ is the (ij)th $(n\times n)$ submatrix of the $(nN\times nN)$ block matrix P_t^{-1}, $P_t = \left[P_t^{(ij)}\right]_{i,j=1}^{N}$.

(b) The fusion error covariance $P_t^{sub} \overset{def}{=} \operatorname{cov}\left\{e_t^{sub}, e_t^{sub}\right\}$, $e_t^{sub} = x_t - \hat{x}_t^{sub}$ is given by

$$P_t^{sub} = \sum_{i,j=1}^{N} c_t^{(i)} P_t^{(ij)} c_t^{(j)^T}. \tag{12}$$

Equations (10)-(12) defining the unknown weights $c_t^{(i)}$ and fusion error covariance P_t^{sub} depend on the local covariances $P_t^{(ii)}$, which determined by (8) and the local cross-covariances

$$P_t^{(ij)} = \operatorname{cov}\left\{e_t^{(i)}, e_t^{(j)}\right\},\quad i,j=1,\dots,N,\quad i\neq j \tag{13}$$

given in Theorem 2.

Theorem 2. *The local cross-covariances (13) satisfy the following differential equations:*

$$\begin{aligned}&\dot{P}_s^{(ij)} = \tilde{F}_s^{(i)} P_s^{(ij)} + P_s^{(ij)} \tilde{F}_s^{(j)^T} + \tilde{Q}_s,\quad t-\Delta\le s\le t,\\ &\tilde{F}_s^{(i)} = F_s - K_s^{(i)} H_s^{(i)},\quad i,j=1,\dots,N,\quad i\neq j\end{aligned} \tag{14}$$

with the horizon initial conditions $P_{t-\Delta}^{(ij)} = P_{t-\Delta}$ and gains $K_s^{(i)}$ determined by (7) and (8), respectively.

The derivation of (14) is given in Appendix.

Thus, equations (8)-(14) completely define the DFRHF.

Remark 1. The LRHKFs $\hat{x}_t^{(i)}$, $i=1,\dots N$ are separated for different types of sensors, i.e., each local estimate $\hat{x}_t^{(i)}$ is found independently of other estimates. Therefore, the LRHKFs can be implemented in parallel for different sensors (2).

Remark 2. We may note, that the local error covariances $P_t^{(ij)}$, $i,j=1,\dots N$ and weights $c_t^{(i)}$ may be pre-computed, since they do not depend on the sensor measurements $y_{t-\Delta}^{t}$, but only on the noise statistics Q_t and $R_t^{(i)}$, and the system matrices F_t, G_t, $H_t^{(i)}$ which are the part of system model (1), (2). Thus, once the measurement schedule has been settled, the real-time implementation of the DFRHF requires only the computation of the LRHKFs $\hat{x}_t^{(i)}$, $i=1,\dots N$ and the final suboptimal fusion estimate $\hat{x}_t^{sub}$.

V. Example

A. Tracking error model

There are two basic control approaches to solving the mobile robot's motion task: stabilization to a fixed posture and tracking of the reference trajectory. For nonholonomic mobile robot systems, the trajectory-tracking problem is easier to solve and more natural than posture stabilization. The state of the tracking error can be expressed in the frame of the real robot [14], as shown in Fig. 1.

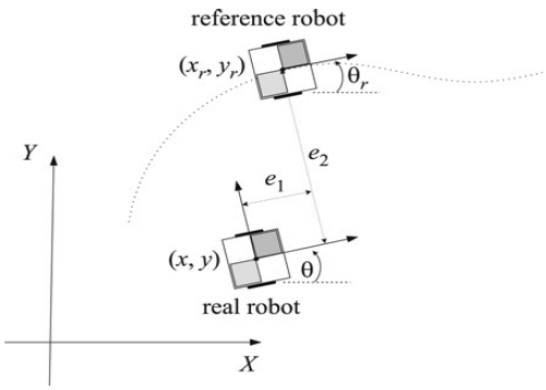

Fig. 1 Robot following error transformation

In Fig. 1 the reference robot is an imaginary robot that ideally follows the reference path. In contrast, the real robot (when compared to the reference robot) has some error when following the reference path. This linearized tracking error model is described by

$$\dot{x}_t = \begin{bmatrix} 0 & \omega_r & 0\\ -\omega_r + 0.5\delta_t & 0 & v_r + \delta_t\\ 0 & 0 & 0\end{bmatrix} x_t + \begin{bmatrix}1\\1\\1\end{bmatrix}\xi_t,\quad t\ge 0, \tag{15}$$

where $x_t = \left[x_{1,t}\ \ x_{2,t}\ \ x_{3,t}\right]^T$, and $x_{1,t}$ is the x-position error, $x_{2,t}$ is y-position error, $x_{3,t}$ is the robot heading angle error,

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v_r is the tangential velocity, and ω_r is angular velocity. The obtained linearization is shown to be controllable [15], [16] as long as the trajectory does not come to a stop. Thus, the reference inputs v_r and ω_r are constant (linear and circular paths), δ_t is an uncertain model parameter, and ξ_t is a white Gaussian noise. Here, $v_r = 0.1643\ m/s$, $\omega_r = 0.3788\ rad/s$, the system noise intensity Q_t is 0.02^2 and $\delta_t = 1$ on the interval $2 \le t \le 6$. The horizon length of the LRHKFs is taken as $\Delta_1 = 0.4$, $\Delta_2 = 0.5$ and $\Delta_3 = 0.6$.

The second coordinate related to the y-position error between real and reference robot is observable through the measurement model containing three identical local sensors with different accuracy given by

$$\begin{aligned} y_t^{(1)} &= H^{(1)}x_t + w_t^{(1)}, \quad H^{(1)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \\ y_t^{(2)} &= H^{(2)}x_t + w_t^{(2)}, \quad H^{(2)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \\ y_t^{(3)} &= H^{(3)}x_t + w_t^{(3)}, \quad H^{(3)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \end{aligned} \tag{16}$$

where $w_t^{(1)}$, $w_t^{(2)}$ and $w_t^{(3)}$ are cross-correlated white Gaussian noises with zero-mean and stationary intensities $R_t^{(ij)}$, $i, j = 1,2,3$. The parameters are subjected to

$$R_t^{(i)} = 0.01^2, \quad i = 1,2,3 \quad R_t^{(12)} = R_t^{(21)} = 0.02^2,$$
$$R_t^{(13)} = R_t^{(31)} = 0.018^2, \quad R_t^{(23)} = R_t^{(32)} = 0.015^2.$$

Two fusion receding horizon filters the CFRHF $\left(\hat{x}_t^{opt}, P_t^{opt}\right)$ and DFRHF $\left(\hat{x}_t^{sub}, P_t^{sub}\right)$, and two fusion non-receding horizon filters the centralized Kalman filter (CKF) and decentralized Kalman filter (DKF) for the system model with uncertainty δ_t are compared. Here the model is considered to show robustness of the receding horizon filters against uncertainty [11].

B. Results and Analysis

The behavior of the CFRHF and DFRHF estimates $\left(\hat{x}_t^{opt}, \hat{x}_t^{sub}\right)$ and their fusion error covariances $\left(P_t^{opt}, P_t^{sub}\right)$ is studied. We focus on the mean square errors (MSEs)

$$P_{22,t}^{opt} = E\left[\left(x_{2,t} - \hat{x}_{2,t}^{opt}\right)^2\right], \; P_{22,t}^{sub} = E\left[\left(x_{2,t} - \hat{x}_{2,t}^{sub}\right)^2\right] \tag{17}$$

for the second coordinate $x_{2,t}$, which is called y-position error, because the uncertainty δ_t appears in (15) only in this coordinate. Simulation results of other coordinates $x_{1,t}$ and $x_{3,t}$ are similar. The point of interest is the comparison of the optimal and suboptimal receding horizon estimates $\left(\hat{x}_{2,t}^{opt}, \hat{x}_{2,t}^{sub}\right)$ of the y-position error, and corresponding MSEs (17).

The MSEs are shown in Figures 2-4. Fig. 2 presents two pairs of the estimates of y-position error using the receding horizon and non-receding horizon fusion filters under the uncertainty $\delta_t = 1$ The first pair includes the CFRHF and DFRHF, and the second one contains the CKF and DKF [10].

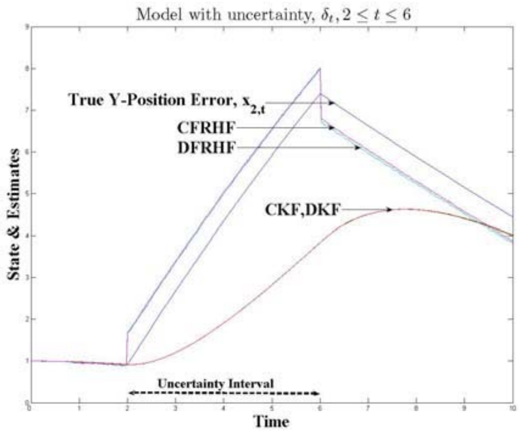

Fig. 2 The y-position error $x_{2,t}$ and its estimates using CFRHF, DFRHF, CKF and DKF

In Fig. 2 we observe that around the uncertainty interval $2 \le t \le 6$, the receding horizon filters (CFRHF, DFRHF) demonstrate good performance compared to the non-receding horizon filters CKF and DKF. This is confirm the general agreement with robustness of the receding horizon strategy. Also in each pair of filters, the centralized versions give more accurate estimates than the decentralized one. However, these differences are negligible.

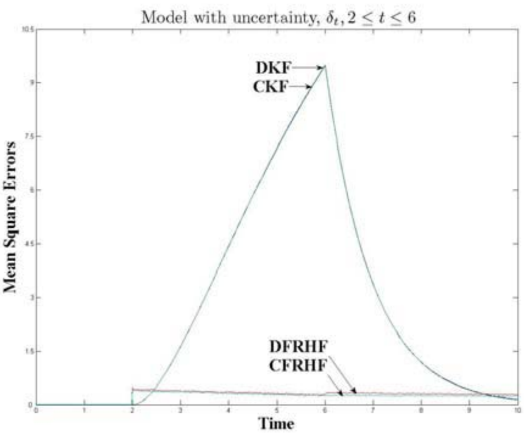

Fig. 3 MSE comparison for $x_{2,t}$ with uncertainty $\delta_t = 1$

The estimation accuracy of the filters can be more clearly compared through MSEs in Fig. 3. Here the actual MSEs are calculated through Monte-Carlo method with 1000 runs. In Fig.3 we see that within the uncertainty interval $2 \le t \le 6$, the MSEs of the non-receding horizon filters CKF and DKF are remarkably large. However, the differences between all optimal and suboptimal filters are negligible outside of the uncertainty interval. This means that for our example the application of the DFRHF can produce good results in real-time processing requirements.

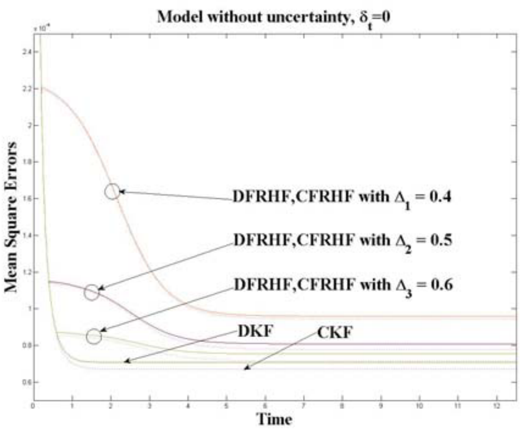

Fig. 4 MSE comparison for $x_{2,t}$ without uncertainty $\delta_t = 0$

Fig. 4 shows the filter performances of the CFRHF and DFRHF as a function of the horizon length Δ. Since the uncertainty does not appear, the actual MSEs can be calculated directly through (6)-(8), (12)-(14) without Monte-Carlo simulations as in Fig. 3. The results in Fig. 4 demonstrate that the receding horizon filters (CFRHF and DFRHF) reach to the non-receding horizon Kalman filters (CKF and DKF) with increasing the horizon length Δ.

VI. Conclusion

In this paper, a new distributed receding horizon filter for mobile robot systems in cross-correlated sensor environment is proposed. It represents the weighted sum of the LRHKFs. Each LRHKF is fused by the minimum mean square error criterion. The matrix weights depend on the cross-covariances between the LRHKFs. The key differential equations for them are derived.

Furthermore, the DFRHF has the parallel structure and allows parallel processing of observations making it reliable since the rest faultless sensors can continue to the fusion estimation if some sensors occur faulty. Taking into account tracking error model of the mobile robot systems, the proposed DFRHF gives good tracking results under the uncertainty. Simulation analysis and comparison with the optimal CFRHF verifies the effectiveness of the proposed DFRHF.

Appendix

Derivation of equations (14)

According to (8) the local errors $e_s^{(i)}$ and $e_s^{(j)}$, $i \ne j$ satisfy the following differential equations on the receding horizon interval $s \in [t-\Delta;\ t]$:

$$\dot{e}_s^{(i)} = \tilde{F}_s^{(i)} e_s^{(i)} + B_s^{(i)} \eta_s, \quad B_s^{(i)} = \begin{bmatrix} -K_s^{(i)} & 0 & G_s \end{bmatrix},$$

$$\dot{e}_s^{(j)} = \tilde{F}_s^{(j)} e_s^{(j)} + B_s^{(j)} \eta_s, \quad B_s^{(j)} = \begin{bmatrix} 0 & -K_s^{(j)} & G_s \end{bmatrix},$$

$$\eta_s = \begin{bmatrix} w_s^{(i)^T} & w_s^{(j)^T} & \xi_s^T \end{bmatrix}^T \in \mathbb{R}^{m_i+m_j+r}, \quad t-\Delta \le s \le t,$$

where η_s is the composite white noise with intensity matrix

$$Q_s^{(\eta)} = \begin{bmatrix} R_s^{(ii)} & R_s^{(ij)} & 0 \\ R_s^{(ji)} & R_s^{(jj)} & 0 \\ 0 & 0 & Q_s \end{bmatrix}, \quad R_s^{(ii)} \equiv R_s^{(i)}, \quad R_s^{(ij)} = R_s^{(ji)^T}.$$

Then the cross-covariance $P_s^{(ij)}$ represents expectation of the product $E\left[e_s^{(i)} e_s^{(j)^T} \right]$, satisfying the following differential equation [17, p.166]:

$$\dot{P}_s^{(ij)} = \frac{d}{ds} E\left[e_s^{(i)} e_s^{(j)^T} \right] = \tilde{F}_s^{(i)} P_s^{(ij)} + P_s^{(ij)} \tilde{F}_s^{(j)^T} + B_s^{(i)} Q_s^{(\eta)} B_s^{(j)^T}.$$

And after simple manipulations with the item

$$B_s^{(i)} Q_s^{(\eta)} B_s^{(j)^T} = K_s^{(i)} R_s^{(ij)} K_s^{(j)^T} + G_s Q_s G_s^T,$$

we obtain (14).

This completes the proof of Theorem 2.

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