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Filteristic Soft Lattice Implication Algebras

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Abstract—Applying the idea of soft set theory to lattice implication algebras, the novel concept of (implicative) filteristic soft lattice implication algebras which related to (implicative) filter(for short, $(IF\text{-})F$-soft lattice implication algebras) are introduced. Basic properties of $(IF\text{-})F$-soft lattice implication algebras are derived. Two kinds of fuzzy filters (i.e.$(\in, \in \vee q_k)((\overline{\in}, \overline{\in} \vee \overline{q_k}))$-fuzzy (implicative) filter) of $\mathscr{L}$ are introduced, which are generalizations of fuzzy (implicative) filters. Some characterizations for a soft set to be a $(IF\text{-})F$-soft lattice implication algebra are provided. Analogously, this idea can be used in other types of filteristic lattice implication algebras (such as fantastic (positive implicative) filteristic soft lattice implication algebras).

Keywords—Soft set; (implicative) filteristic lattice implication algebras; fuzzy (implicative) filters; $((\in, \in \vee q_k))$ $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy (implicative) filters.

I. Introduction

IN In order to research the many-valued logical system whose propositional value is given in a lattice, in 1993, Xu[1] firstly established the lattice implication algebras by combining lattice and implication algebras, and investigated many useful structures[2], [3], [4]. This logical algebra has been extensively investigated by several researchers, and many elegant results are obtained, collected in the monograph[4].

Because of various uncertainties typical for complicated problems in economics, engineering and environment, they can't be successfully solved by existing theories such as theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [12]. Molodtsov[12] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov(1999) introduced a novel concept called soft sets as a new mathematical tools for dealing with uncertainties. The soft set theory is free from many difficulties that has been troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. Research works on soft sets are very active and progressing rapidly in these years. Maji[16] discussed the application of soft set theory to a decision-making problems. They also investigated some operations on the theory of soft sets. In 2001, Maji[14]et al. investigated the fuzzification of a soft set and obtained many useful results on fuzzy soft set. Aktas and Cagman[12] related soft sets to groups, they defined soft groups, derive some basic properties, and showed that soft groups extended fuzzy groups. Jun[18], [19] applied the soft set theory to the BCK-algebras, investigated soft BCK-subalgebras and soft ideals, introduced the notion of $\in$-soft set and q-soft set, and gave characterizations for subalgebras and ideals. Furthermore, Feng et al.[17] applied soft set theory to the study of semirings and initiated the notion called soft semirings. Zhan,et al.[22] applied soft set to BL-algebras, initiated the notion (implicative)filteristic soft BL-algebras.

The concept of fuzzy set was introduced by Zadeh(1965)[5]. Since then this idea has been applied to other algebraic structures such as groups, semigroups, rings, modules, vector spaces and topologies. Some scholars[8], [20], [21] applied this fuzzification to the filter in lattice implication algebras, too. They further to introduce relative fuzzy filter such as fuzzy (positive) implicative filter, fuzzy fantastic filter and investigated some properties. The idea of fuzzy point and 'belongingness' and 'quasi-coincidence' with a fuzzy set were given by Pu et al.[6]. A new type of fuzzy subgroup (viz $(\in, \in \vee q)$-fuzzy subgroup) was introduced in[9]. In fact, $(\in, \in \vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup. The idea of fuzzy point and 'belongingness' and 'quasi-coincidence' with a fuzzy set have been applied some important algebraic system[10], [11]. Liu [7], [8] investigate the interval-valued $(\in, \in \vee q)$-fuzzy lattice implication subalgebras and fuzzy filters, respctively.

The aim of this paper is to apply the idea of soft set theory to lattice implication algebras, and introduce the (implicative) filteristic soft lattice implication algebras which related to (implicative) filter(for short,$(IF\text{-})$ F- soft lattice implication algebras). Basic properties of $(IF\text{-})F$-soft lattice implication algebras are investigated. We introduce the notion of $(\in, \in \vee q_k)((\overline{\in}, \overline{\in} \vee \overline{q_k}))$-fuzzy (implicative)filters, which are generalizations of fuzzy (implicative) filter. we provide characterizations for a soft set to be an $(IF\text{-})F$-soft lattice implication algebra. Analogously, this idea can be used in other types lattice implication algebras such as fantastic filteristic lattice implication algebras, positive implicative filteristic soft lattice implication algebras. We hope that it will be of great use to provide theoretical foundation to design intelligent information processing systems.

II. Basic results on lattice implication algebras

Definition 2.1: [1] Let $(L, \vee, \wedge, O, I)$ be a bounded lattice with an order-reversing involution $'$, the greatest element I and the smallest element O, and

$$\rightarrow: L \times L \longrightarrow L$$

be a mapping. $\mathscr{L} = (L, \vee, \wedge, ', \rightarrow, O, I)$ is called a lattice implication algebra if the following conditions hold for any $x, y, z \in L$:

(I_1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$,

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(I_2) $x \to x = I$,
(I_3) $x \to y = y' \to x'$,
(I_4) $x \to y = y \to x = I$ implies $x = y$,
(I_5) $(x \to y) \to y = (y \to x) \to x$,
(l_1) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$,
(l_2) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$.

In this paper, denote $\mathscr{L}$ as a lattice implication algebra([4])$(L, \vee, \wedge, ', \to, O, I)$.

Definition 2.2: [4] A non-empty subset F of a lattice implication algebra $\mathscr{L}$ is called a **filter** of $\mathscr{L}$ if it satisfies

(F1) $I \in F$.

(F2) $(\forall x \in F)(\forall y \in L)(x \to y \in F \Rightarrow y \in F)$.

Definition 2.3: [4] A non-empty subset F of a lattice implication algebra $\mathscr{L}$ is called an **implicative filter** of $\mathscr{L}$ if it satisfies

(F1) $I \in F$.

(F2) $(\forall x, y, z \in L)(x \to (y \to z \in F$ and $x \to y \in F \Rightarrow x \to z \in F)$.

A fuzzy subset of a nonempty set X is defined as a mapping from X to $[0, 1]$, where $[0, 1]$ is the usual interval of real numbers.

Definition 2.4: [2] A fuzzy subset μ of $\mathscr{L}$ is said to be a **fuzzy filter** if, for any $x, y \in L$,

(1) $\mu(I) \geq \mu(x)$,

(2) $\mu(y) \geq min\{\mu(x), \mu(x \to y)\}$.

Definition 2.5: [2] A fuzzy subset μ of $\mathscr{L}$ is said to be a **fuzzy implicative filter** if, for any $x, y \in L$,

(1) $\mu(I) \geq \mu(x)$,

(2) $\mu(x \to z) \geq min\{\mu(x \to (y \to z)), \mu(x \to y)\}$.

A fuzzy set μ of a lattice implication algebra $\mathscr{L}$ of the form: when $y = x$,$\mu(y) = t \in (0, 1]$; in otherwise, $\mu(t) = 0$. This fuzzy set is said to be a **fuzzy point** with support x and value t and is denoted by x_t.

For a fuzzy point x_t and a fuzzy set μ in $\mathscr{L}$, Pu and Liu[6] gave meaning to the symbol $x_t \theta \mu$, where $\theta \in \{\in, q, \in \vee q, \in \wedge q\}$.

For a fuzzy point x_t is said to be belong to (resp. be quasi-coincident with) a fuzzy set A, written as $x_t \in \mu$ (resp. $x_t q \mu$) if $\mu(x) \geq t$ (resp. $\mu(x) + t > 1$). If $x_t \in \mu$ or (resp. and) $x_t q \mu$, then we write $x_t \in \vee q \mu$. The symbol $\overline{\in \vee q}$ means $\in \vee q$ doesn't hold.

III. (IF-) F-Soft lattice Implication Algebras

Molodtsov [12] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathbf{P}(U)$ denote the power set of U and $A \subset E$.

Definition 3.1: [12] A pair (F, A) is called a **soft set** over U, where F is a mapping $F : A \to \mathbf{P}(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For any $x \in A$, $F(x)$ may be considered as the set of x-approximate elements of the soft set (F, A).

In 2003, Maji[14] defined operations **and,** $\cap$**, or** $\cup$ which were later termed as basic intersection, basic union, and union by D. Pei[24]. We are taking the following definitions from [24].

Definition 3.2: Let (F, A) and (G, B) be any two soft sets over a lattice implication algebras $\mathscr{L}$.

(1)The **basic intersection** of two soft sets (F, A) and (G, B) is defined as the soft set $(H, C) = (F, A) \wedge (G, B)$, where $C = A \times B$ and $H(a, b) = F(a) \cap G(b)$ for any $(a, b) \in A \times B$.

(2)The **intersection** of soft sets (F, A) and (G, B) over a common universe U is defined as the soft set $(H, C) = (F, A) \cap (G, B)$, where $C = A \cap B$, and $H(c) = F(c) \cap G(c)$ for any $c \in C$.

(3) The **union** (H, C) of two soft sets (F, A) and (G, B) is defined as the soft set $(H, C) = (F, A) \cup (G, B)$, where $C = A \cup B$ and $H(c) = F(c)$ when $c \in A \setminus B$; $H(c) = G(c)$ when $c \in B \setminus A$; $H(c) = F(c) \cup G(c)$ when $c \in A \cap B$.

Definition 3.3: Let (F, A) be a nonempty soft set over a lattice implication algebras $\mathscr{L} = (L, \vee, \wedge, ', \to, O, I)$. Then

(1) (F, A) is a called a F**-soft lattice implication algebra** if $F(t)$ is a filter of $\mathscr{L}$ for any $t \in A$. For our convenience, the empty set $\emptyset$ is regarded as a filter of $\mathscr{L}$.

(2) (F, A) is a called a IF**-soft lattice implication algebra** if $F(t)$ is an implicative filter of $\mathscr{L}$ for any $t \in A$. For our convenience, the empty set $\emptyset$ is regarded as an implicative filter of $\mathscr{L}$.

Example 3.1: Let $L = \{O, a, b, c, d, I\}$, the Hasse diagram of L and its implication operator $\to$ and negation operator $'$ be defined in EXAMPLE 2.1.4 in [4] Then $\mathscr{L} = (L, \vee, \wedge, ', \to, O, I)$ is a lattice implication algebra.

(1) Let (F, A) be a soft set over $\mathscr{L}$, where $A = \{I, a, b\}$ and the set-valued function $F : A \to \mathbf{P}(L)$ defined by $F(t) = \{x \in L | x \vee t = I\}$. Then $F(I) = L$, $F(a) = \{I, b, c\}$, $F(b) = \{I, a\}$ are all filters of $\mathscr{L}$. Therefore (F, A) is a F-soft lattice implication algebra over $\mathscr{L}$.

(2) Let (F, A) be a soft set over $\mathscr{L}$, where $A = \{c, d\}$ and $F : A \to \mathbf{P}(L)$ the set-valued function defined by $F(t) = \{y \in \mathscr{L} | t \to y \in \{a, b\}\}$, then $F(c) = \{O, a, d\}, F(d) = \{O, c\}$ aren't filters of $\mathscr{L}$. Therefore (F, A) is not a F-soft lattice implication algebra of $\mathscr{L}$.

(3) Let (F, A) be a soft set over $\mathscr{L}$, where $A = \{c, d\}$ and $F : A \to \mathbf{P}(L)$ the set-valued function defined by $F(t) = \{y \in \mathscr{L} | y \to t \in \{a, c, d\}\}$, then $F(c) = \{a, I\}, F(d) = \{b, c, I\}$ are filters of $\mathscr{L}$. Therefore (F, A) is a F-soft lattice implication algebra of $\mathscr{L}$.

Example 3.2: Let $L = \{O, a, b, I\}$, its implication operator $\to$ and negation operator $'$ be defined as Table 2. Then $\mathscr{L} = (L, \vee, \wedge, ', \to, O, I)$ is a lattice implication algebra.

Let (F, A) be a soft set over $\mathscr{L}$, where $A = (0, 1]$ and $F : A \to \mathbf{P}(L)$ the set-valued function defined by $F(t) = L$ when $t \in (0, 0.5]$; $F(t) = \{I, a\}$ when $t \in (0.5, 0.9]$; $F(t) = \emptyset$ when $t \in (0.9, 1]$.

Then $F(t)$ is an implicative filter of $\mathscr{L}$ for $t \in A$. Therefore, (F, A) is an IF-soft lattice implication algebra over $\mathscr{L}$.

Theorem 3.1: Let (F, A) and (G, B) be two F-soft lattice implication algebras over $\mathscr{L}$, then $(F, A) \cap (G, B)$is also a F-soft lattice implication algebra over $\mathscr{L}$ if $A \cap B \neq \emptyset$.

Proof: Let $(F, A) \cap (G, B) = (H, C)$, where $C = A \cap B \neq \emptyset$ and $H(c) = F(c) \cap G(c)$ for any $c \in C$. We have $F(c)$ and $G(c)$ are two filters of $\mathscr{L}$, hence $H(c) = F(c) \cap G(c)$ is a filter of $\mathscr{L}$ or $H(c) = \emptyset$. That is, $(H, C) = (F, A) \cap (G, B)$ is a F-soft lattice implication algebra over $\mathscr{L}$. ■

Theorem 3.2: Let (F, A) and (G, B) be two F-soft lattice implication algebras over $\mathscr{L}$, then $(F, A) \cup (G, B)$is also a

F-soft lattice implication algebra over $\mathscr{L}$ if $A \cap B = \emptyset$.

Proof: Let $(F, A) \cup (G, B) = (H, C)$ and $A \cap B = \emptyset$, where $C = A \cap B = \emptyset$. We have $c \in A \backslash B$ or $c \in B \backslash A$ for any $c \in C$. If $c \in A \backslash B$, then $H(c) = F(c)$, it follows that $H(c)$ is a F-soft lattice implication algebra over $\mathscr{L}$. Similarly, we have $H(c)$ is a F-soft lattice implication algebra over $\mathscr{L}$ for any $c \in B \backslash A$. Therefore $H(c)$ is a F-soft lattice implication algebra over $\mathscr{L}$. That is (H, C) is a F-soft lattice implication algebra over $\mathscr{L}$. ■

Theorem 3.3: Let (F, A) and (G, B) be two F-soft lattice implication algebras over $\mathscr{L}$. Then $(F, A) \wedge (G, B)$ is also a F-soft lattice implication algebra over $\mathscr{L}$.

Proof: Let $(H, C) = (F, A) \wedge (G, B)$, where $C = A \times B$ and $H(x_1, x_2) = F(x_1) \cap F(x_2), (x_1, x_2) \in A \times B$. Now, $F(x_1)$ and $F(x_2)$ are two filters of $\mathscr{L}$, so $F(x_1) \cap G(x_2)$ is also a filter of $\mathscr{L}$. Hence (H, C) is a F-soft lattice implication algebra over $\mathscr{L}$. ■

Definition 3.4: Let (F, A) be a F-soft lattice implication algebra over $\mathscr{L}$.

(1) (F, A) is called the trivial F-soft lattice implication algebra over $\mathscr{L}$ if $F(x) = \{I\}$ for any $x \in A$.

(2) (F, A) is called the whole F-soft lattice implication algebra over $\mathscr{L}$ if $F(x) = L$ for any $x \in A$.

Example 3.3: In Example 3.1, let (F, A) be a soft set over $\mathscr{L} = (L, \vee, \wedge, ', \rightarrow, O, I)$.

(1) $A = \{I\}$ and $F : A \rightarrow \mathbf{P}(L)$ the set-valued function defined by $F(x) = \{y \in L | x \leq y\}$, then $F(I) = \{I\}$ is a filter of $\mathscr{L}$. Hence (F, A) is a trivial F-soft lattice implication algebra over $\mathscr{L}$.

(2) $A = \{I\}$ and $F : A \rightarrow \mathbf{P}(L)$ the set-valued function defined by $F(x) = \{y \in L | x \vee y = I\}$, then $F(I) = L$ is a filter of $\mathscr{L}$. Hence (F, A) is a whole F-soft lattice implication algebra over $\mathscr{L}$.

Let $\mathscr{L}_1 = (L_1, \vee, \wedge, ', \rightarrow, O, I), \mathscr{L}_2 = (L_2, \vee, \wedge, ', \rightarrow, O, I)$ be two lattice implication algebras and $f : \mathscr{L}_1 \rightarrow \mathscr{L}_2$ a mapping of lattice implication algebras. If (F_1, A) and (F_2, B) are soft set over $\mathscr{L}_1$ and $\mathscr{L}_2$, respectively. Then $(f(F_1), A)$ is a soft set over $\mathscr{L}_2$, where $f(F_1) : A \rightarrow \mathbf{P}(L_2)$ defined by $f(F_1)(x) = f(F(x))$ for any $x \in A$. And $(f^{-1}(F_2), B)$ is a soft set over $\mathscr{L}_1$, where $f^{-1}(F_2) : B \rightarrow \mathbf{P}(L_1)$ is defined by $f^{-1}(F_2)(y) = f^{-1}(F_2(y))$ for any $y \in B$.

Theorem 3.4: Let $\mathscr{L}_1, \mathscr{L}_2$ be lattice implication algebras and $f : \mathscr{L}_1 \rightarrow \mathscr{L}_2$ be an lattice implication homomorphism. If (F_2, B) is a F-soft lattice implication algebra over $\mathscr{L}_2$, then $(f^{-1}(F_2), B)$ is a F-soft lattice implication algebra over $\mathscr{L}_1$.

Proof: Since (F_2, B) is a F-soft lattice implication algebra over $\mathscr{L}_2$, then $F_2(y)$ is a filter for any $y \in B$ and so $f(I) = I^* \in F_2(y)$, where I and I^* are the greatest elements of $\mathscr{L}_1$ and $\mathscr{L}_2$, respectively. It follows that $I = f^{-1}(I^*) \in f^{-1}(F_2(y))$.

If $x_1, x_1 \rightarrow y_1 \in f^{-1}(F_2)(y)$ for any $y \in B$, then $f(x_1), f(x_1 \rightarrow y_1) \in F_2(y)$. Since $F_2(y)$ is a filter of $\mathscr{L}_2$ and f is a lattice implication homomorphism, we have $f(y_1) \in F_2(y)$, that is, $y_1 \in f^{-1}(F_2)(y)$. Therefore, $f^{-1}(F_2)(y)$ is a filter of $\mathscr{L}_1$ and so $(f^{-1}(F_2), B)$ is a F-soft lattice implication algebra over $\mathscr{L}_1$. ■

Theorem 3.5: Let $f : \mathscr{L}_1 \rightarrow \mathscr{L}_2$ be an lattice implication homomorphism and (F, A) and (G, B) are F-soft lattice implication algebras over $\mathscr{L}_1$ and $\mathscr{L}_2$, respectively.

(1) If $F(x) = D - Ker(f)$ for any $x \in A$, then $(f(F), A)$ is a trivial F-soft lattice implication algebra over $\mathscr{L}_2$.

(2) If f is onto and (F, A) is whole, then $(f(F), A)$ is a whole F-soft lattice implication algebra over $\mathscr{L}_2$.

(3) If $G(y) = F(\mathscr{L}_1)$ for any $y \in B$, then $(f^{-1}(G), B)$ is F-soft lattice implication algebra over $\mathscr{L}_1$.

(4) If f is injective and (G, B) is trivial, then $(f^{-1}(G), B)$ is a trivial F-soft lattice implication algebra over $\mathscr{L}_1$.

Proof: (1) Assume that $F(x) = D - Ker(f)$ for any $x \in A$, then $f(F)(x) = f(F(x)) = \{I_2\}$ for any $x \in A$, where I_2 is the greatest element of $\mathscr{L}_2$. Obviously, $f(F(x))$ is a filter of $\mathscr{L}$ for any $x \in A$, that is $(f(F), A)$ is a trivial F-soft lattice implication algebra over $\mathscr{L}_2$.

(2) Assume that f is onto and (F, A) is whole, then $F(x) = \mathscr{L}_1$ and $f(\mathscr{L}_1) = \mathscr{L}_2$for any $x \in A$ and so $f(F)(x) = f(F(x)) = \mathscr{L}_2$. Obviously, $f(F)(x) = \mathscr{L}_2$ is a filter of $\mathscr{L}_2$ for any $x \in A$, that is $(f(F), A)$ is a whole F-soft lattice implication algebra over $\mathscr{L}_2$.

(3) Suppose that $G(y) = f(\mathscr{L}_1)$ for any $y \in B$. Then $f^{-1}(G)(y) = f^{-1}(G(y)) = f^{-1}(f(\mathscr{L}_1))\mathscr{L}_1$ for any $y \in B$. Hence $(f^{-1}(G), B)$ is F-soft lattice implication algebra over $\mathscr{L}_1$.

(4) Let f is injective and (G, B) is trivial, then $G(y) = I$ for any $y \in B$ and so $f^{-1}(G)(y) = f^{-1}(G(y)) = f^{-1}(I) = D - ker(f) = \{I_1\}$ for any $y \in B$. Therefore $f^{-1}(G)(y)$ is a filter of $\mathscr{L}_1$ for any $y \in B$, where I_1 is the greatest element of $\mathscr{L}_1$. It follows that $(f^{-1}(G), B)$ is a trivial F-soft lattice implication algebra over $\mathscr{L}_1$. ■

IV. (IF-)F-Soft lattice Implication Algebras in Fuzzy Context

In this section, firstly, we discuss the relations between $(IF$-$)F$-soft lattice implication algebras and fuzzy (implicative) filters. Secondly, we generalized fuzzy filters of $\mathscr{L}$, initiating the notion of $(\in, \in \vee q_k)$-fuzzy (implicative) filter ($(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy (implicative) filter) of $\mathscr{L}$, their equivalent characterizations are derived. At last, we discuss the relations between $(IF$-$)$ F-soft lattice implication algebras and $(\in, \in \vee q_k)$-fuzzy (implicative) filter.

Given a fuzzy set μ in a lattice implication algebra $\mathscr{L} = (L, \vee, \wedge, ', \rightarrow, O, I)$ and $A \subseteq [0, 1]$, consider two set-valued functions $F : A \rightarrow \mathbf{P}(L)$, defined by $F(t) = \{x \in L | x_t \in \mu\}$ and $F_q : A \rightarrow \mathbf{P}(L)$, defined by $F_q(t) = \{x \in L | x_t q \mu\}$. Then (F, A) and (F_q, A) are two soft set over $\mathscr{L}$. In fact, (F, A) and (F_q, A) is called $\in$-soft set and q-soft set in [19], respectively.

Theorem 4.1: Let μ be a fuzzy set of $\mathscr{L}$ and $(F, (0, 1])$ be a soft set over $\mathscr{L}$. Then

(1)$(F, (0, 1])$ is a F-soft lattice implication algebra if and only if μ is a fuzzy filter of $\mathscr{L}$.

(2)$(F, (0, 1])$ is a IF-soft lattice implication algebra if and only if μ is a fuzzy implicative filter of $\mathscr{L}$.

Proof: (1) Suppose that μ is a fuzzy filter of $\mathscr{L}$ and let $x \in (0, 1]$. If $x \in F(t)$, then $x_t \in \mu$, i.e. $I \in F(t)$. Let $x, x \rightarrow y \in F(t)$ for any $t \in (0, 1]$, then $x_t \in \mu$ and $(x \rightarrow y)_t \in \mu$, i.e. $\mu(x) \geq t$ and $\mu(x \rightarrow y) \geq t$. It follows that $\mu(y) \geq min\{\mu(x), \mu(x \rightarrow y)\} \geq t$, and so $y_t \in \mu$, i.e.

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$y \in F(t)$. Therefore $F(t)$ is a filter of $\mathscr{L}$ for any $t \in (0,1]$. Hence $(F,(0,1])$ is a F-soft lattice implication algebra over $\mathscr{L}$.

Conversely, assume that $(F,(0,1])$ is a F-soft lattice implication algebra over $\mathscr{L}$. If there exists $a \in L$ such that $\mu(I) < \mu(a)$, then we can choose $t \in (0,1]$ such that $\mu(I) < t \leq \mu(a)$ and so $I_t \notin \mu$, i.e. $I \notin F(t)$, contradiction. Thus $\mu(I) \geq \mu(x)$ for any $x \in L$. Suppose that there exist $a, b \in L$ such that $\mu(b) < min\{\mu(a \to b), \mu(a)\}$, we choose $t \in (0,1]$ such that $\mu(b) < t \leq min\{\mu(a \to b), \mu(a)\}$. It follows that $a \to b \in F(t)$ and $a \in F(t)$, but $b \notin F(t)$, which contradicts with $F(t)$ is a filter of $\mathscr{L}$. Hence $\mu(y) \geq min\{\mu(x \to y), \mu(x)\}$ for any $x, y \in L$. Therefore μ is a fuzzy filter of $\mathscr{L}$.

(2) The case for (2) can be similarly proved. ■

Theorem 4.2: Let μ be a fuzzy set of $\mathscr{L}$ and $(F_q,(0,1])$ be a soft set over $\mathscr{L}$. Then

(1) $(F_q,(0,1])(F_q(t) \neq \emptyset, t \in (0,1])$ is a F-soft lattice implication algebra if and only if μ is a fuzzy filter of $\mathscr{L}$.

(1) $(F_q,(0,1])(F_q(t) \neq \emptyset, t \in (0,1])$ is a IF-soft lattice implication algebra if and only if μ is a fuzzy implicative filter of $\mathscr{L}$.

Proof: (1) Assume that μ is a fuzzy filter of $\mathscr{L}$ and let $t \in (0,1]$ such that $F_q(t) \neq \emptyset$. If $I \notin F_q(t)$, i.e. $I_t \overline{q} \mu$ and so $\mu(I) + t < 1$. It follows that $\mu(x) + t \leq \mu(I) + t < 1$ for any $x \in L$, so that $F_q(t) = \emptyset$. This is a contradiction and so $I \in F_q(t)$. Let $x, y \in L$ be such that $x \to y \in F_q(t)$ and $x \in F_q(t)$, then $(x \to y)_t q \mu$ and $x_t q \mu$, i.e. $\mu(x \to y) + t > 1$ and $\mu(y) + t > 1$. Hence $\mu(y) + t \geq min\{\mu(x \to y), \mu(y)\} + t = min\{\mu(x \to y) + t, \mu(y) + t\} > 1$. We have $y \in F_q(t)$ for any $t \in (0,1]$. Therefore $F_q(t)$ is a filter of $\mathscr{L}$ for any $t \in (0,1]$, i.e. $(F_q,(0,1])$ is a F-soft lattice implication algebra over $\mathscr{L}$.

Conversely, assume that (F_q, A) is a F-soft lattice implication algebra over $\mathscr{L}$. If there exists $a \in L$ such that $\mu(I) < \mu(a)$, then $\mu(I) + t \leq 1 < \mu(a) + t$ for some $t \in (0,1]$. Thus $a_t q \mu$ and $F_q(t) \neq \emptyset$. We have $F_q(t)$ is a filter of $\mathscr{L}$ for any $t \in (0,1]$. Hence $I \in F_q(t)$ and $I_t q \mu$, i.e. $\mu(I) + t > 1$, this is impossible, and so $\mu(I) \geq \mu(x)$ for any $x \in L$. Suppose that there exist $a, b \in L$ such that $\mu(b) < min\{\mu(a \to b), \mu(a)\}$, then $\mu(a) + s \leq 1 < min\{\mu(a \to b), \mu(a)\} + s$ for some $s \in (0,1]$. It follows that $(a \to b)_s q \mu$ and $a_s q \mu$, that is, $a \to b \in F_q(s)$ and $a \in F_q(s)$. It follows from $F_q(s)$ is a filter that $b \in F_q(t)$. So $\mu(a) + s > 1$, contradiction. Therefore $\mu(y) \geq min\{\mu(x \to y), \mu(x)\}$ for any $x, y \in L$. Hence μ is a fuzzy filter of $\mathscr{L}$.

(2) The case for (2) can be similarly proved. ■

In what follows, let k denote an arbitrary element of $[0,1)$ unless otherwise specified. To say $x_t q_k \mu$, we mean $\mu(x) + t + k > 1$. To say $x_t \in \vee q_k \mu$, we mean $x_t \in \mu$ or $x_t q_k \mu$. For $\alpha \in \{\in, \in \vee q_k\}$, to say $x_t \overline{\alpha} \mu$, we mean $x_t \alpha \mu$ doesn't hold.

Definition 4.1: A fuzzy set μ in $\mathscr{L}$ is said to be an $(\in, \in \vee q_k)$-fuzzy filter of $\mathscr{L}$ if it satisfies the following:

(1) $x_t \in \mu$ implies $I_t \in \vee q_k \mu$,

(2) $x_t \in \mu$ and $(x \to y)_r \in \mu$ imply $y_{min\{t,r\}} \in \vee q_k \mu$, for any $x, y \in L$ and $t, r \in (0,1]$.

Definition 4.2: A fuzzy set μ in $\mathscr{L}$ is said to be an $(\in, \in \vee q_k)$-fuzzy implicative filter of $\mathscr{L}$ if it satisfies the following:

(1) $x_t \in \mu$ implies $I_t \in \vee q_k \mu$,

(2) $(x \to (y \to z))_t \in \mu$ and $(x \to y)_r \in \mu$ imply $(x \to z)_{min\{t,r\}} \in \vee q_k \mu$, for any $x, y \in L$ and $t, r \in (0,1]$.

Example 4.1: (1) In Example 3.1, we define a fuzzy set μ of $\mathscr{L}$ as following:

$$\mu(I) = 0.45, \mu(a) = 0.8, \mu(b) = \mu(c) = \mu(d) = \mu(O) = 0.3.$$

It is routine to verify that μ is an $(\in, \in \vee q_{0.2})$-fuzzy filter. But μ is neither a fuzzy filter nor an $(\in, \in \vee q)$-fuzzy filter of $\mathscr{L}$ since $\mu(I) = 0.45 \mu(a) = 0.8$, and $a \leq I$, $c_{0.5 \in \mu}$ but $I_{0.5} \overline{\in \vee q} \mu$.

(2) In Example 3.1, we define a fuzzy set G of $\mathscr{L}$ as following:

$$G(I) = G(a) = 0.7, G(b) = 0.6, G(d) = 0.3, G(O) = G(c) = 0.2.$$

It is routine to verify that G is an $(\in, \in \vee q_{0.6})$-fuzzy implicative filter.

Theorem 4.3: A fuzzy set μ in $\mathscr{L}$ is an $(\in, \in \vee q_k)$-fuzzy filter of $\mathscr{L}$ if and only if it satisfies the following, for any $x, y, z \in L$

(1) $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\}$

(2) $\mu(y) \geq min\{\mu(x \to y), \mu(x), \frac{1-k}{2}\}$.

Proof: Let μ be an $(\in, \in \vee q_k)$-fuzzy filter, Assume that $\mu(I) < min\{\mu(x), \frac{1-k}{2}\}$. Then $\mu(I) < r \leq min\{\mu(x), \frac{1-k}{2}\}$ for some $r \in (0, \frac{1-k}{2}]$. If $\mu(x) < \frac{1-k}{2}$, then $\mu(I) < r \leq \mu(x)$. Hence $x_r \in \mu$ and $I_r \notin \mu$. Furthermore, $\mu(I) + r < r + r = 2r < 1 - k$, that is, $I_r \overline{q_k} \mu$, thus $I_r \overline{\in \vee q_k} \mu$, contradiction. If $\mu(x) \geq \frac{1-k}{2}$, then $\mu(I) < r \leq \frac{1-k}{2}$, hence $x_{\frac{1-k}{2}} \in \mu$, but $I_{\frac{1-k}{2}} \notin \mu$. Therefore $\mu(I) + \frac{1-k}{2} \leq 1 - k$, that is, $I_{\frac{1-k}{2}} \overline{q_k} \mu$. Therefore, $y_{\frac{1-k}{2}} \overline{\in \vee q_k} \mu$, contradiction. Hence $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\}$ for any $x, y \in L$.

Assume that (2) doesn't hold, then there exist $x, y \in L$ such that $\mu(y) < min\{\mu(x \to y), \mu(x), \frac{1-k}{2}\}$. If $min\{\mu(x \to y), \mu(x)\} < \frac{1-k}{2}$, then $\mu(y) < min\{\mu(x \to y), \mu(x)\}$. Hence $\mu(y) < t \leq min\{x \to y), \mu(x)\}$ for some $t \in (0, \frac{1-k}{2}]$. It follows that $(x \to y)_t \in \mu$ and $x_t \in \mu$, but $y_t \notin \mu$. Moreover, $\mu(y) + t < 2t < 1 - k$ and so $y_t \overline{\in \vee q_k} \mu$, contradiction. If $min\{\mu(x \to y), \mu(x)\} \geq \frac{1-k}{2}$, then $\mu(x) \geq \frac{1-k}{2}$ and $\mu(x \to y) \geq \frac{1-k}{2}$. It follows that $x_{\frac{1-k}{2}} \in \mu$ and $(x \to y)_{\frac{1-k}{2}} \in \mu$. Since μ is an $(\in, \in \vee q_k)$-fuzzy filter of $\mathscr{L}$, we have $y_{\frac{1-k}{2}} \in \vee q_k \mu$, $\mu(y) < min\{\mu(x \to y), \mu(x), \frac{1-k}{2}\} = \frac{1-k}{2}$. And so $y_{\frac{1-k}{2}} \notin \mu$, also $\mu(y) + \frac{1-k}{2} < \frac{1-k}{2} \times 2 = 1 - k$. That is $y_{\frac{1-k}{2}} \overline{q_k} \mu$. Hence $y_{\frac{1-k}{2}} \overline{\in \vee q_k} \mu$, contradiction. Therefore $\mu(y) \geq min\{\mu(x \to y), \mu(x), \frac{1-k}{2}\}$ for any $x, y \in L$.

Conversely, let μ be a fuzzy set in $\mathscr{L}$ satisfying two conditions. Let $x, y \in L$ and $t, r \in (0, \frac{1-k}{2}]$ be such that $x_t \in \mu$, then $\mu(x) \geq t$ and so $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\} \geq min\{t, \frac{1-k}{2}\}$. If $t \leq \frac{1-k}{2}$, then $\mu(x) \geq t$ and $I_t \in \mu$. If $t > \frac{1-k}{2}$, then $\mu(I) \leq \frac{k-1}{2}$ and so $\mu(I) + t > \frac{1-k}{2} + \frac{1-k}{2} = 1 - k$. Hence $\mu(I) + k + t > 1$, i.e. $I_t q_k \mu$, we have $I_t \in \vee q_k \mu$.

Let μ satisfying $\mu(y) \geq min\{\mu(x \to y), \mu(x), \frac{1-k}{2}\}$ for any $x, y \in L$. Let $x, y \in L$ and $t, r \in (0, \frac{1-k}{2}]$ be such that $x_t \in \mu$ and $(x \to y)_r \in \mu$, then $\mu(x) \geq t, \mu(x \to y) \geq r$. We have $\mu(y) \geq min\{\mu(x \to y), \mu(x), \frac{1-k}{2}\} \geq min\{t, r, \frac{1-k}{2}\}$. If $min\{t, r\} \leq \frac{1-k}{2}$, then $\mu(y) \geq min\{t, r\}$, it follows that $y_{min\{t,r\}} \in \mu$ and so $y_{min\{t,r\}} \in \vee q_k \mu$. If $min\{t, r\} > \frac{1-k}{2}$,

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i.e. $t \geq \frac{1-k}{2}$ and $r \geq \frac{k-1}{2}$, we have $\mu(y) > \frac{1-k}{2}$, $\mu(y) + min\{t,r\} > \frac{1-k}{2} + \frac{1-k}{2} = 1-k$. It follows that $y_{min\{t,r\}} \in \vee q_k \mu$. Therefore μ is an $(\in, \in \vee q_k)$-fuzzy filter of $\mathscr{L}$. ■

Corollary 4.1: (see [21]) Let μ be a fuzzy subset of $\mathscr{L}$. Then μ is an $(\in, \in \vee q)$-fuzzy filter of $\mathscr{L}$ if and only μ satisfies following

(1)$(\forall x \in L)(\mu(I) \geq min\{\mu(x), 0.5\})$,

(2)$(\forall x, y \in L)(\mu(y) \geq min\{\mu(x), \mu(x \to y), 0.5\}$.

Theorem 4.4: A fuzzy set μ in $\mathscr{L}$ is an $(\in, \in \vee q_k)$-fuzzy implicative filter of $\mathscr{L}$ if and only if it satisfies the following, for any $x, y, z \in L$

(1) $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\}$

(2) $\mu(x \to z) \geq min\{\mu(x \to (y \to z)), \mu(x \to y), \frac{1-k}{2}\}$.

Proof: It is similarly proved as Theorem 4.3. ■

Corollary 4.2: (see [21]) Let μ be a fuzzy subset of $\mathscr{L}$. Then μ is an $(\in, \in \vee q)$-fuzzy implicative filter of $\mathscr{L}$ if and only μ satisfies following

(1)$(\forall x \in L)(\mu(I) \geq min\{\mu(x), 0.5\})$,

(2)$(\forall x, y \in L)(\mu(x \to y) \geq min\{\mu(x \to (y \to z)), \mu(x \to y), 0.5\}$.

Theorem 4.5: Let μ be a fuzzy set of $\mathscr{L}$ and $(F, (0, \frac{1-k}{2}])$ be a soft set. Then $(F, (0, \frac{1-k}{2}])$ is a F-soft lattice implication algebra over $\mathscr{L}$ is and only if μ is an $(\in, \in \vee q_k)$-fuzzy filter of $\mathscr{L}$.

Proof: Let μ be an $(\in, \in \vee q_k)$-fuzzy filter of $\mathscr{L}$. For any $t \in (0, \frac{1-k}{2}]$, we have $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\}$ for any $x \in F(t)$ by Theorem 4.3. Hence $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\} \geq min\{t, \frac{1-k}{2}\} = t$, which implies $I_t \in \mu$ and so $I \in F(t)$. If $x \to y \in F(t), x \in F(t)$. That is $\mu(x \to y) \geq t, \mu(x) \geq t$. Therefore, we have $\mu(y) \geq min\{\mu(x \to y), \mu(x), \frac{1-k}{2}\} = t$, i.e. $y_t \in \mu$, $y \in F(t)$. Therefore $F(t)$ is a filter of $\mathscr{L}$ for any $t \in (0, \frac{1-k}{2}])$, i.e. $(F, (0, \frac{1-k}{2}])$ is a F-soft lattice implication algebra over $\mathscr{L}$.

Conversely, assume that $(F, (0, \frac{1-k}{2}])$ is a F-soft implication algebra over $\mathscr{L}$. If there exist $a \in L$ such that $\mu(I) < min\{\mu(a), \frac{k-1}{2}\}$, then $\mu(I) < t \leq min\{\mu(a), \frac{1-k}{2}\}$ for some $t \in (0, \frac{1-k}{2}]$. It follows that $I_t \notin \mu$, i.e. $I \notin F(t)$, which contradicts with $F(t)$ is a filter of $\mathscr{L}$. Hence $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\}$ for any $x \in L$. If there exists $a, b \in L$ such that $\mu(b) < min\{\mu(x \to y), \mu(a), \frac{1-k}{2}\}$, then there exist $t \in (0, \frac{1-k}{2}]$ such that $\mu(b) < t \leq min\{\mu(a \to b), \mu(a), \frac{1-k}{2}\}$ which implies $a \to b \in F(t)$ and $a \in F(t)$, it follows that $b \in F(t)$, i.e $\mu(b) \geq t$, but $\mu(b) < t$, contradiction. Therefore, $\mu(y) \geq min\{\mu(x \to y), \mu(x), \frac{1-k}{2}\}$ hold for any $x, y \in L$. And so μ is an $(\in, \in \vee q_k)$-fuzzy filter$\mathscr{L}$. ■

Corollary 4.3: Let μ be a fuzzy set of $\mathscr{L}$ and $(F, (0, 0.5])$ be a soft set. Then $(F, (0, 0.5])$ is a F-soft lattice implication algebra over $\mathscr{L}$ is and only if μ is an $(\in, \in \vee q)$-fuzzy filter of $\mathscr{L}$.

Theorem 4.6: Let μ be a fuzzy set of $\mathscr{L}$ and $(F, (0, \frac{1-k}{2}])$ be a soft set. Then $(F, (0, \frac{1-k}{2}])$ is a IF-soft lattice implication algebra over $\mathscr{L}$ is and only if μ is an $(\in, \in \vee q_k)$-fuzzy implicative filter of $\mathscr{L}$.

Proof: Let μ be an $(\in, \in \vee q_k)$-fuzzy implicative filter of $\mathscr{L}$. For any $t \in (0, \frac{1-k}{2}]$, we have $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\}$ for any $x \in F(t)$. Hence $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\} \geq min\{t, \frac{1-k}{2}\} = t$, which implies $I_t \in \mu$ and so $I \in F(t)$. If $x \to (y \to z) \in F(t), x \to y \in F(t)$. That is $\mu(x \to (y \to z) \geq t, \mu(x \to y) \geq t$. Therefore, we have $\mu(x \to z) \geq min\{\mu(x \to (y \to z), \mu(x \to y), \frac{1-k}{2}\} = t$, and so $(x \to z)_t \in \mu$ i.e. $x \to z \in F(t)$. Therefore $F(t)$ is an implicative filter of $\mathscr{L}$ for any $t \in (0, \frac{1-k}{2}])$, i.e. $(F, (0, \frac{1-k}{2}])$ is a IF-soft lattice implication algebra over $\mathscr{L}$.

Conversely, assume that $(F, (0, \frac{1-k}{2}])$ is an IF-soft implication algebra over $\mathscr{L}$. If there exist $a \in L$ such that $\mu(I) < min\{\mu(a), \frac{k-1}{2}\}$, then $\mu(I) < t \leq min\{\mu(a), \frac{1-k}{2}\}$ for some $t \in (0, \frac{1-k}{2}]$. It follows that $I_t \notin \mu$, i.e. $I \notin F(t)$, which contradicts with $F(t)$ is an implicative filter of $\mathscr{L}$. Hence $\mu(I) \geq min\{\mu(x), \frac{1-k}{2}\}$ for any $x \in L$. If there exists $a, b, c \in L$ such that $\mu(a \to c) < min\{\mu(a \to (b \to c)), \mu(a \to b), \frac{1-k}{2}\}$, then there exist $t \in (0, \frac{1-k}{2}]$ such that $\mu(a \to c) < t \leq min\{\mu(a \to (b \to c)), \mu(a \to b), \frac{1-k}{2}\}$ which implies $a \to (b \to c) \in F(t)$ and $a \to b \in F(t)$, it follows that $a \to c \in F(t)$, i.e $\mu(a \to c) \geq t$, but $\mu(a \to c) < t$, contradiction. Therefore, $\mu(x \to z) \geq min\{\mu(x \to (y \to z)), \mu(x \to y), \frac{1-k}{2}\}$ hold for any $x, y, z \in L$. And so μ is an $(\in, \in \vee q_k)$-fuzzy implicative filter of $\mathscr{L}$. ■

Corollary 4.4: Let μ be a fuzzy set of $\mathscr{L}$ and $(F, (0, 0.5])$ be a soft set. Then $(F, (0, 0.5])$ is a IF-soft lattice implication algebra over $\mathscr{L}$ is and only if μ is an $(\in, \in \vee q_k)$-fuzzy implicative filter of $\mathscr{L}$.

Definition 4.3: A fuzzy subset μ on $\mathscr{L}$ is said to be an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy filter, if it satisfies, for any $x, y \in L$, $t, r \in (\frac{1-k}{2}, 1]$:

(1) $I_t \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q_k} \mu$,

(2) if $y_{min\{t,r\}} \overline{\in} \mu$, then $x_t \overline{\in} \vee \overline{q_k} \mu$ or $(x \to y)_r \overline{\in} \vee \overline{q_k} \mu$.

Definition 4.4: A fuzzy subset μ on $\mathscr{L}$ is said to be an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy implicative filter, if it satisfies, for any $x, y \in L$, $t, r \in (\frac{1-k}{2}, 1]$:

(1) $I_t \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q_k} \mu$,

(2) if $(x \to z)_{min\{t,r\}} \overline{\in} \mu$, then $(x \to (y \to z)_t \overline{\in} \vee \overline{q_k} \mu$ or $(x \to y)_r \overline{\in} \vee \overline{q_k} \mu$.

Example 4.2: In Example 3.1, we define a fuzzy set μ as follows:

$$\mu(O) = 0.4, \mu(I) = \mu(b) = \mu(c) = 0.9, \mu(a) = \mu(d) = 0.3.$$

It is routine to verify μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_{0.2}})$-fuzzy (implicative) filter of $\mathscr{L}$.

Theorem 4.7: Let μ be a fuzzy subset of $\mathscr{L}$, then μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy filter of $\mathscr{L}$ if and only if for any $x, y \in L$,

(1)$max\{\mu(I), \frac{1-k}{2}\} \geq \mu(x)$,

(2)$max\{\mu(y), \frac{1-k}{2}\} \geq min\{\mu(x), \mu(x \to y)\}$.

Proof: Assume that there exists $x \in L$ such that $max\{\mu(I), \frac{1-k}{2}\} < \mu(x) = t$. Then $t \in (\frac{1-k}{2}, 1]$ and $I_t \overline{\in} \mu$. It follows that $x_t \overline{\in} \vee \overline{q_k} \mu$. Hence $\mu(x) < t$ or $\mu(x) + t + k \leq< 1$, we have $t \leq \frac{1-k}{2}$ for $\mu(x) = t$, contradiction. Therefore, $max\{\mu(I), \frac{1-k}{2}\} \geq \mu(x)$, (1) is valid.

Assume that there exist $x, y \in L$ such that $max\{\mu(y), \frac{1-k}{2}\} < min\{\mu(x), \mu(x \to y)\} = t$, then $\mu(y) < t$ and $t \in (\frac{1-k}{2}, 1]$. It follows that $y_t \overline{\in} \mu$ and so $x_t \overline{\in} \vee \overline{q_k} \mu$ or $(x \to y)_t \overline{\in} \vee \overline{q_k} \mu$. But $x_t \in \mu$ and $(x \to y)_t \in \mu$. Hence $x_t \overline{q_k} \mu$ or $(x \to y)_t \overline{q_k} \mu$. It follows that $\mu(x) \geq t$ and $\mu(x) + t + k \leq 1$, $\mu(x \to y) = t$ and $\mu(x \to y) + t + k \leq 1$, we have that $t \leq \frac{1-k}{2}$, contradiction. Therefore, (2) holds.

Conversely, assume that there exist $x \in L$ and $t, r \in (\frac{1-k}{2}, 1]$ such that $I_t \overline{\in} \mu$, but $x_t \overline{\overline{\in} \vee \overline{q_k}} \mu$, then $\mu(I) < t, \mu(x) \geq t$ and $\mu(x) + t + k \geq 1$. Therefore, $\mu(x) \geq \frac{1-k}{2}$. Thus $max\{\mu(I), \frac{1-k}{2}\} < max\{t, \frac{1-k}{2}\} \leq max\{\mu(x), t\} = \mu(x)$, contradiction. That is, $I_t \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q_k} \mu$.

Assume (2) holds and let $y_{min\{t,r\}} \overline{\in} \mu$, then $\mu(y) < min\{t, r\}$. There are two cases to be discussed.

(a) If $\mu(y) \geq min\{\mu(x), \mu(x \to y)\}$, then $min\{t, r\} > min\{\mu(x), \mu(x \to y)\}$. It follows that $\mu(x) < t$ or $\mu(x \to y) < r$, that is, $x_t \overline{\in} \mu$ or $(x \to y)_r \overline{\in} \mu$. Of course, $x_t \overline{\in} \vee \overline{q_k} \mu$ or $(x \to y)_r \overline{\in} \vee \overline{q_k} \mu$.

(b) If $\mu(y) < min\{\mu(x), \mu(x \to y)\}$, then $\frac{1-k}{2} \geq min\{\mu(x), \mu(x \to y)\}$. Assume that $x_t \overline{\overline{\in} \vee \overline{q_k}} \mu$ and $(x \to y)_r \overline{\overline{\in} \vee \overline{q_k}} \mu$, then $\mu(x) \geq r$ and $\mu(x) + r + k > 1$, $\mu(x \to y) \geq r$ and $\mu(x \to y) + r + k > 1$. It follows that $\mu(x) > \frac{1-k}{2}$ and $\mu(x \to y) > \frac{1-k}{2}$. Hence $min\{\mu(x), \mu(x \to y)\} > \frac{1-k}{2}$, which contradicts with $min\{\mu(x), \mu(x \to y)\} \leq \frac{1-k}{2}$. Therefore, $x_t \overline{\in} \vee \overline{q_k} \mu$ or $(x \to y)_r \overline{\in} \vee \overline{q_k} \mu$. ■

Theorem 4.8: Let μ be a fuzzy subset of $\mathscr{L}$, then μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy implicative filter of $\mathscr{L}$ if and only if for any $x, y \in L$,

(1)$max\{\mu(I), \frac{1-k}{2}\} \geq \mu(x)$,

(2)$max\{\mu(x \to z), \frac{1-k}{2}\} \geq min\{\mu(x \to (y \to z)), \mu(x \to y)\}$.

Proof: It is similarly proved as Theorem 4.7. ■

Corollary 4.5: Let μ be a fuzzy subset of $\mathscr{L}$, then μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy implicative filter of $\mathscr{L}$ if and only if for any $x, y \in L$,

(1)$max\{\mu(I), 0.5\} \geq \mu(x)$,

(2)$max\{\mu(x \to z), 0.5\} \geq min\{\mu(x \to (y \to z)), \mu(x \to y)\}$.

Theorem 4.9: Let μ be a fuzzy set of $\mathscr{L}$ and $(F, (\frac{1-k}{2}, 1])$ be a soft set. Then $(F, (\frac{1-k}{2}, 1])$ is a F-soft lattice implication algebra over $\mathscr{L}$ if and only if μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy filter of $\mathscr{L}$.

Proof: Let $(F, (\frac{1-k}{2}, 1])$ is a F-soft lattice implication algebra over $\mathscr{L}$ for any $t \in (\frac{1-k}{2}, 1]$. If there exists $a \in L$ such that $\mu(a) > max\{\mu(I), \frac{1-k}{2}\}$, then $\mu(a) \geq t > max\{\mu(I), \frac{1-k}{2}\}$ for some $t \in (\frac{1-k}{2}, 1]$ and so $\mu(I) < t$, i.e. $I \notin F(t)$, contradiction. Hence $max\{\mu(I), \frac{1-k}{2}\} \geq \mu(x)$ for any $x \in L$. If there exist $a, b \in L$ such that $min\{\mu(a \to b), \mu(a) \geq t > max\{\mu(b), \frac{1-k}{2}\}$ for some $t \in (\frac{1-k}{2}, 1]$. Thus $(a \to b)_t \in \mu$ and $a_t \in \mu$, i.e. $a \to b \in F(t)$ and $a \in F(t)$, it follows that $b \in F(t)$, but $\mu(b) < t$, i.e. $b \notin F(t)$, contradiction. Therefore $max\{\mu(y), \frac{1-k}{2}\} \geq min\{\mu(x), \mu(x \to y)\}$ for any $x, y \in L$. That is μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy filter of $\mathscr{L}$.

Conversely, let μ be an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy filter of $\mathscr{L}$. For any $t \in (\frac{1-k}{2}, 1]$, we have $\mu(x) \leq max\{\mu(I), \frac{1-k}{2}\}$ for $x \in F(t)$. Therefore $\mu(x) \geq t > \frac{1-k}{2}$, $t \leq \mu(x) \leq \max\{\mu(I), \frac{1-k}{2}\} = \mu(I)$ and so $I_t \in \mu$, i.e. $I \in F(t)$. Let $x, t \in L$ be such that $x \to y \in F(t)$ and $x \in F(t)$, then $(x \to y)_t \in \mu$ and $x_t \in \mu$, i.e. $\mu(x \to y) \geq t$ and $\mu(x) \geq t$. It follows from (2) that $\frac{1-k}{2} < t \leq min\{\mu(x \to y), \mu(x)\} \leq max\{\mu(y), \frac{1-k}{2}\}$, we have $max\{\mu(y), \frac{1-k}{2}\} = \mu(y)$, which implies $t \leq \mu(y)$, i.e. $y \in F(t)$. Therefore $F(t)$ is a filter for any $t \in (\frac{1-k}{2}, 1]$ and so $(F, (\frac{1-k}{2}, 1])$ is a F-soft lattice implication algebra over $\mathscr{L}$. ■

Corollary 4.6: Let μ be a fuzzy set of $\mathscr{L}$ and $(F, (0.5, 1])$ be a soft set. Then $(F, (0.5, 1])$ is a F-soft implication algebra over $\mathscr{L}$ is and only if μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filter of $\mathscr{L}$.

Theorem 4.10: Let μ be a fuzzy set of $\mathscr{L}$ and $(F, (\frac{1-k}{2}, 1])$ be a soft set. Then $(F, (\frac{1-k}{2}, 1])$ is a IF-soft lattice implication algebra over $\mathscr{L}$ if and only if μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy implicative filter of $\mathscr{L}$.

Proof: It is similarly proved as Theorem 4.9. ■

Corollary 4.7: Let μ be a fuzzy set of $\mathscr{L}$ and $(F, (0.5, 1])$ be a soft set. Then $(F, (0.5, 1])$ is a IF-soft implication algebra over $\mathscr{L}$ is and only if μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy implicative filter of $\mathscr{L}$.

V. CONCLUSION

Soft sets are related to fuzzy sets and rough sets. It applied to some algebraic structures. In this paper, we introduce the (implicative) filteristic soft lattice implication algebras which related with (implicative) filter(for short, $(IF$-$)$ F- soft lattice implication algebras). Basic properties of $(IF$-$)F$-soft lattice implication algebras are investigated. We introduce the notion of $(\in, \in \vee q_k)((\overline{\in}, \overline{\in} \vee \overline{q_k}))$-fuzzy (implicative) filters, which are generalizations of fuzzy (implicative) filter. we provide characterizations for a soft set to be a $(IF$-$)F$-soft lattice implication algebra. Analogously, this idea can be applied in other structures (such as positive implicative filters, ultra-filter, fantastic filter, and so on), analogously. It will be of great use to provide theoretical foundation to design intelligent information processing systems.

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