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Relative Mapping Errors of Linear Time Invariant Systems Caused By Particle Swarm Optimized Reduced Order Model

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***Abstract*—**The authors present an optimization algorithm for order reduction and its application for the determination of the relative mapping errors of linear time invariant dynamic systems by the simplified models. These relative mapping errors are expressed by means of the relative integral square error criterion, which are determined for both unit step and impulse inputs. The reduction algorithm is based on minimization of the integral square error by particle swarm optimization technique pertaining to a unit step input. The algorithm is simple and computer oriented. It is shown that the algorithm has several advantages, e.g. the reduced order models retain the steady-state value and stability of the original system. Two numerical examples are solved to illustrate the superiority of the algorithm over some existing methods.

***Keywords*—**Order reduction, Particle swarm optimization, Relative mapping error, Stability.

I. Introduction

IN the analysis and design of complex systems, it is often necessary to simplify a high order system. The use of a reduced order model makes it easier to implement analysis, simulations and control system designs. Here we consider the system in the form of a transfer function. To establish a transfer function of lower order, numerous methods have been proposed [1-6]. In spite of the significant number of methods available, no approach always gives the best results for all systems. Almost all methods, however, aim at accurate reduced models for a low computational cost.

The concept of determining the mapping error of the linear time invariant dynamic system by a simplified model, as one of the application of the reduced order modeling was suggested by Layer [7-8], in which the mapping was expressed by means of the integral square error (ISE) criterion. A special calculation algorithm to compute the maximum value of this criterion was also discussed in [8].

Further, numerous methods of order reduction are also available in the literature [9-16], which are based on the minimization of the ISE criterion. However, a common feature in these methods [9-15] is that the values of the denominator coefficients of the low-order system (LOS) are chosen arbitrarily by some stability preserving methods such as dominant pole, Routh approximation methods, etc. and then the numerator coefficients of the LOS are determined by minimization of the ISE. In [16], Howitt and Luss suggested a technique, in which both the numerator and denominator coefficients are considered to be free parameters and are chosen to minimize the ISE in impulse or step responses.

Recently, particle swarm optimization (PSO) technique appeared as a promising algorithm for handling the optimization problems. PSO is a population based stochastic optimization technique, inspired by social behavior of bird flocking or fish schooling [17]. PSO shares many similarities with Genetic Algorithm (GA); like initialization of population of random solutions and search for the optimal by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. One of the most promising advantage of PSO over GA is its algorithmic simplicity, as it uses a few parameters and easy to implement. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles.

In the present work, the authors present an algorithm for order reduction based on minimization of the ISE by PSO pertaining to a unit step input. The relative mapping errors between the original and LOS are also determined and plotted with respect to time for both unit step and impulse inputs. The comparison between the proposed and other well known existing order reduction techniques is also shown in the present work. In the following sections, the algorithm is described in detail and the same is used in solving two numerical examples.

II. Reduction Algorithm

Let the transfer function of the original high-order system (HOS) of order 'n' be :

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{a_o + a_1 s + a_2 s^2 + ... + a_{n-1} s^{n-1}}{b_o + b_1 s + b_2 s^2 + ... + b_{n-1} s^{n-1} + s^n} \quad (1)$$

and let the same of low-order system (LOS) of order 'r' to be

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synthesized is :

$$G_r(s) = \frac{c_0 + c_1 s + ... + c_{r-1} s^{r-1}}{d_o + d_1 s + d_2 s^2 + ... + d_{r-1} s^{r-1} + s^r} \quad , r < n \qquad (2)$$

The deviation of the LOS response from the original system response is given by the error index 'E', known as the integral square error (ISE), which is given by [4] :

$$E = \int_0^\infty [y(t) - y_r(t)]^2 \, dt \qquad (3)$$

where, $y(t)$ and $y_r(t)$ are the unit step responses of original and reduced order systems.

The PSO method is a population based search algorithm where each individual is referred to as particle and represents a candidate solution. Each particle flies through the search space with an adaptable velocity that is dynamically modified according to its own flying experience and also the flying experience of the other particles. In PSO, each particle strives to improve itself by imitating traits from their successful peers. Further, each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it. The position corresponding to the best fitness is known as pbest and the overall best out of all the particles in the population is called gbest [18].

In a d-dimensional search space, the best particle updates its velocity and positions with following equations :

$$v_{id}^{n+1} = w v_{id}^n + c_1 r_1^n (p_{id}^n - x_{id}^n) + c_2 r_2^n (p_{gd}^n - x_{id}^n) \qquad (4)$$

$$x_{id}^{n+1} = x_{id}^n + v_{id}^{n+1} \qquad (5)$$

where,
w = inertia weight.
c_1, c_2 = cognitive and social acceleration, respectively.
r_1, r_2 = random numbers uniformly distributed in the range (0, 1).

The i-th particle in the swarm is represented by a d-dimensional vector $X_i = (x_{i1}, x_{i2}, \ldots\ldots, x_{id})$ and its velocity is denoted by another d-dimensional vector $V_i = (v_{i1}, v_{i2}, \ldots\ldots, v_{id})$. The best previously visited position of the i-th particle is represented by $P_i = (p_{i1}, p_{i2}, \ldots\ldots, p_{id})$.

In PSO, each particle moves in the search space with a velocity according to its own previous best solution and its group's previous best solution. The velocity update in particle swarm consists of three parts; namely momentum, cognitive and social parts. The balance among these parts determines the performance of a PSO algorithm [19]. The parameters c_1 & c_2 determine the relative pull of pbest and gbest and the parameters r_1 & r_2 help in stochastically varying these pulls. In the above equations (4) and (5), superscripts denote the iteration number. Fig. 1 shows the position updates of a particle for a two-dimensional parameter space.

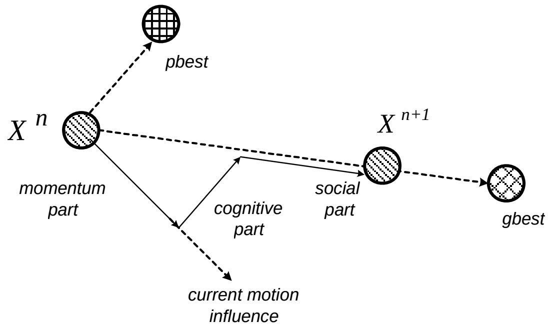


Fig. 1 Position updates in PSO for a two dimensional parameter space.

In the present study, PSO is employed to minimize the objective function 'E' as given in (3), and the parameters to be determined are the coefficients of the numerator and denominator polynomials of the LOS as given in (2), subject to the following conditions :

(i) To have a stable reduced order model, it follows from the Routh test that :

$$d_i > 0 \ ; \ i = 0, 1, 2, ..., (r-1) \qquad (6)$$

(ii) To eliminate any steady state error in the approximation, the condition is :

$$d_0 = \frac{b_0}{a_0} c_0 \qquad (7)$$

In Table I, the specified parameters for the PSO algorithm used in the present study are given. The computational flow chart of the proposed algorithm is shown in Fig. 2.

TABLE I
PARAMETERS USED FOR PSO ALGORITHM

Parameters	Value
Swarm Size	20
Max. Generations	100
c_1, c_2	2.0, 2.0
w_{start}, w_{end}	0.9, 0.4

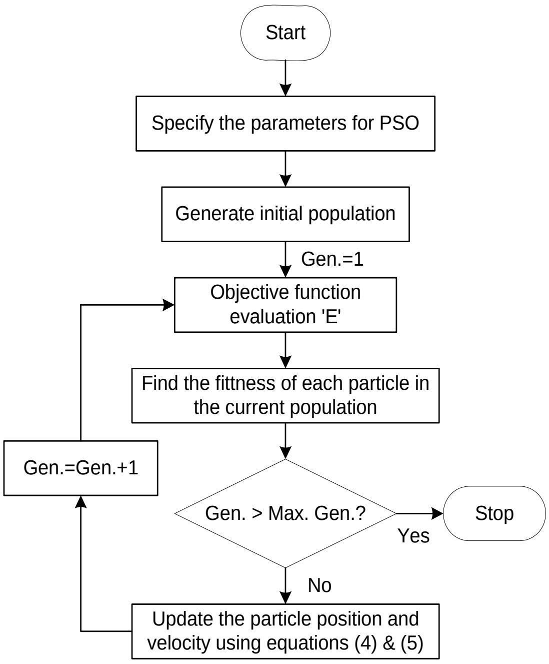


Fig. 2 Flowchart of PSOAlgorithm.

III. RELATIVE MAPPING ERRORS

The relative mapping errors of the original system relative to its LOS are expressed by means of the relative integral square error criterion, which are given by [20] :

$$I = \int_0^\infty [\ g(t) - \tilde{g}(t)]^2 \ dt \Big/ \int_0^\infty g^2(t) \ dt \tag{8}$$

$$J = \int_0^\infty [\ r(t) - \tilde{r}(t)]^2 dt \Big/ \int_0^\infty [r(t) - r(\infty)]^2 dt \tag{9}$$

where, $g(t)$ and $r(t)$ are the impulse and step responses of original system, respectively, and $\tilde{g}(t)$ and $\tilde{r}(t)$ are that of their approximants.

In this paper, both the relative mapping errors 'I' and 'J' are calculated and plotted with respect to time for the proposed reduction algorithm. These relative mapping errors are also compared in the tabular form for the proposed reduction algorithm and the other well-known existing order reduction techniques.

IV. NUMERICAL EXAMPLES

Two numerical examples are chosen from the literature for the comparison of the low-order system (LOS) with the original high-order system (HOS).

Example-1. Consider a sixth-order system taken from Layer [8] :

$$G_6(s) = \frac{6s^4 + 50s^3 + 196s^2 + 418s + 434}{s^6 + 12s^5 + 71s^4 + 256s^3 + 575s^2 + 804s + 585} \tag{10}$$

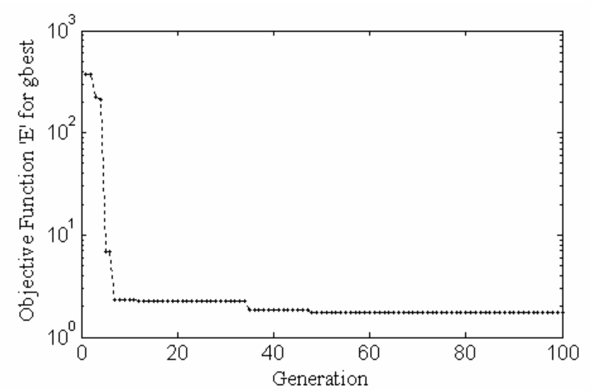
By using the proposed algorithm, the following reduced second-order model is obtained :

$$G_2(s) = \frac{5.27473}{s^2 + 3.0508s + 7.1088} \tag{11}$$

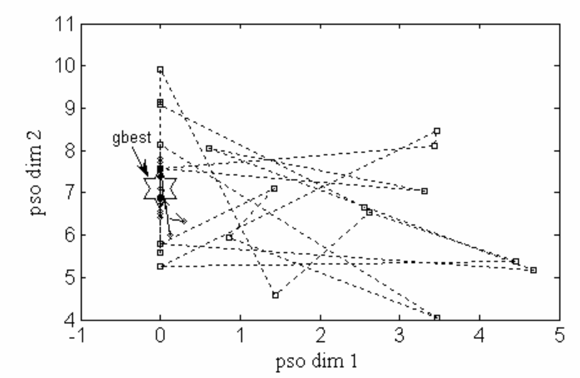
A comparison of the proposed algorithm with Layer [8] for a second-order reduced model is given in Table II. Fig. 3(a)–(f) presents diagrams of convergence of the objective function 'E' for gbest, movement of the particles in the PSO algorithm, step and impulse responses of $G_6(s)$ and $G_2(s)$, and characteristic of the relative mapping errors 'I' and 'J', respectively.

TABLE II
COMPARISON OF REDUCED ORDER MODELS

Method of order reduction	**Reduced Models**	**I**	**J**
Proposed Algorithm	$\frac{5.27473}{s^2 + 3.0508s + 7.1088}$	4.17431 x 10^{-3}	1.73755 x 10^{-3}
Layer [8]	$\frac{6}{s^2 + 3.66s + 7.78}$	3.36294 x 10^{-3}	1.37127 x 10^{-2}



(a)



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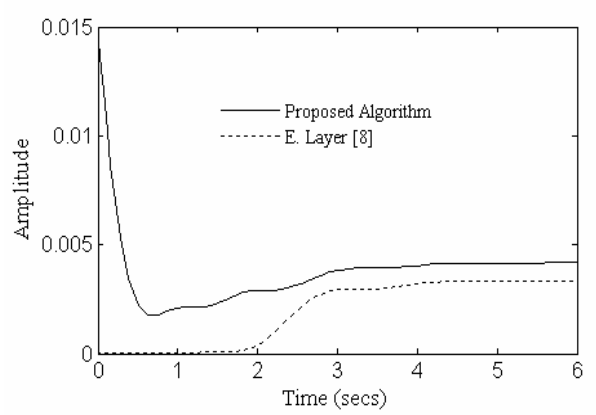
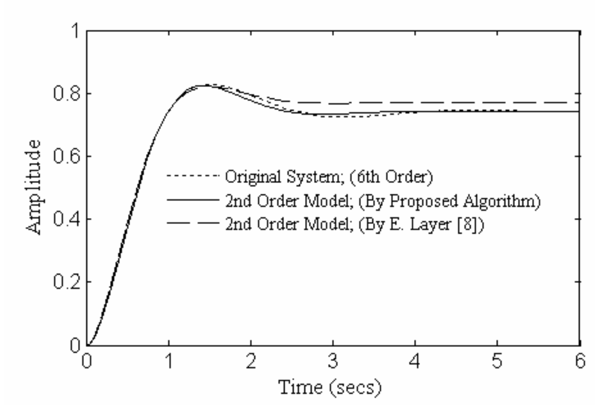


(e)



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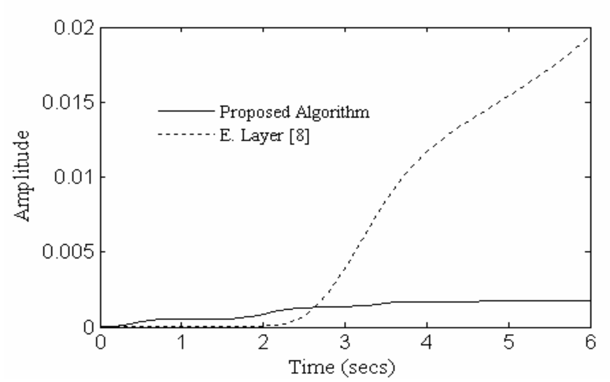
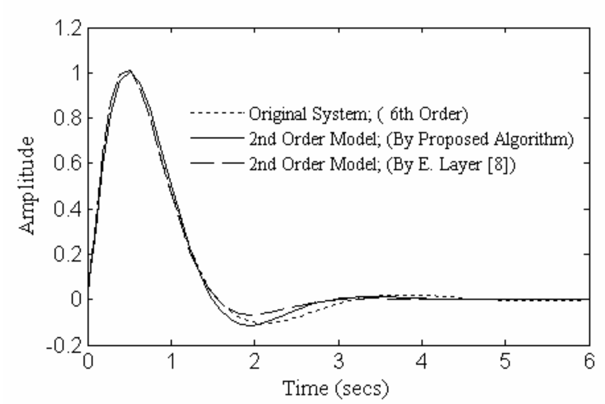


(f)



(d)

Fig. 3 (a) Convergence of objective function 'E' for gbest. (b) Movement of the particles in PSO algorithm. (c) Step response. (d) Impulse response. (e) Characteristic of relative mapping error, 'I' and (f) Characteristic of relative mapping error, 'J'.

Example-2. Consider a eighth-order system [21] described by the transfer function :

$$G_8(s) = \frac{a(s)}{b(s)} \tag{12}$$

where,

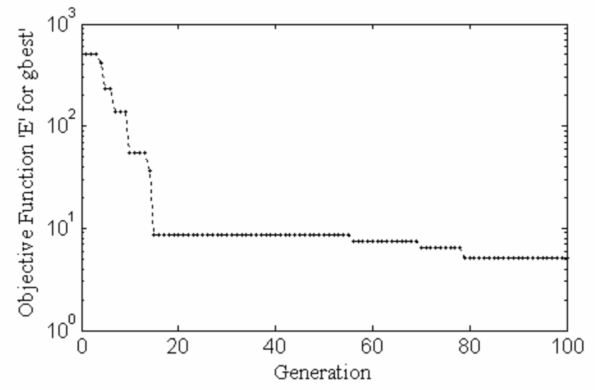
$$a(s) = 18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320$$

$$b(s) = s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320$$

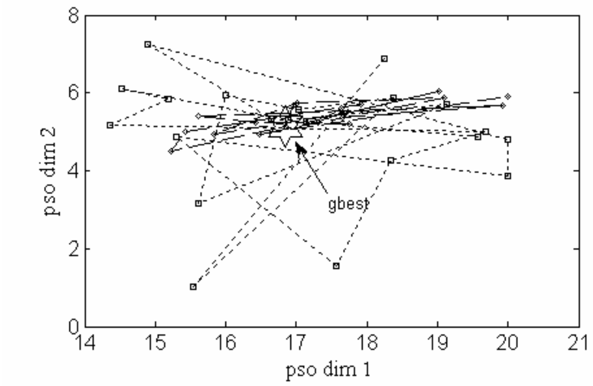
By using the proposed algorithm, the following reduced second-order model is obtained :

$$G_2(s) = \frac{16.8517\,s + 5.1379}{s^2 + 6.8976\,s + 5.1379} \tag{13}$$

A comparison of the proposed algorithm with the other well known existing order reduction techniques for a second-order reduced model is given in Table III. Figure 4 (a)–(f) presents diagrams of convergence of the objective function 'E' for gbest, movement of the particles in the PSO algorithm, step and impulse responses of $G_8(s)$ and $G_2(s)$, characteristics of the relative mapping errors 'I' and 'J', respectively.



(a)



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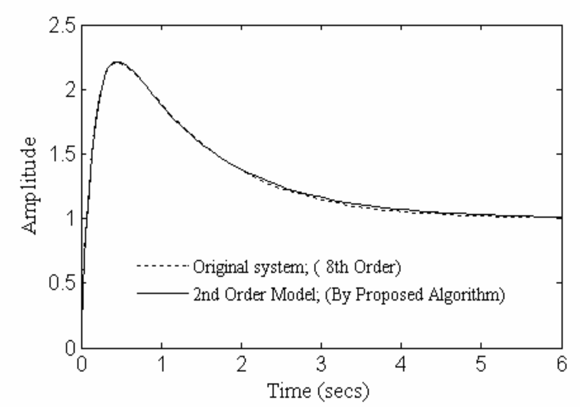

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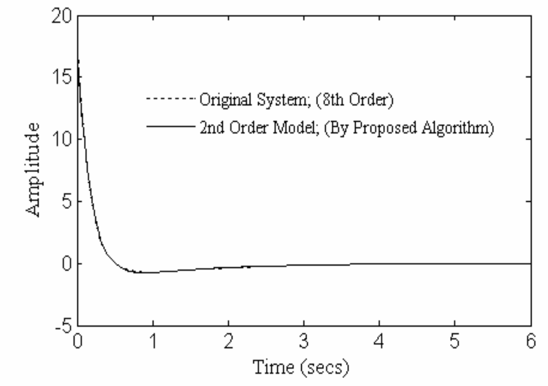

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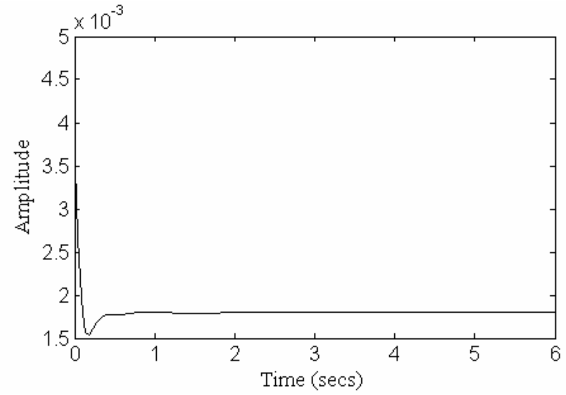

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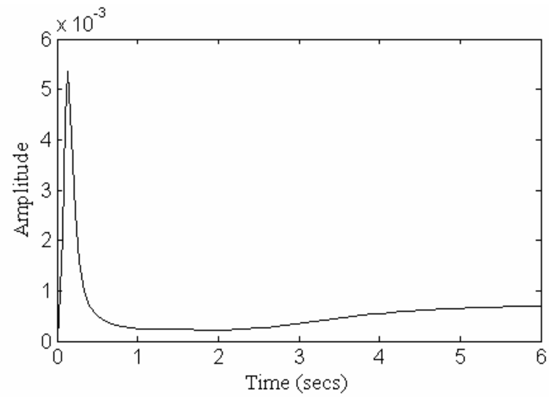

(f)

Fig. 4 (a) Convergence of objective function 'E' for gbest. (b) Movement of the particles in PSO algorithm. (c) Step response. (d) Impulse response. (e) Characteristic of relative mapping error, 'I' and (f) Characteristic of relative mapping error, 'J'.

TABLE III
COMPARISON OF REDUCED ORDER MODELS

Method of order reduction	Reduced Models	I	J
Proposed Algorithm	$\frac{16.8517s + 5.1379}{s^2 + 6.8976s + 5.1379}$	1.80078×10^{-3}	6.91635×10^{-4}
Mukherjee *et al.* [5]	$\frac{11.3909s + 4.4357}{s^2 + 4.2122s + 4.4357}$	8.99334×10^{-2}	3.88109×10^{-2}
Mukherjee and Mishra [12]	$\frac{7.0903s + 1.9907}{s^2 + 3s + 2}$	2.86389×10^{-1}	1.83434×10^{-1}
Mittal *et al.* [15]	$\frac{7.0908s + 1.9906}{s^2 + 3s + 2}$	2.86362×10^{-1}	1.83413×10^{-1}
Shamash [21]	$\frac{6.7786s + 2}{s^2 + 3s + 2}$	3.02978×10^{-1}	1.90469×10^{-1}
Hutton and Friedland [22]	$\frac{1.98955s + 0.43184}{s^2 + 1.17368s + 0.43184}$	7.59574×10^{-1}	1.307654
Krishna-murthy and Seshadri [23]	$\frac{155658.6152s + 40320}{65520s^2 + 75600s + 40320}$	7.24657×10^{-1}	1.127673
Pal [24]	$\frac{151776.576s + 40320}{65520s^2 + 75600s + 40320}$	7.29677×10^{-1}	1.126099
Chen *et al.* [25]	$\frac{0.72046s + 0.36669}{s^2 + 0.02768s + 0.36669}$	1.031795	4.918133
Gutman *et al.* [26]	$\frac{4[133747200s + 203212800]}{85049280s^2 + 552303360s + 812851200}$	3.64418×10^{-1}	9.38578×10^{-1}
Lucas [27]	$\frac{6.7786s + 2}{s^2 + 3s + 2}$	3.02978×10^{-1}	1.90469×10^{-1}
Prasad and Pal [28]	$\frac{17.98561s + 500}{s^2 + 13.24571s + 500}$	7.88491×10^{-1}	9.94796×10^{-1}
Safonov *et al.* [29]	$\frac{16.96s + 4.729}{s^2 + 7.028s + 5.011}$	1.36169×10^{-3}	4.01631×10^{-3}

V. CONCLUSIONS

An optimization algorithm for order reduction and its application for determining the relative mapping errors of linear time invariant dynamic systems, has been presented. The reduction algorithm is based on minimization of the integral square error by particle swarm optimization technique pertaining to a unit step input. The algorithm has been implemented in Matlab 7.0.1 on a Pentium-IV processor. The matching of the unit step and impulse responses is assured reasonably well in the algorithm. The algorithm is simple, rugged and computer oriented.

The relative step and impulse mapping errors between the original and low order systems are also determined and plotted with respect to time. A comparison of these mapping errors for the proposed reduction algorithm and the other well known existing order reduction techniques is also given, as shown in Tables II and III, from which it is clear that the proposed reduction algorithm compares well with the other techniques of model order reduction.

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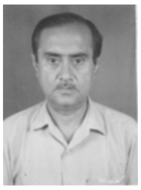

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