

# Robot Cell Planning

Allan Tubaileh, Ibrahim Hammad, and Loay Al Kafafi

**Abstract**—A new approach to determine the machine layout in flexible manufacturing cell, and to find the feasible robot configuration of the robot to achieve minimum cycle time is presented in this paper. The location of the input/output location and the optimal robot configuration is obtained for all sequences of work tasks of the robot within a specified period of time. A more realistic approach has been presented to model the problem using the robot joint space. The problem is formulated as a nonlinear optimization problem and solved using Sequential Quadratic Programming algorithm.

**Keywords**—Robotics, Layout.

## I. INTRODUCTION

**F**LEXIBLE Manufacturing Cells (FMCs) have been widely implemented in modern factories. For an efficient utilization of the material handling system used to serve the machines, the layout of the FMC must take into account the performance characteristics of the material handling system.

In developing the layout of FMC served by an industrial robot, the location of L/UL points is determined by taking into consideration the reachability and mobility criteria of the robot. Using these considerations, machines are located within the feasible and achievable region of the robot.

The optimal cell layout is obtained by minimizing the cycle time of the robot joints required to perform a sequence of travel. Minimizing the cycle time of the robot will enhance the production rate of the manufacturing system and increase the life of the robot. Many researchers were concerned in optimizing the travel time of the robot joints between a group of machines work sites, and in determining the optimum sequence of the robot between different worksites. Fenton et al [2], presented a method to obtain the optimal robot configurations for minimum travel time and minimum joint displacement between two positions. Dissanayake and Gal [1], propose a method to obtain the optimum sequence of travel between work sites, robot configuration and robot base location such that the total time of travel is optimized. Mata and Tubaileh [3], presented two algorithms to find the layout of machines and the feasible robot configurations corresponding to machine work sites, such that the travel time and robot joint displacement are optimized. In their study, robot operational characteristics and machine dimension and orientation are considered in developing the final cell layout.

Manuscript received Feb. 28, 2007.

A. Tubaileh (corresponding author, email:tubaileh@birzeit.edu).

I. Hammad (e-mail:ihammad@birzeit.edu).

L. Al Kafafi is with Mechanical Engineering Department, Birzeit Univ., P. O. Box 14, Birzeit, Palestine (phone: +972-2-2982115; fax: +972-2-2982984; e-mail: lkafafi@birzeit.edu).

All previously mentioned works concerned in robot workstation planning, assume constant joint velocity to determine the travel time between two work sites. This approximation is reasonable to some extent, but is not accurate, since it is well known in robotic control, that the robot starts and ends its trajectory with zero joint velocity.

Unlike other previous works which assumed that the joint velocities are constant, this paper provide a more practical technique used in real industrial robots to control the velocity of the joints at the start and end of robot trajectory, and controlling the maximum permitted velocity and acceleration of each joint. In order to achieve this, the robot joint displacement is modeled as function of time. This function is approximated to a third order polynomial. By determining the coefficients of these polynomials, the robot motion can be controlled in an implicit manner while developing the layout of the cell.

## II. CONSIDERATIONS OF ROBOT CELL PLANNING

The layout of the robotic manufacturing cells depends in the following factors:

### A. Robotic Operational Characteristics

The robot workspace and limits of joints movements must be considered in a early stages of machine layout.

### B. Material Flow between Machines

The material flow between machines affects the sequence of travel of the robot between all machines work sites. The number of trips that the robot must perform between two pairs of machines during a specified period of time is provided by a flow matrix.

### C. Cycle Time

It is required to find the optimum layout of machines and the feasible robot configuration, such that the total cycle time of the robot between all work sites is minimized.

This approximation is not normally applicable in real operation conditions, since a constant end effector velocity, which is a normal operation condition, does not implies a constant joint velocities. In addition, these formulations do not allow the user to control the velocity and acceleration of the joints, such as to start and end robot motion with zero joint velocity and to limit the velocity and acceleration of the robot to a maximum value. In order to satisfy this condition, this paper models the joint displacement between a pair of machines work sites J and k as a third order polynomial of time

$$q_i^{jk} = a_i^{jk} + b_i^{jk}(t_i^{jk}) + c_i^{jk}(t_i^{jk})^2 + d_i^{jk}(t_i^{jk})^3 \quad (1)$$

Where  $q_i^{jk}$  is the displacement function of the  $i$ th joint between positions  $j$  and  $k$ ,  $t_i^{jk}$  is the time required by joint  $i$  to travel between positions  $j$  and  $k$ , and  $a_i^{jk}$ ,  $b_i^{jk}$ ,  $c_i^{jk}$  and  $d_i^{jk}$  are the coefficient of the polynomial of the  $i$ th joint.

The polynomial must satisfy the following conditions:

$$t = 0 \quad q_i^{jk}(0) = q_i^j \quad \dot{q}_i^{jk}(0) = 0 \quad (2)$$

$$t = t_i^{jk} \quad q_i^{jk}(t_i^{jk}) = q_i^k \quad \dot{q}_i^{jk}(t_i^{jk}) = 0 \quad (3)$$

$$q_i^{jk}(t_i^{jk}) = q_i^{kh}(0) \quad (4)$$

Where  $q_i^j$ ,  $q_i^k$  and  $q_i^h$  are the joint coordinates corresponding to the worksites  $j$ ,  $k$  and  $h$  respectively. Applying this procedure to all joints, the robot configuration at each work site will be determined.

#### D. Dimension and orientations of machines

### III. PROBLEM FORMULATION

In order to simplify the problem formulation, an approximated method is used to describe the objects found in the FMC. Machines of rectangular or irregular shapes are modeled by means of evenly separated circles of identical radius. The number and radius of circles are selected such that the circles will envelop all corners of the machine. The orientation of the machine is represented by the angle between the  $x$ -axis and the line that passes through the centre of the machine and the L/UL point, see Fig. 2.

Overlapping between a pair of machines is prevented if the distance between the circles that belong to different machines is equal or exceeds the summation of radii of both circles. While, collisions between machines and robot arm is avoided by limiting the distance,  $D_{ik}$ , between the  $i$ th link and the centre of the  $k$ th circle which represent the machine, see Fig. 1.

The location of the robot end at the  $j$ th machine work-sites is represented in the Cartesian space by the vector  $\vec{x}^j = \{x_1^j, x_2^j, \dots, x_m^j\}$ . Although, in order to address the constraints proposed to avoid machine overlapping as well as the constraints implemented to avoid collision between robot arms and machines, it would be necessary to find the coordinates of the geometric centers of circles used to represent a machine.

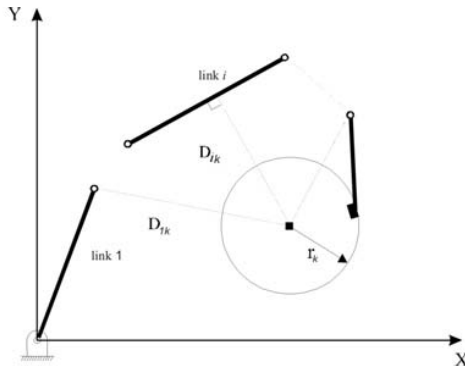


Fig. 1 Distance calculation from the centre of the machine to each robot link

The Cartesian coordinates of the centre of each circle  $(x_k^j, y_k^j)$  shown in Fig. 2, where  $k=1, \dots, \psi(j)$  is the number of circles that model machine  $j$ , can be stated through a simple trigonometric relation as function of input/output point, machine orientation  $\alpha^j$  and machine radius  $r^j$ .

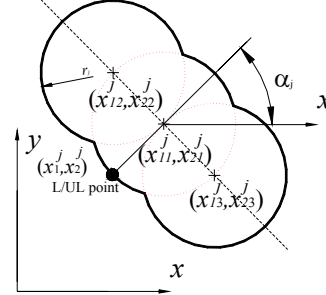


Fig. 2 Calculation of the coordinates of circle centers

Consider a robot of  $n$  degree-of-freedom with its base fixed at the origin of the Cartesian reference frame system. The robot will visit a set of  $N$  work sites located at the counter of each machine, the location of which is described by L/UL points, denoted by  $\vec{x}^j \in R^m$ ,  $j=1, \dots, N$ , and machines in the floor of the cell  $\alpha^j$ , where  $m$  is the Cartesian workspace of the robot.

Therefore, the machine layout in robot cells is addressed as follows:

$$\text{Min} \sum_{k=1}^N \sum_{j=1}^N f_{jk}(t_i^{jk}) \quad , \quad i=1, \dots, n \quad (5)$$

where  $t_i^{jk}$  is the travel time done by  $i$ th joint between the  $j$ th and  $k$ th robot configuration.

Subjected to:

$$\vec{x}^j = FK(\vec{q}^j, \vec{b}) \quad (6)$$

$$q_i^{jk} = a_i^{jk} + b_i^{jk}(t_i^{jk}) + c_i^{jk}(t_i^{jk})^2 + d_i^{jk}(t_i^{jk})^3 \quad (7)$$

$$q_i^{jk}(0) = q_i^j \quad q_i^{jk}(t_i^{jk}) = q_i^k \quad (8)$$

$$\dot{q}_i^{jk}(0) = 0 \quad \dot{q}_i^{jk}(t_i^{jk}) = 0 \quad (9)$$

$$q_i^{jk}(t_i^{jk}) = q_i^{kh}(0) \quad j=1, \dots, N, \quad k=1, \dots, N, \quad h=1, \dots, N \quad (10)$$

$$\dot{q}_i^j \leq \dot{q}_{\max} \quad j=1, \dots, N \quad (11)$$

$$q_{\min}^j \leq q_i^j \leq q_{\max}^j \quad j=1, \dots, N \quad (12)$$

$$\sum_{k=1}^{\Psi(j)} \sum_{p=1}^{\Psi(i)} (x_{lk}^j - x_{lp}^i)^2 + \dots + (x_{mk}^j - x_{mp}^i)^2 \geq (r^j + r^i + d^{jl})^2 \quad (13)$$

$$\sum_{i=1}^n \sum_{k=1}^{\Psi(j)} [D_{ik}(\vec{q}^j)]^2 \geq (r^j)^2 \quad j=1, \dots, N \quad (14)$$

where (5) is the objective function which intend to optimize the cycle time of all robot joints for all possible sequences of travel represented by the flow matrix  $f_{jk}$ . Constraint (6) states the direct kinematics relationship between the position of the robot end and its joint coordinates, which enable one to model

the problem in the robot joint space. Each robot joint between to pair of machines is represented as a third order polynomial of time as declared by Constraints (7), such that there is a different polynomial for each displacement of the  $i$ th joint between a pair of worksites. The coefficient of the polynomial is a design variable which is to be found by the optimization problem. Robot configuration at each worksite is defined by constraints (8). Constraints (10) insures that at each machine worksite the robot starts its motion with the same configuration it ends its travel from the previous machine, where  $h$  is the machine site visited after machine  $k$ . Constraints (9), insures that the robot starts and ends its travel with a zero joint velocities, while constraint (11) limits the velocity of each joint to a predetermined value. The range of operation of joint movement is limited by constraint (12). Constraint (13) avoids machine overlapping, where  $d^{jl}$  is the minimum clearance distance between machine  $j$  and  $l$ . Finally, constraint (14) insures a feasible robot configuration by avoiding collisions between robot arms and machines.

#### IV. NUMERICAL EXAMPLES

The layout problem is formulated as a nonlinear optimization algorithm as presented by equations (5-14). The sequential Quadratic Programming Algorithm (SQP) described by [5] was proven to be efficient for this type of problems. A FORTRAN subroutine available in the NAG Library is used to solve this problem.

In order to illustrate the efficiency of the proposed algorithm and to demonstrate its practical implementation, the layout problem was solved for a planar two degree-of-freedom robot. The approach can be correctly applied for a workstation served by SCARA robot, since the location of the machine worksites in the cell floor is determined by the major two revolute joints. The result of varies machine layout served by a planar robot is shown in Table I. All problems presented in the table started by unfeasible initial solution, since it is very difficult to provide the program with a time and polynomial variables that produce a feasible robot a configurations and a machine layout. Fig. 3 shows the layout of 5 machines served by a planar robot solved by the proposed algorithm. Each figure represents the final layout developed for a different sequence of travel indicated by the numbers inside the figures.

TABLE I  
RESULT OF TWO DIMENSIONAL CELL LAYOUT

Problem	Travel Sequence	FOV Total Cycle Time(sec.)	CPU(sec.)
1	1-2-3-4-5-1	5.494	4.5
2	1-5-3-2-4-1	3.997	4.2
3	1-3-4-2-5-1	3.722	6.3
4	5-2-1-4-3-5	4.523	5.3
5	4-2-1-3-5-4	3.547	3.8

It is remarkable that there is only one robot configuration at each machine worksite, that is, the robot configurations produced by two different joint functions at a worksite are identical.

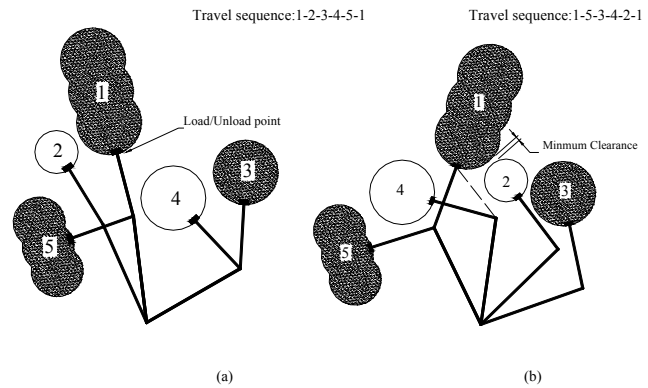


Fig. 3 Machine layout in a robotic cell

Fig. 4 demonstrates the effectiveness of the proposed approach to control the joint motion for the example represented in Fig. 3a. It can be observed that the robot has one configuration at each machine worksite, this can be shown in Fig. 4a, which shows that each joint displacement function starts with the same joint coordinates by which the previous function ends its travel at a given work site. Furthermore, Fig. 4b shows the robot has a zero velocity at the start and end of its travel between a pair of machines. The same procedure can be applied to control the acceleration of the joints and to limit the max permitted velocity of the robot joints.

#### V. CONCLUSION

This paper presents a method to find the layout of machines and the feasible robot configurations at each machine worksites, such that the total cycle time is optimized. The problem is addressed using the joint coordinates system, and take into account the performance characteristics of the employed robot.

The major advantage of this new method can be summarized in addressing the joint space as a nonlinear polynomial of the time, unlike other previous researches where the joint velocity is assumed to be constant.

#### REFERENCES

- [1] Dissanayake, M. W. M. G., and Gal, J. A., 1994, Workstation planning for redundant manipulators. *International Journal of Production Research*, 32(5), 1105-1118.
- [2] Fenton, R.G., Poon, D., Davies, S.P., Robotic Workcell cycle time Optimization Using Computer Graphics. *Annals of the CIRP*, 41, 1992, 509-512.
- [3] Mata, V., and Tubaileh, A., The Machine Layout problem of robot cells. *International Journals of Production Research*, 36,(5), 1273-1292.
- [4] Nelson, B., and Donath, M., 1990, Optimizing the location of assembly tasks in a manipulator's workspace. *Journal of Robotics Systems*, 7(6), 791-811.
- [5] Schittkowski, K., 1986, NLPQL: A FORTRAN subroutine solving constrained nonlinear programming problems. *Annals of operations Research*, 5, 485-500.

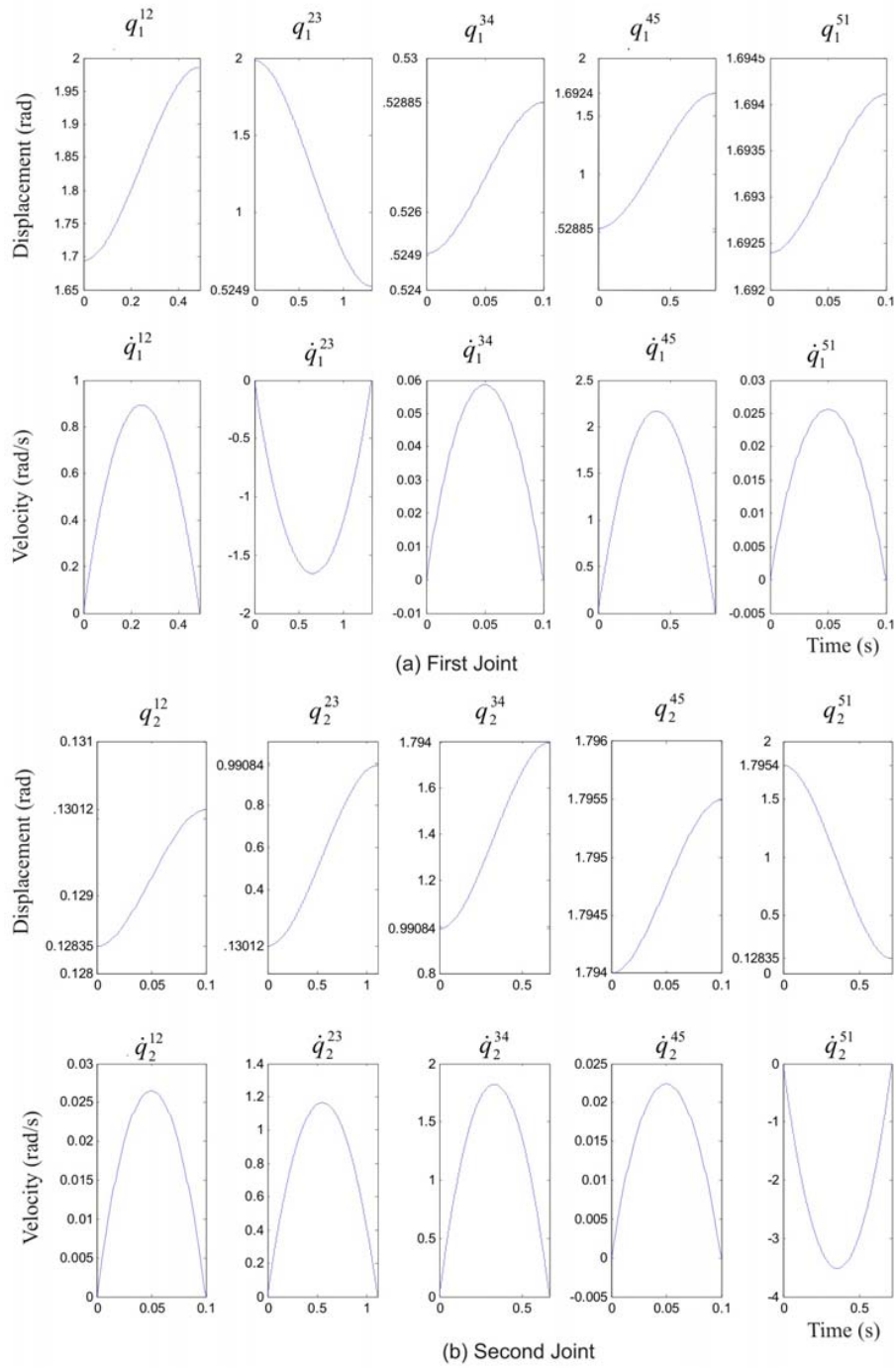


Fig. 4 Joint displacement and velocity of the cell layout shown in Fig. 3a