

# Kinematic and Dynamic Analysis of a Lower Limb Exoskeleton

Tawakal Hasnain Baluch, Adnan Masood, Javaid Iqbal, Umer Izhar, Umar Shahbaz Khan

**Abstract**—This paper will provide the kinematic and dynamic analysis of a lower limb exoskeleton. The forward and inverse kinematics of proposed exoskeleton is performed using Denevit and Hartenberg method. The torques required for the actuators will be calculated using Lagrangian formulation technique. This research can be used to design the control of the proposed exoskeleton.

**Keywords**—Dynamic Analysis, Exoskeleton, Kinematic Analysis, Lower Limb, Rehabilitation Robotics

## I. INTRODUCTION

THE concept of exoskeletons was started in 1960s and Hardiman was the first power assist system [1]. Main purpose of this project was to make soldiers carry heavy loads for long distances. Research of exoskeletons has a history of about four decades [2], [3]. Lower limb exoskeleton can be used for providing strength during walking, to carry heavy loads or aiding the disabled people for walking [4]. Lower limb exoskeletons can also help disabled people by decreasing contractures, pressure sores, spasticity, and osteoporosis and will also improve cardiopulmonary functions [5]. Design aspects of exoskeletons include degree of freedom (DOF), range of motion (ROM), Torque and velocity requirements for joints, Actuation type, Weight and Inertia and Kinematic considerations.

Comparative studies of available exoskeletons based upon the above factors were provided by Massimo and Aaron [6]. Yanjun Zhao [7] has done the mechanical modeling and simulation of lower limb exoskeleton. Kinematics analysis of lower limb exoskeleton with three joints was done by Hongbo WANG and Zengguang HOU [8].

According to the comparative study in [6] minimum degree of freedom for hip, knee and ankle is one (flexion/extension). In proposed exoskeleton two degrees of freedom for hip has been selected (Flexion/Extension and Internal/External rotation) so that the person wearing the exoskeleton can turn in both directions.

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Single degree of freedom has been chosen for both knee and ankle. Complete kinematics analysis with forward and inverse kinematics, along with dynamic analysis using Lagrangian technique to calculate the torques required for the actuators will be provided.

## II. STRUCTURE OF THE PROPOSED EXOSKELETON

Below is the free body diagram for one leg of the proposed exoskeleton (Figure 1). Exoskeleton will have total eight degrees of freedom with four for each leg. We have two hip joints, one knee joint and one ankle joint for each leg. All joints will be revolute joints.  $l_1$ ,  $l_2$  and  $l_3$  represent lengths of thigh, calf and feet respectively. Z axis represents the rotational axis for each joint.

Joint 1 in fig. 1 is for internal/external rotation of the proposed exoskeleton while joint 2 connects the link equivalent to human thigh with link 2. Similarly joint 2 connect the link 1 and 2 while joint 3 connects the link 2 and 3.

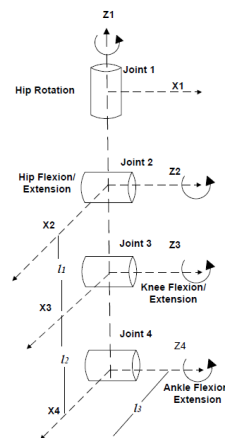


Fig. 1 Free Body Diagram

Kinematics and dynamics of only one leg is presented here. Same expression for joint angles and torques can be used to solve the second leg.

## III. KINEMATICS ANALYSIS

Link parameters of the proposed exoskeleton is found in Table I, where  $\alpha_{i-1}$ ,  $a_{i-1}$ ,  $d_i$  and  $\theta_i$  represents link twist, link length, link offset and joint angle respectively.

TABLE I  
LINK PARAMETERS AS PER DH NOTATION

No.	Joint	Movement	$\alpha_{i-1}$	$a_{i-1}$	$d_i$	$\theta_i$
1	Hip	Rotation	0	0	0	$\theta_1$
2	Hip	F/E	-90	0	0	$\theta_2$
3	Knee	F/E	0	$l_1$	0	$\theta_3$
4	Ankle	F/E	0	$l_2$	0	$\theta_4$

\*F/E Flexion/Extension

\*Ranges of  $\theta_i$  will be restricted as per human joints

Homogenous transformation of link parameters as per DH notation can be calculated as per following relation

$${}^{i-1}_i\mathbf{T} = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i s\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i c\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where  $s\theta = \sin\theta$ ,  $c\theta = \cos\theta$ ,  $s\alpha = \sin\alpha$  and  $c\alpha = \cos\alpha$

Now transformations from hip to foot as per equation (1) will be given as

$${}^0_1\mathbf{T} = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1_2\mathbf{T} = \begin{bmatrix} c2 & -s2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s2 & -c2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3\mathbf{T} = \begin{bmatrix} c3 & -s3 & 0 & l_1 \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3_4\mathbf{T} = \begin{bmatrix} c4 & -s4 & 0 & l_2 \\ s4 & c4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4_{\text{foot}}\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation matrix from hip to foot will be calculated as

$${}^0_{\text{foot}}\mathbf{T} = {}^0_1\mathbf{T} \cdot {}^1_2\mathbf{T} \cdot {}^2_3\mathbf{T} \cdot {}^3_4\mathbf{T} \cdot {}^4_{\text{foot}}\mathbf{T} \quad (2)$$

$${}^0_{\text{foot}}\mathbf{T} = \begin{bmatrix} c1c234 & -c1c234 & -s1 & c1c234l_3 + c1c23l_2 + c1c2l_1 \\ slc234 & -slc234 & -c1 & slc234l_3 + slc23l_2 + slc2l_1 \\ -c234 & c^*234 & 0 & -s234l_3 - s23l_2 - s2l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where

$$c234 = \cos(\theta_1 + \theta_2 + \theta_3)$$

$$s234 = \sin(\theta_1 + \theta_2 + \theta_3)$$

$$c23 = \cos(\theta_1 + \theta_2)$$

$$s23 = \sin(\theta_1 + \theta_2)$$

$$c^*234 = \cos(\theta_4 - \theta_2 - \theta_3)$$

#### A. Forward Kinematics

For kinematics simplicity we can write  ${}^0_{\text{foot}}\mathbf{T}$  as

$${}^4_{\text{foot}}\mathbf{T} = \begin{bmatrix} r11 & r12 & r13 & p1 \\ r21 & r22 & r23 & p2 \\ r31 & r32 & r33 & p3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The last column of the above matrix represents the position and orientation of the exoskeleton in x and y coordinates respectively.

$$\begin{aligned} p_1(x) &= c1c234l_3 + c1c23l_2 + c1c2l_1 \\ p_2(y) &= slc234l_3 + slc23l_2 + slc2l_1 \end{aligned} \quad (4)$$

First and second derivative of the above equations will give the velocity and acceleration of the proposed exoskeleton.

#### B. Inverse Kinematics

Now that all the variables of equation (3) are known, joint angles can be obtained by using a simple technique of multiplying each side of transformation equation (2) by an inverse which will separate out the variables and make the equation more solvable. To find  $\theta_1$  we will multiply  ${}^0_1\mathbf{T}^{-1}$  to L.H.S of the equation (2) which will give us

$${}^0_1\mathbf{T}^{-1} \cdot {}^0_{\text{foot}}\mathbf{T} = {}^1_2\mathbf{T} \cdot {}^2_3\mathbf{T} \cdot {}^3_4\mathbf{T} \cdot {}^4_{\text{foot}}\mathbf{T} \quad (5)$$

From equation (5) we can derive the equation

$$\begin{aligned} c1p_1 + s1p_1 &= ((c2c3 - s2s3)c4 + (-s2c3 - c2s3)s4)l_3 \\ &+ ((c2c3 - s2s3))l_2 + c2l_1 \end{aligned} \quad (6)$$

From the above equation we can derive  $\theta_1$  as

$$\theta_1 = A \tan 2\left(\frac{d_1}{\zeta_1}, \pm \sqrt{1 - \frac{d_1^2}{\zeta_1^2}}\right) - A \tan 2(p_1, p_2) \quad (7)$$

where

$$\begin{aligned} d_1 &= ((c2c3 - s2s3)c4 + (-s2c3 - c2s3)s4)l_3 \\ &+ ((c2c3 - s2s3))l_2 + c2l_1 \end{aligned}$$

$$\zeta_1 = \sqrt{p_1^2 + p_2^2} \quad (8)$$

Using the same technique we find  $\theta_2$  as

$$\theta_2 = A \tan 2(k_1, k_2) - A \tan 2(d_2, \pm \sqrt{1 - \frac{d_2^2}{\zeta_2^2}}) \quad (9)$$

Where

$$\begin{aligned} k_1 &= c_1 r_{11} + s_1 r_{21} \\ k_2 &= r_{31} \\ d_2 &= c_2 k_1 - s_2 k_2 \\ \zeta_2 &= \sqrt{k_1^2 + k_2^2} \end{aligned} \quad (10)$$

Similarly we can find  $\theta_4$  as

$$\theta_4 = A \tan 2(s_4, c_4) \quad (11)$$

Now we are left with  $\theta_3$  which we can find by taking elements (1,4) and (3,4) of equation (2). Adding them we get

$$s_3 a_1 + c_3 a_2 = d_3$$

Using above equation  $\theta_3$  is given as

$$\theta_3 = A \tan 2\left(\frac{d_3}{\zeta_3}, \pm \sqrt{1 - \frac{d_3^2}{\zeta_3^2}}\right) - A \tan 2(a_2, a_1) \quad (12)$$

where

$$\begin{aligned} a_1 &= -c_2 c_4 l_3 + s_2 s_4 l_3 - s_2 c_4 l_3 - c_2 s_4 l_3 - c_2 l_2 \\ a_2 &= (-s_2 c_4 l_3 - c_2 s_4 l_3 + c_2 c_4 l_3 - s_2 s_4 l_3 \\ &\quad - s_2 l_2 + c_2 l_2) - s_2 l_1 + c_2 l_1 \\ d_3 &= c_1 p_1 + s_1 p_2 + p_3 + s_2 l_1 - c_2 l_1 \\ \zeta_3 &= \sqrt{a_1^2 + a_2^2} \end{aligned} \quad (13)$$

#### IV. DYNAMIC ANALYSIS

For the sake of simplification an assumption has been made that torque required for two hip joints (Flexion/Extension and Rotation) is same. So we need to calculate only three expressions for torque. Lagrangian formulation technique is been used which is actually an energy based approach towards dynamics.

According to Lagrangian formulation technique, firstly we calculate kinetic and potential energy ( $k$  and  $u$  respectively) of the individual links using the relation

$$k_i = \frac{1}{2} m_i v_i^T v_i + \frac{1}{2} w_i^T C_i I_i w_i \quad (14)$$

$$u_i = -m_i g^T P_i + u_{ref} \quad (15)$$

where  $m$ ,  $v$ ,  $w$ ,  $I$  and  $P$  are mass, linear velocity of link's center, angular velocity of link's center, Inertia tensor, gravity vector and vector locating the center of mass of  $i$ th link respectively. Total kinetic and potential energy is the sum of energies of all links. Equation for the expression of torque using Lagrangian formulation is

$$\tau_i = \frac{d}{dt} \left[ \frac{\partial k}{\partial \dot{\theta}_i} \right] - \frac{\partial k}{\partial \theta_i} + \frac{\partial u}{\partial \theta_i} \quad (16)$$

From equation (14) and (15)

$$\begin{aligned} k &= \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 (\dot{\theta}_1 + \dot{\theta}_2)^2 \\ &\quad + \frac{1}{2} m_3 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_3 l_2^2 (2\dot{\theta}_1 + 2\dot{\theta}_2 + \dot{\theta}_3)^2 + m_2 l_1 l_2 \dot{\theta}_1 \\ &\quad (\dot{\theta}_1 + \dot{\theta}_2) \cos \theta_2 + m_3 l_1 l_3 \dot{\theta}_1 (2\dot{\theta}_1 + 2\dot{\theta}_2 + \dot{\theta}_3) \cos \theta_2 \\ &\quad + \frac{1}{2} I_{zz_1} \dot{\theta}_1^2 + \frac{1}{2} I_{zz_2} (\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2} I_{zz_3} (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)^2 \end{aligned} \quad (17)$$

$$\begin{aligned} u &= m_1 g l_1 \sin \theta_1 + m_1 g l_1 + m_2 g l_2 \sin(\theta_1 + \theta_2) + m_2 g l_1 \sin \theta_1 \\ &\quad + m_2 g (l_1 + l_2) + m_3 g l_3 \sin(\theta_1 + \theta_2 + \theta_3) + m_3 g (l_1 + l_2) \sin(\theta_1 + \theta_2) + m_3 g \\ &\quad (l_1 + l_2 + l_3) \end{aligned} \quad (18)$$

Using the above expressions of  $k$  and  $u$  we can find joint torques  $\tau_1, \tau_2$  and  $\tau_3$  as follows

$$\begin{aligned} \tau_1 &= (m_1 l_1 c_1 + m_2 l_1^2 + m_2 l_2 c_2^2 + m_3 l_1^2 + 4m_3 l_2 c_2^2 + I_{zz_1} \\ &\quad + I_{zz_2} + I_{zz_3}) \ddot{\theta}_1 + (m_2 l_2 c_2^2 + 4m_3 l_2 c_2^2 + I_{zz_2} + I_{zz_3}) \\ &\quad \ddot{\theta}_2 + (2m_3 l_2 c_2^2 + I_{zz_3}) \ddot{\theta}_3 + (m_2 l_1 l_2 c_2 \ddot{\theta}_2 + 2m_2 l_1 l_2 c_2 \ddot{\theta}_1 + \\ &\quad 4m_3 l_1 l_3 c_3 \ddot{\theta}_1 + 2m_3 l_1 l_3 c_3 \ddot{\theta}_2 + m_3 l_1 l_3 c_3 \ddot{\theta}_3) \cos \theta_2 - (m_2 l_1 l_2 c_2 \\ &\quad \dot{\theta}_1^2 + 2m_2 l_1 l_2 c_2 \dot{\theta}_1 \dot{\theta}_2 + 4m_3 l_1 l_3 c_3 \dot{\theta}_1 \dot{\theta}_2 + 2m_3 l_1 l_3 c_3 \dot{\theta}_2^2 \\ &\quad + m_3 l_1 l_3 c_3 \dot{\theta}_2 \dot{\theta}_3) \sin \theta_2 + m_1 g l_1 \cos \theta_1 + m_2 g l_2 \\ &\quad \cos(\theta_1 + \theta_2) + m_2 g l_1 \cos \theta_1 + m_3 g l_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ &\quad + m_3 g (l_1 + l_2) \cos(\theta_1 + \theta_2) \end{aligned} \quad (19)$$

$$\begin{aligned} \tau_2 = & (m_2 l_{c_2}^2 + 4m_3 l_{c_2}^2 + I_{zz_2} + I_{zz_3}) \ddot{\theta}_1 + (m_2 l_{c_2}^2 + 4m_3 l_{c_2}^2 \\ & + I_{zz_2} + I_{zz_3}) \ddot{\theta}_2 + (2m_3 l_{c_2}^2 + I_{zz_3}) \ddot{\theta}_3 + (m_2 l_1 l_{c_2} \ddot{\theta}_1 + \\ & 2m_3 l_1 l_{c_3} \ddot{\theta}_1) \cos \theta_2 + (m_2 l_1 l_{c_2} \dot{\theta}_1^2 + 2m_3 l_1 l_{c_3} \dot{\theta}_1^2 + m_3 l_1 \\ & l_3 \dot{\theta}_1 \dot{\theta}_3) \sin \theta_2 + m_2 g l_{c_2} \cos(\theta_1 + \theta_2) + m_3 g l_{c_3} \\ & \cos(\theta_1 + \theta_2 + \theta_3) + m_3 g (l_1 + l_2) \cos(\theta_1 + \theta_2) \end{aligned} \quad (20)$$

And

$$\begin{aligned} \tau_3 = & (2m_3 l_{c_2}^2 + I_{zz_3})(\ddot{\theta}_1 + \ddot{\theta}_2) + (m_3 l_{c_2}^2 + I_{zz_3}) \ddot{\theta}_3 \\ & + m_3 l_1 l_{c_3} \ddot{\theta}_3 \cos \theta_2 - m_3 l_1 l_{c_3} \dot{\theta}_2 \dot{\theta}_3 \sin \theta_2 + m_3 g l_{c_3} \\ & \cos(\theta_1 + \theta_2 + \theta_3) \end{aligned} \quad (21)$$

## V. RESULTS

To verify the above equations, kinetic and potential energies of the proposed exoskeleton have been plotted. To determine the mass, height, location of the center of the mass and rotational velocity of lower limb links and joint following data has been used from [9] and [10].

- 1) Mass of the shank ( $m_1$ ) is 22.36% of total mass of person.
- 2) Mass of the Thigh ( $m_2$ ) is 8.78% of total mass of person.
- 3) Mass of the foot ( $m_3$ ) is 3.66% of total mass of person.
- 4) Height of the shank ( $l_1$ ) is 26% of the total height of the person.
- 5) Height of the Thigh ( $l_2$ ) is 24% of the total height of the person.
- 6) Height of the foot ( $l_3$ ) is 12% of the total height of the person.
- 7) Location of the center of the mass for shank ( $l_{c1}$ ) is 43.3% from the start of the link.
- 8) Location of the center of the mass for thigh ( $l_{c2}$ ) is 43.3% from the start of the link.
- 9) Location of the center of the mass for foot ( $l_{c3}$ ) is 42.9% from the start of the link.
- 10) Rotational inertia for all links =  $m_i * l_{ci}^2$
- 11) Rotational velocity for knee joint is 2.6m/s for normal walking speed. Same value of velocity has been taken for hip and ankle joints.

Kinetic and potential energies have been calculated for the following two conditions:

- 1) Mass of 65 kg is constant while height in meters varies.
- 2) Height of 1.6 meter is constant while mass in kg varies.

Plot for both conditions are in fig. (2) and (3) respectively.

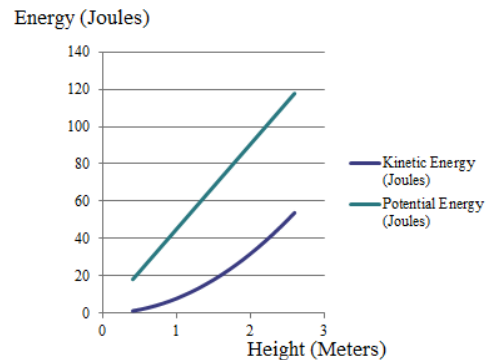


Fig. 2 Kinetic and Potential Energies with 65kg Mass

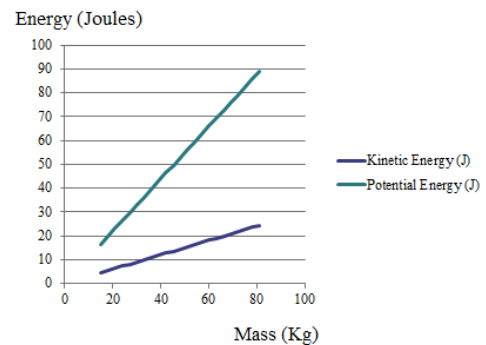


Fig. 3 Kinetic and Potential Energies with 1.6 meter height

As we can see from the fig. 2 and 3 that there is a linear relationship between Kinetic/Potential energies and the Mass/height of the person which validates the above equations.

## VI. CONCLUSION

Lower limb exoskeleton with eight degrees of freedom has been proposed with four DOFs for each leg. Forward and Inverse kinematics of the proposed exoskeleton has been performed along with Dynamic analysis using Lagrangian formulation technique to calculate the expressions for the joint torques. The equations have been validated by plotting the kinetic and potential energies equations of the proposed exoskeleton. Now the expressions for the joint angles and torques can be used for structure designing as well as for the control design of the exoskeleton.

## REFERENCES

- [1] O. B.J. Makinson, "Research and Development Prototype for Machine Augmentation of Human Strength and Endurance, Hardiman I Project Schenectady, NY," General Electric Report S-71-1056, 1971.
- [2] A. M. Dollar, "Lower Extremity Exoskeletons and Active Orthoses: Challenges and State-of-the-Art," IEEE Transactions on Robotics, pp. 144-158, Feb 2008.
- [3] H. P. Crowell, "Human Engineering Design Guidelines for a Powered, Full Body Exoskeleton," U.S. Army Res Lab, Aberdeen Proving Ground, MD ARL-TN-60, Jul 1995.

- [4] Han Yali, Wang Xingsong, "Kinematics Analysis of Lower Extremity Exoskeleton," Control and Decision Conference, Yantai, China, 2-4 July 2008.
- [5] Nelson Costa†, Darwin G. Caldwell "Control of a Biomimetic Soft-actuated 10DoF Lower Body Exoskeleton," The first IEEE/RAS-EMBS International Conference on Biomedical Robotics and Biomechatronics, Pisa, Italy, 20-22 Feb, 2006.
- [6] Massimo Cenciari, Aaron M. Dollar, "Biomechanical Considerations in the Design of Lower Limb Exoskeletons," IEEE International Conference on Rehabilitation Robotics (ICORR), Zurich, Switzerland, June 29-July 1 2011.
- [7] Yanjun Zhao, Cheng Xu, Yufei Luo, Yaping Wang, "Design Modeling and Simulation of Human lower Extremity Exoskeleton," Control and Decision Conference, Yantai, China., 2-4 July 2008.
- [8] Hongbo WANG, Hongtao LIU, Xiaohua SHI, Zengguang HOU, "Design and Kinematics of a Lower Limb Rehabilitation Robot," 2nd International Conference on Biomedical Engineering and Informatics, Tianjin, China., 17-19 Oct, 2009.
- [9] Rudolfs Drillis, Renato Contini, Maurice Bluestein, "Body Segment Parameters A Survey of Measurement Techniques". D. Knudson, Fundamentals of Biomechanics, New York: Kluwer Academic/Plenum Publishers, 2003.