International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:5, No:1, 2011

HPM Solution of Momentum Equation for Darcy-Brinkman Model in a Parallel Plates Channel Subjected to Lorentz Force

Asghar Shirazpour , Seyed Moein Rassoulinejad-Mousavi , Hamid Reza Seyf

***Abstract*—**In this paper an analytical solution is presented for fully developed flow in a parallel plates channel under the action of Lorentz force, by use of Homotopy Perturbation Method (HPM). The analytical results are compared with exact solution and an excellent agreement has been observed between them for both Couette and Poiseuille flows. Moreover, the effects of key parameters have been studied on the dimensionless velocity profile.

***Keywords*—**Lorentz Force,Porous Media, Homotopy Perturbation method

I. Introduction

FULLY developed flow in porous saturated channel is an important subject for researchers due to its wide applications in different industries such as electronic cooling and solar collectors. There are comprehensive investigations about this topic in references [1-8]. Kaviany [9] numerically studied laminar flow in a porous channel bounded by two isoflux parallel plates by use of Brinkman extended Darcy model. Vafai and Kim [10] using this model, analytically considered forced convection in thermally fully developed flow between two flat plates. Amiri and Vafai [11] numerically considered the effects of non-thermal equilibrium and dispersion on the fully developed flow and heat transfer characteristic in a channel filled with a porous medium with variable porosity. Hung et al. [12] considered fully developed forced convection in a homogeneous porous medium and obtained a closed form solutions for the temperature distributions in the transverse direction. Nield et al. [13] investigated forced convection in a channel filled with fluid-saturated porous medium with isothermal or isoflux boundaries. Haji-Sheikh and Vafai [14] used Brinkman's model to analyze flow and heat transfer in porous media imbedded inside various-shaped ducts and presented an exact solution for both rectangular and circular ducts.

At first the application of using Lorentz force for controlling an electrically conducting fluid over a flat plate was presented by Gailitisand Lielausis [15]. Recently it is an interesting topic for many researchers. For examples Pantokratoras [16] presented exact solution for clear fully developed flow. Pantokratoras et al [17] studied the effect of using porous media in a parallel plates channel and investigated the velocity profile for both Poiseuille and Couette flows. Also, Magyari [18] discussed about their solution which has been presented as a comment paper.

Homotopy Perturbation Method (HPM) is a novel methods that introduced by He [19]-[20]. Recently, it has been implemented by many researchers to find an analytical solution of linear and non-linear ordinary or partial differential equations. For instance, Dehghan and Shakeri [21] by use of this method solved Partial differential equation arising in modeling of flow in porous media. Siddiqui et al. Ganji and sadighi [22] solved nonlinear heat transfer and porous media equations by use of HPM and variational iteration methods, and found a very good agreement between these two methods and exact solution. Biazar et al. [23] solved general form of porous medium equation by HPM and compared the results with the Adomian decomposition. They observed good agreement between approximate methods. For more literature review about HPM, an interested reader may refer to [24]-[27].

The aim of this paper is to present an analytical solution using homotopy perturbation method for fully developed flow in a porous saturated channel subjected to Lorentz force for both Couette and Poiseuille flows.

II. Governing equations

The fully develop flow of a viscous and incompressible fluid through parallel plate channel is defined as:

$$-\frac{dp}{dx}+\mu_{eff}\frac{d^2u}{dy^2}-\frac{\mu u}{K}=0 \qquad (1)$$

Where x denotes horizontal direction, y is the vertical coordinate, u is the fluid velocity, K is the permeability of the porous media, $-\frac{dp}{dx}$ is the negative pressure gradient, μ is the dynamic viscosity of fluid and μ_{eff} denotes the effective viscosity of the fluid.

By adding the Lorentz Force to the equation it would be changed as follows:

$$-\frac{dp}{dx}+\mu_{eff}\frac{d^2u}{dy^2}-\frac{\mu u}{K}+\frac{\pi j_0 M_0}{8}\exp(-\frac{\pi}{h}y)=0 \qquad (2)$$

a,* Islamic Azad University, Karaj branch, Iran
e-mail: a-shirazpour@kiau.ac.ir
b,c Islamic Azad University, Karaj branch, Young Researchers Club. Iran

Where j_0 is the applied current density in the electrode, M_0 is the magnetization of the permanent magnet, *a* is the width of magnets and electrodes.

The associated boundary conditions are *u(0)=0* , *u(h)=0* for Poiseuille and *u(0)=0* , *u(h)=u_w* for Couette flow.

The non-dimensional form of eq.2 by defining $U=\frac{u}{u_r}, Y=\frac{y}{h}$ is:

$$-\frac{1}{u_r}\frac{dp}{dx}+\frac{\mu_{eff}}{h^2}\frac{d^2U}{dY^2}-\frac{\mu U}{K}+\frac{\pi j_0 M_0}{8u_r}\exp(-\frac{\pi h}{a}Y)=0 \quad (3)$$

For the sake of brevity we convert the above equation and boundary conditions to the following form:

$$A+\frac{d^2U}{dY^2}-DaU+Q\exp(-BY)=0 \quad (4)$$

u(0)=0 , *u(1)=0* (Poiseuille flow)

u(0)=0 , *u(1)=1* (Couette flow)

Where

$$A=-\frac{h^2}{\mu_{eff}u_r}\frac{dp}{dx},\ Q=\frac{\pi j_0 M_0 h^2}{8u_r\mu_{eff}},\ B=\frac{\pi h}{a},\ Da=\frac{\mu h^2}{\mu_{eff}K}$$

where the Q declares the balance between the electromagnetic forces and viscous forces, *B* is the ratio between the channel height (*h)* and the characteristic length (*a/π)* of the Riga plate and Da is the Darcy number.

III. IMPLEMENTING OF THE HPM

The first step to solve Eq(4) is to define the linear operator which is the linear part of the Eq(4)

$$L(\zeta)=\frac{d^2\zeta}{dY^2}-\zeta\, Da \quad (5)$$

Where ζ is an auxiliary function.

The second step is to guess an arbitrary initial approximation which satisfies the boundary condition as follows:

$$U_{ini}(Y)=Y^3-Y \quad (6)$$

Where subscript *ini* refers to an initial approximation of Eq(4). According Eq. (4), (6) and HPM the following Homotopy equation would be constructed as:

$$D(g,p)=(1-p)\left[\frac{d^2U}{dY^2}-DaU-\frac{d^2U_{ini}}{dY^2}-DaU_{ini}\right]+p\left[A+\frac{d^2U}{dY^2}-DaU+Q\exp(-BY)\right]=0 \quad (7)$$

So, by applying the perturbation technique we obtain a system of equations with *n+1* differential equations to be solved simultaneously when *n* is the order of *p*. So we have:

Zeroth-order:

$$\frac{d^2}{dY^2}g_0(Y)-Da\,g_0(Y)-12Y-Da(2Y^3-Y)=0 \quad (8a)$$

First order:

$$-Da\,g_1(Y)+\frac{d^2}{dY^2}g_1(Y)+12Y+Da(2Y^3-Y)+Q\,e^{-BY}+A=0 \quad (8b)$$

And the boundary conditions are:

$g_i(0)=0, g_i(1)=0 \quad i\geq 0$ poiseuille flow

$g_i(0)=0, g_i(1)=1 \quad i=0 \ and \ g_i(0)=0, g_i(1)=0 \quad i\succ 0$ Couette flow

Solving Eqs. (8a-b) with corresponding boundary conditions, the following functions can be obtained successively.

$$g_0(Y)=Y^3-Y \quad (9a\text{-}b)$$

$$g_1(\bar{y})=\frac{-(e^{-\sqrt{Da}(Y-1)}Q\,Da+e^{-\sqrt{Da}Y}AB+2e^{-\sqrt{Da}}Da^2Y^3+e^{-\sqrt{Da}}Da\,B^2+...)}{(-Da+B^2)(e^{-\sqrt{Da}}-e^{\sqrt{Da}})Da}$$

Finally, by summing up the results, and $p\to 1$ we write the velocity profile as:

$$U(Y)=\sum_{i=0}^{1}g_i(Y) \quad (10)$$

Equation (10) is the analytical solution of the problem by use of Homotopy perturbation method.

IV. RESULTS

The results consist of two parts. At first as a cross-check for the analysis, HPM results are compared with the exact solution and then the effect of some key parameter have been investigated on the velocity profile for both Couette and Poiseuille flow.

Figures 1 and 2 show the effect of *B* on the dimensionless velocity for Couette and Poiseuille flows respectively. It is clear from the figure that variation of *B* has a significant influence on the velocity profile. Also it can be seen that by decreasing the value of *B* the impact of Lorentz Force increases which is owing to an exponentially decrease with *y*.

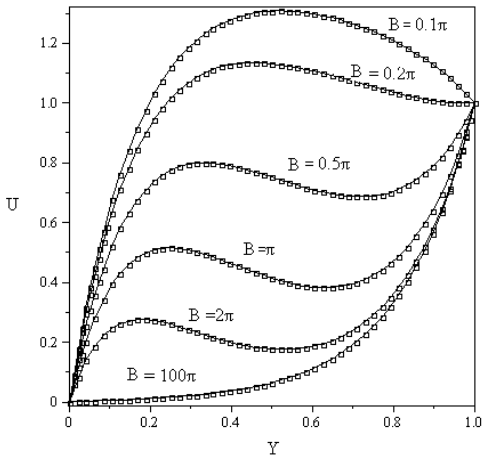

Fig.1. Effect of *B* on velocity profile when Da=30, A=0, Q=50; solid line HPM results; circles exact results for Couette flow

(17a)

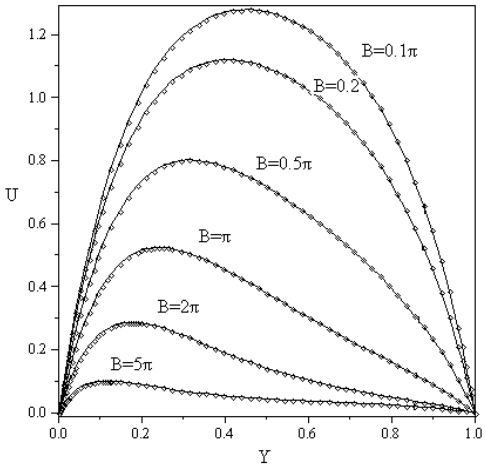

Fig. 2 Effect of *B* on velocity profile when *Da*=30, *A*=1, *Q*=50; solid line HPM results; circles exact results for Poiseuille flow

Figures 3 and 4 illustrate the effect of Chandrasekhar number (*Q*) on velocity profile when Darcy number is constant and equal to 100, the parameters B=π and A=0,1 where A=0 denotes pure Couette and A=1 for Poiseuille flows respectively. The figures depict that value of velocity increases by increasing *Q*. More over it is obvious that velocity capes tend to bottom wall of the channel which would be more intensively by growing the Lorentz force.

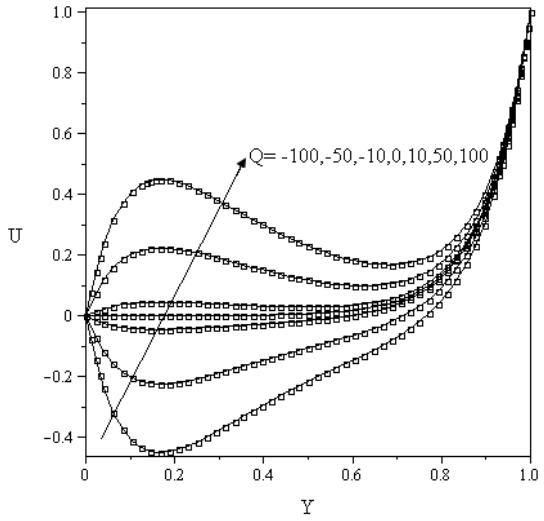

Fig. 3 Effect of *Q* on the velocity profile when *Da*=100 and *A*=0, *B*= π; solid line HPM results; circles exact results for Couette flow

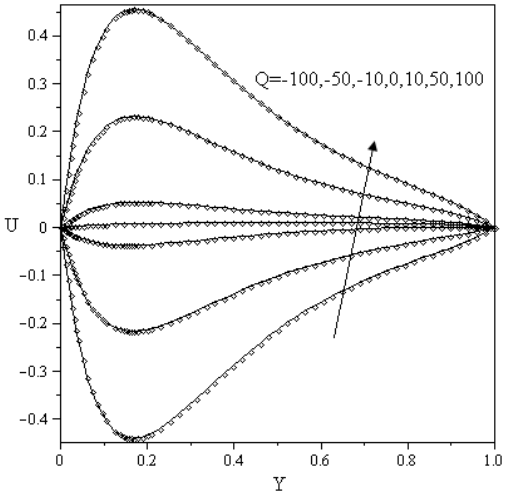

Fig. 4 Effect of *Q* on the velocity profile when *Da*=100 and *A*=1, *B*= π; solid line HPM results; circles exact results for Poiseuille flow

In figures 5 and 6 the influence of Darcy number is considered for. The velocity shape exemplifies that the maximum value of velocity falls by rising Da. It shows that the summit of velocity shoots up when Da increase tends to clear fluid. It means that reduction of the permeability of the porous medium leads to decreasing fluid velocity through the channel. Also it can be seen that the maximum value of velocity for the same values of *Da*, *Q*, *B*, and *A*, is much more in the Couette flow.

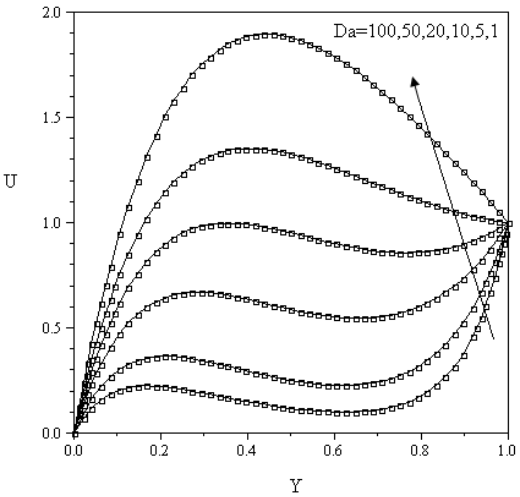

Fig.5 Effect of Da on velocity profile when and *A*=0 *Q*=50 *B*= π; solid line HPM results; circles exact results for Couette flow

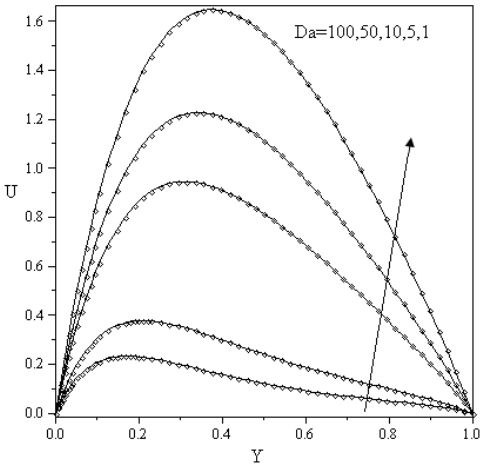

Fig.6 Effect of *Da* on velocity profile when and *A*=1 *Q*=50 *B*= π; solid line HPM results; circles exact results for Poiseuille flow

V. Conclusion

In the present work the application of Homotopy perturbation method as a novel method for solving differential equations has been considered to obtain velocity profile in a porous saturated parallel plats channel under the influence of horizontal Lorentz force and spectacular consent between exact and HPM results is observed. In addition the effect of all existed parameters in Eq.4 including *Q*, *Da*, and *B* has been investigated in the presence of constant pressure gradient. The most important conclusions can be drawn

- HPM is a reliable method for solving the equation and a remarkable agreement is obtained.
- The summit of the velocity profile for Couette is higher than Poiseuille flow in all cases.
- Increasing the effect of Lorentz force leads to increasing the velocity maxima in both types of flows.

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Asghar Shirazpour received his B.S. in mechanical engineering from university of Iran University of Science & Technology and the M.S. degree in 1976 from university of Pretoria, South Africa. His fields of interest involve Combustion in internal engines, Air pollution Controlling and Porous media.

Seyed Moein Rassoulinejad Mousavi was born in Ahwaz, Iran, in 1988. He received his B.S. degree in Mechanical engineering from Karaj Azad University in 2010. His current research interests include Flow and heat transfer in porous media, Heat pipes science and technology, Mathematical modeling and Analytical solution of differential equations.

Hamid Reza Seyf is a graduated B.S student in the division of Thermal/Fluid Science of Mechanical Engineering Department at Karaj Azad University, Iran. His current research interests are CFD, Electronic cooling, Porous Media, Nanofluid Science and Technology, Mathematical Modeling and Two phase flow.