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Observer Design for Chaos Synchronization of Time-delayed Power Systems

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Abstract**—**The global chaos synchronization for a class of time-delayed power systems is investigated via observer-based approach. By employing the concepts of quadratic stability theory and generalized system model, a new sufficient criterion for constructing an observer is deduced. In contrast to the previous works, this paper proposes a theoretical and systematic design procedure to realize chaos synchronization for master-slave power systems. Finally, an illustrative example is given to show the applicability of the obtained scheme.

Keywords**—**Chaos; Synchronization; Quadratic stability theory; Observer

I. INTRODUCTION

A chaotic system is a highly complex dynamic nonlinear system and its response exhibits a number of specific characteristics, including an excessive sensitivity to the initial conditions, broad Fourier transform spectra, and fractal properties of the motion in phase space. Since the pioneering work of Pecora and Carroll in 1990 [1], chaos synchronization has received increasing attention over the last few years. Chaos synchronization can be applied in the vast areas of physics and engineering systems such as in chemical reactions, power converters, biological systems, information processing, especially in secure communication [2-8]. Therefore, various effective methods have been proposed in the past decades to achieve the synchronization of chaotic systems, such as impulsive control [9, 10], linear feedback control [11], variable structure control [12-14], optimal control [15], digital redesign control [16], backstepping control [17, 18], and so on.

On the other hand, power system has been widely used and studied in the industry. In the past years, many classes of power systems have been found with rich phenomena of chaos. Study of chaos and its control in power systems is with considerable importance from the point of view of avoiding undesired behaviors such as power blackout. Kopell and Washburn [19] used Menikov's technique to analyze chaotic motions in the two-degree-of-freedom swing equations. Abed and Varaiya [20] employed the Hopf bifurcation theory to explain nonlinear chaotic behaviors in power systems. Chen et al. [21] used a single model equation to analyze the qualitative chaotic behaviors of a single-machine-infinite-bus (SMIB) power system. Shahverdiv et al. [22] investigated chaos synchronization for such SMIB power systems via numerical simulations to estimate the coupling strengths between the master and slave systems.

In this paper, the global synchronization for the state trajectories of two SMIB power systems described by delay differential equations (DDE) is investigated. The type of power systems described by ordinary differential equations (ODE) can be though as a special case of DDE. By employing the quadratic stability theory [23] and the concept of the generalized model [24], a new design scheme for observer-based controller can be derived. Compared with the previous works [21, 22], avoiding using numerical analysis, this paper proposes a theoretical and systematic design procedure to realize complete synchronization. This paper is organized as follows. Section 2 describes the dynamics of a DDE power system. In Section 3, the synchronization problem for master-slave DDE power systems is formulated. Then a new sufficient condition is proposed. Numerical simulations those confirm the validity and feasibility of the proposed method are shown in Section 4. Finally, conclusions are presented in Section 5.

II. DYNAMICS OF THE DDE POWER SYSTEM

Consider the classical DDE power system. The equation governing the motion of this DDE in terms of the variable θ is given by [22].

$$M\ddot{\theta} + D\dot{\theta} + P_{\max}\sin\theta = P_m + P_\tau \tag{1}$$

where M is the moment of inertia, D is the damping constant, $P_{\max}$ is the maximum power of generator, $P_m = A\sin wt$ is the power of the machine, $P_\tau = r\sin(R\theta(t-\tau))$ is the delayed feedback with a constant delay time τ, and A, w, r, R are all constant. System (1) can be rewritten as a system of first order equations as follows.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -cx_2 - \beta\sin x_1 + f\sin wt + \varepsilon\sin(R(x_1(t-\tau)) \end{aligned} \tag{2}$$

where $x_1 = \theta, x_2 = \dot{\theta}, c = \dfrac{D}{M}, \beta = \dfrac{P_{\max}}{M}, f = \dfrac{A}{M}, \varepsilon = \dfrac{r}{M}$. Let $x = [x_1 \quad x_2]^T \in R^2$, the vector form of the system (2) with the available output y is

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$$\dot{x} = Ax + f(x) + f_\tau(x(t-\tau))$$
$$y = Cx \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -c \end{bmatrix}; f(x) = \begin{bmatrix} 0 \\ -\beta\sin(x_1) + f\sin(wt) \end{bmatrix};$$
$$f_\tau(x(t-\tau)) = \begin{bmatrix} 0 \\ \varepsilon\sin(Rx_1(t-\tau)) \end{bmatrix}; C \in R^{1\times 2}$$

The dynamics of this system has been extensively studied in [22] for a space range of the amplitude of the term R. In particular, for the parameter values of $\tau = 0.6, c = 2, \beta = 2, \varepsilon = 3, w = 5, f = 5$ and $R = 5$, this DDE power system displays chaotic behavior. Figures 1-2 show the chaotic attractor and state responses, respectively, with initial condition of $x_1(t_0) = 1, \forall t_0 \in [-\tau, 0]; x_2(0) = 1$.

III. Problem Formulation and Main Results

In this section, our goal is to investigate chaos synchronization for two identical DDE power systems coupled by an observer-based feedback controller. For this purpose, consider the following master and slave DDE systems.

$$\dot{x}_m = Ax_m + f(x_m) + f_\tau(x_m(t-\tau))$$
$$y_m = Cx_m \tag{4}$$

and

$$\dot{x}_s = Ax_s + f(x_s) + f_\tau(x_s(t-\tau)) + L(y_s - y_m)$$
$$y_s = Cx_s \tag{5}$$

where state vectors of the master and slave systems are denoted with x_m and x_s, respectively. $C = [c_1 \quad c_2] \in R^{1\times 2}$ is the output gain matrix and (A, C) is observable. $L \in R^{2\times 1}$ is the coupling vector designed to guarantee the synchronization between Systems (4) and (5). Define the synchronization error between System (5) and System (4) as $e = [e_1 \quad e_2]^T = [x_{s1} - x_{m1} \quad x_{s2} - x_{m2}]^T$, then the error dynamics is governed by the following equation.

$$\dot{e} = Ae + f(x_s) - f(x_m) + f_\tau(x_s(t-\tau)) - f_\tau(x_m(t-\tau)) + LCe$$
$$= (A + LC + F(t))e(t) + F_\tau(t)e(t-\tau) \tag{6}$$

where

$$F(t) = \begin{bmatrix} 0 & 0 \\ \dfrac{-\beta[\sin(x_{s1}) - \sin(x_{m1})]}{x_{s1} - x_{m1}} & 0 \end{bmatrix};$$
$$F_\tau(t) = \begin{bmatrix} 0 & 0 \\ \dfrac{\varepsilon[\sin(Rx_{s1}(t-\tau)) - \sin(Rx_{m1}(t-\tau))]}{x_{s1}(t-\tau) - x_{m1}(t-\tau)} & 0 \end{bmatrix} \tag{7}$$

According to the differential mean-value theorem, one has

$$\sin(x_{s1}) - \sin(x_{m1}) = \cos(\eta_1)\cdot(x_{s1} - x_{m1}),$$
$$\eta_1 \in [x_{s1}, x_{m1}] \text{ or } [x_{m1}, x_{s1}] \tag{8.a}$$

and

$$\sin(Rx_{s1}(t-\tau)) - \sin(Rx_{m1}(t-\tau))$$
$$= \cos(\eta_2)\cdot(R)\cdot(x_{s1}(t-\tau) - x_{m1}(t-\tau)), \tag{8.b}$$
$$\eta_2 \in [Rx_{s1}(t-\tau), Rx_{m1}(t-\tau)] \text{ or } [Rx_{m1}(t-\tau), Rx_{s1}(t-\tau)]$$

, thus $F(t)$ and $F_\tau(t)$ can be simplified as

$$F(t) = \begin{bmatrix} 0 & 0 \\ -\beta\cos(\eta_1) & 0 \end{bmatrix}; F_\tau(t) = \begin{bmatrix} 0 & 0 \\ \varepsilon R\cos(\eta_2) & 0 \end{bmatrix} \tag{9}$$

The considered goal of this paper is that for any given master-slave DDE power systems as (4) and (5), a sufficient condition for the observer-based controller is proposed such that the trajectories of $x_m(t)$ and $x_s(t)$, wherever the choice of initial states, satisfy $\|x_m(t) - x_s(t)\| \to 0$ as $t \to \infty$.

As a sequence, to achieve synchronization via the observer-based controller is equivalent to select the coupling matrix L such that the error dynamics (6) is asymptotically stable at the origin. The following lemma will be applied to prove the main theorem of this paper.

Lemma 1 [23]: Define the Hamiltonian matrix

$$H =: \begin{bmatrix} A - LC & \eta I_2 \\ -M^T M & -(A - LC)^T \end{bmatrix}; \text{ where } M = \begin{bmatrix} 0 & 0 \\ |\beta| + |\varepsilon R| & 0 \end{bmatrix} \tag{10}$$

Assume that (i) $(A - LC, M)$ and $(A - LC)$ are observable and stable, respectively, and (ii) H with $\eta > 1$ has no eigenvalues on the imaginary axis. Then the algebraic Riccati equation (ARE)

$$(A - LC)^T P + P(A - LC) + \eta PP + M^T M = 0 \tag{11}$$

has a positive definite solution P.

Proof: It is an immediate result of the work of Doyle et al.[23] and hence is omitted.

Lemma 1 [24]: The error system (6) is asymptotically stable independent of time delay τ if and only if the following 'generalized system model'

$$\dot{e} = (A + LC + F(t))e(t) + F_\tau(t)ze(t), \tag{12}$$

is asymptotically stable. Where z is the complex number; $z = \exp(j\sigma) = \cos(\sigma) + j\sin(\sigma), \forall \sigma \in [0, 2\pi]$ and $|z| = 1$.

Theorem 1: Suppose the coupling matrix L is selected such that $(A - LC, M)$ and $(A - LC)$ are observable and stable, respectively. If the Hamiltonian matrix H with some $\eta > 1$ as defined in (10) has no eigenvalues on the imaginary axis, then the master-slave system defined in (4) and (5) achieves global chaos synchronization.

Proof: Suppose the coupling matrix L is selected such that $(A - LC, M)$ and $(A - LC)$ are observable and stable, respectively. According to Lemma 1, if the Hamiltonian matrix (10) has no eigenvalues on the imaginary axis, then the following algebraic Riccati equation (ARE) is obtained.

$$(A - LC)^T P + P(A - LC) + \eta PP + M^T M = 0 \tag{13}$$

where P is a positive definite solution. Now, introducing a

Lyapunov function $V(t)$ as

$$V(t) = e^T(t)Pe(t) \geq 0 \tag{14}$$

It is easily verified that $V(t)$ is a non-negative function over $[0,+\infty)$ and radially unbounded, i.e. $V(t) \to \infty$ as $e(t) \to \infty$. Subsequently, evaluating the time derivative of V along the trajectory of generalized system model (12) in Lemma 2, it yields

$$\begin{aligned}\dot{V} &= \dot{e}^T Pe + e^T P\dot{e} \\ &= \left[(A - LC + F(t))e + F_\tau(t)ze)\right]^T Pe \\ &\quad + e^T P\left[(A - LC + F(t))e + F_\tau(t)ze\right] \\ &= e^T[(A-LC)^T P + P(A-LC)]e + 2e^T P(F(t) + F_\tau(t)z)e \\ &\leq e^T[(A-LC)^T P + P(A-LC)]e + 2\left\|e^T P\right\|\left\|(F(t) + F_\tau(t)z)e\right\|\end{aligned} \tag{15}$$

Furthermore,

$$\begin{aligned}&2\left\|e^T P\right\|\left\|(F(t) + F_\tau(t)z)e\right\| \\ &\leq \left\|e^T P\right\|^2 + \left\|(F(t) + F_\tau(t)z)e\right\|^2 \\ &= e^T PPe + e^T (F(t) + F_\tau(t)z)^T (F(t) + F_\tau(t)z)e \\ &= e^T PPe + (-\beta\cos(\eta_1) + \varepsilon R\cos(\eta_2)z)^2 e_1^2 \\ &\leq e^T PPe + (|\beta| + |\varepsilon R|)^2 e_1^2 = e^T PPe + e^T M^T Me \text{; for } \forall |z| = 1\end{aligned} \tag{16}$$

Therefore, from (15) and (16), it yields

$$\begin{aligned}\dot{V} &\leq e^T[(A-LC)^T P + P(A-LC) + PP + M^T M]e \\ &< e^T[(A-LC)^T P + P(A-LC) + \eta PP + M^T M]e\end{aligned} \tag{17}$$

where $\eta > 1$. Since P is the positive definite solution of ARE(13), one has $\dot{V}(t) < 0$. According to Lyapunov stability theory and Lemma 2, the last inequality $\dot{V}(t) < 0$ indicates $V(t)$ as well as $e(t)$ converge to zero asymptotically. This completes the proof.

Remark 1: Roughly speaking, the eigenvalues of Hamiltonian matrix (10) will not be located on the imaginary axis, if matrix $(A - LC)$ has more negative eigenvalues. Fortunately, since (A, C) is observable, it is easy to select appropriate L such that $(A - LC)$ has desired negative eigenvalues so that the conditions in Theorem 1 are easily satisfied.

IV. Numerical Example

In this section, simulation results are presented to demonstrate the effectiveness of the proposed synchronization scheme. All the simulation procedures are coded and executed using the software of MATLAB. The system parameters are chosen as follows: $\tau = 0.6, c = 2, \beta = 2, \varepsilon = 3, w = 5, f = 5$ and $R = 5$. The initial states of the master system (4) are $x_{m1}(t_0) = 1, \forall t_0 \in [-\tau, 0]; x_{m2}(0) = 1$ and initial states of the slave system (5) are $x_{s1}(t_0) = 0, \forall t_0 \in [-\tau, 0]; x_{s2}(0) = 2$. The output matrix $C = [1 \quad 0]$ is selected. According to Remark 1, the coupling matrix L is chosen as $L = [-19 \quad -70]$ such that the eigenvalues of $(A - LC)$ are -9 and -12. Then the Hamiltonian matrix H with $\eta = 1.1 > 1$ is obtained as

$$H = \begin{bmatrix} -19 & 1 & 1.1 & 0 \\ -70 & -2 & 0 & 1.1 \\ -289 & 0 & 19 & 70 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

It is easy to check that H has no eigenvalues on the imaginary axis. Thus according to Theorem 1, the master and slave systems (4)(5) are globally synchronized. The simulation results are shown in Fig.3. Fig. 3 shows the corresponding state and error responses for the controlled master-slave DDE systems. From the simulation results, it shows that the trajectories of master-slave systems are synchronized and the synchronization error also converges to zero.

V. Conclusion

In this paper, the global synchronization for a class of chaotic time-delayed power systems has been investigated via observer-based approach. By employing the concepts of quadratic stability theory and generalized system model, a new sufficient criterion has been proposed to construct an observer. Numerical simulations have verified the effectiveness of the proposed method.

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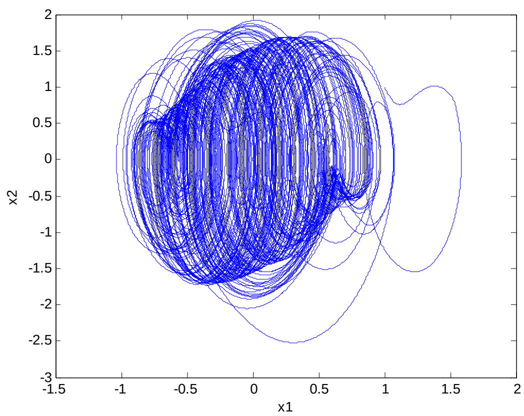

Fig. 1 The phase portrait of the chaotic DDE power system.

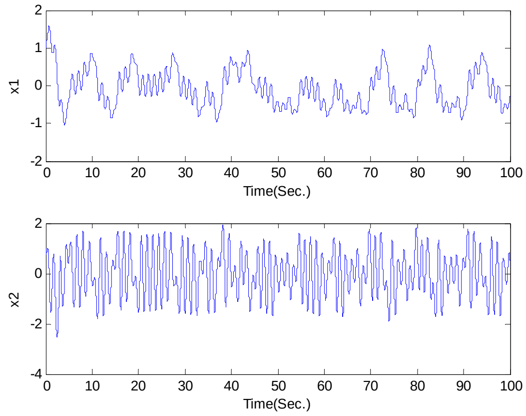

Fig. 2 The time history of the chaotic DDE power system

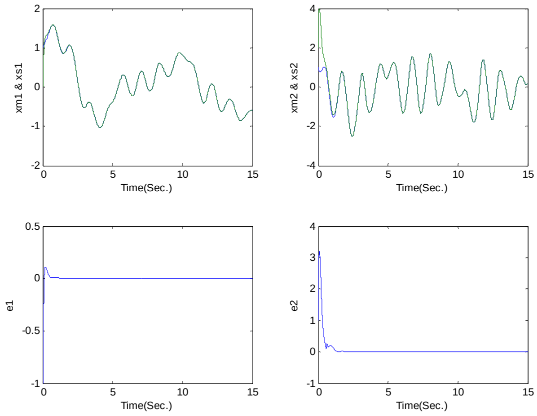

Fig. 3 State and error responses for the controlled master-slave DDE power system.