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Static Single Point Positioning Using The Extended Kalman Filter

I. Sarras, G. Gerakios, A. Diamantis, A. I. Dounis and G. P. Syrcos

***Abstract*—**Global Positioning System (GPS) technology is widely used today in the areas of geodesy and topography as well as in aeronautics mainly for military purposes. Due to the military usage of GPS, full access and use of this technology is being denied to the civilian user who must then work with a less accurate version.

In this paper we focus on the estimation of the receiver coordinates (X, Y, Z) and its clock bias (δt_r) of a fixed point based on pseudorange measurements of a single GPS receiver. Utilizing the instantaneous coordinates of just 4 satellites and their clock offsets, by taking into account the atmospheric delays, we are able to derive a set of pseudorange equations.

The estimation of the four unknowns (X, Y, Z , δt_r) is achieved by introducing an extended Kalman filter that processes, off-line, all the data collected from the receiver. Higher performance of position accuracy is attained by appropriate tuning of the filter noise parameters and by including other forms of biases.

***Keywords*—**Extended Kalman filter, GPS, Pseudorange

I. Introduction

THE Global Positioning System (GPS) is a continuous, all-weather satellite-based navigation and positioning system developed by the U.S. Department of Defence. The system of satellites that makes up the space segment of GPS consists of 24 satellites allocated in six orbital planes. Each satellite transmits two carrier signals on the L1 (1575.42 MHz) and L2 (1227.6 MHz) frequencies that contain the ephemeris data for the determination of the position of the satellites. When the satellite position is known, an authorized user can receive the satellite's transmitted signals and determine the signal propagation time. By using this information, each receiver will be able to compute its ranges to the satellites and correct its clock. The actual measurement is called pseudorange because of the offset of the receiver from true GPS time. A minimum of four simultaneous pseudoranges are necessary for the accurate determination of the receiver position (X, Y, Z) and the clock bias (δt_r) [1,2].

With the implementation of Selective Availability (SA) the system accuracy for civilian applications is intentionally degraded and in addition to the errors due to clock biases, ionospheric and troposheric delays and other unmodeled error sources cause the pseudorange data collected from GPS receivers to be inaccurate. These noisy measurements should be treated appropriately in order to minimize the errors and increase the position accuracy.

A widely used tool in order to get an optimal, in the sense of minimizing the mean squared error, estimate of the state of a system is the Kalman filter. In his seminal paper [9] R.E. Kalman presented a recursive solution to the linear filtering problem. Soon after, the extension to the case of non-linear systems with non-linear measurements was introduced by considering the linearization of the process and of the measured output [3, 4]. Its applicability to various applications is well acknowledged and extensive research is still being carried out. See [13]-[16] for GPS related works and references. Some of the recent developments constitute of the Fuzzy Kalman filter [11,12] and the Unscented Kalman filter [10] that elegantly generalizes the EKF without the need of linearization and Gaussian noise distribution, yielding superior results.

In this work, we are interested in estimating the receiver coordinates and clock bias, combining all the information about noise and bias error sources, using the extended Kalman filter. We show that the application of the EKF produces more accurate estimates of the receiver position.

II. Extended Kalman filter algorithm

As we already mentioned, the extended Kalman filter is employed when the process model and/or the measurement model are represented by a nonlinear equation. In the point positioning problem only the measurement equation is nonlinear so, the main purpose is to find the linearized equations needed for the filter implementation.

The vector that is chosen to be estimated is:

$$\underline{x} = [\ \underline{r} \quad \delta t_r \ ]^T ,$$

where $\underline{r} = (\mathrm{X\ Y\ Z})$ the receiver's position coordinates and δt_r is the receiver clock bias.

As we deal with a fixed-point positioning problem it is simple to derive the model describing our system given by the linear state-space equation:

I. Sarras is with the Laboratoire des Signaux & Systèmes, Université Paris-Sud XI, 3 Joliot-Curie, 91192, Gif-sur-Yvette, (cedex) France. (corresponding author; e-mail: sarras@ lss.supelec.fr).

G. Gerakios, A. Diamantis,A. I. Dounis and G. P. Syrcos are with DSP and Digital Control Lab, Technological Education Institute of Piraeus, Department of Automation, Petrou Ralli and Thivon 250, 12244 Athens, GREECE, (email: rakiosge@yahoo.gr, aidounis@otenet.gr, gsyrkos@teipir.gr).

$$\underline{x}_{k+1} = \Phi_k \cdot \underline{x}_k + \underline{w}_k$$

where Φ_k the state transition matrix is equal to the identity matrix and w is white Gaussian noise expressing the system noise.

The extended Kalman filter mechanism is divided into two steps. First, the time update step where a prediction of the state vector and the error covariance matrix is made taking into account the previous measurement $\underline{z}_k$:

$$\hat{\underline{x}}_{k+1}(-) = \Phi_k \cdot \hat{\underline{x}}_k(+)$$

$$P_{k+1}(\text{-}) = \Phi_k \, P_k(+) \, \Phi_k^{\,T} + Q_k$$

The model for the GPS pseudorange measurement is given by the nonlinear equation:

$$\underline{h}(\underline{x}_k) = \rho_j + b_k$$

where

$$\rho_j = \sqrt{\left(X^j(t) - X\right)^2 + \left(Y^j(t) - Y\right)^2 + \left(Z^j(t) - Z\right)^2} \qquad (1)$$

is the geometric distance, *X, Y, Z* are the position coordinates, X^j, Y^j, Z^j are the position coordinates of satellite j at transmission time and b_k are all errors related to the measurement.

Linearizing the measurement equation $\underline{h}(\underline{x}_k) = \underline{h}(\hat{\underline{x}}_k) + H(\hat{\underline{x}}_k)(\underline{x}_k - \hat{\underline{x}}_k)$ we find the sensitivity matrix $H(\hat{\underline{x}}_k) = \left.\dfrac{\partial \underline{h}}{\partial \underline{x}_k}\right|_{\underline{x}_k = \hat{\underline{x}}_k}$.

$$H_k = \begin{bmatrix} -\dfrac{\left(X^j(t) - X\right)}{\rho_j} & -\dfrac{\left(Y^j(t) - Y\right)}{\rho_j} & -\dfrac{\left(Z^j(t) - Z\right)}{\rho_j} & c \end{bmatrix}$$

Second, the measurement update step where the filter incorporates the predicted values and the information from the measurements to improve the estimated position and clock bias. The measurement update filter equations are:

$$K_k = P_k(\text{-}) \, H_k^{\,T} \, [H_k K_k H_k^{\,T} + R_k]^{-1}$$

$$\hat{\underline{x}}_k(+) = \hat{\underline{x}}_k(\text{-}) + K_k\left(\underline{z}_k - h_k(\hat{\underline{x}}_k(\text{-}))\right)$$

$$P_k(+) = \left(I - K_k H_k\right) P_k(\text{-}),$$

where R_k is the discrete measurement covariance matrix and is diagonal due to uncorrelated measurements.

III. GPS MEASUREMENT MODEL

The fundamental observations that are mostly applied in GPS point positioning problems are pseudorange and phase measurements. In this paper only the former one using the C/A code in L1 frequency will be treated as it is the basis of standard positioning applications.

The model for the pseudorange observations in L1 frequency is given as follows:

$$P_j^{L1} = \rho_j + I_j^P + c \cdot (\delta t_r - \delta^j(t)) + \varepsilon,$$

where P_j^{L1} is the pseudorange in L1 frequency for the satellite j, ρ_j is the geometric range given by Eq.(1), I_j^P the ionospheric delay, *c* is the speed of light, δt_r is the unknown receiver clock bias, $\delta^j(t)$ is the GPS satellite j clock bias, and ε represents time-correlated errors associated with pseudorange such as tropospheric refraction, ephemeris data errors and multipath effect and is supposed to be random Gaussian .

The information for each satellite clock is known and transmitted via the broadcast navigation message in the form of three polynomial coefficients a_0, a_1, a_2 with a reference time t_e which is also known. The equation

$$\delta^j = a_0 + a_1 \cdot (t_S - t_e) + a_2 \cdot (t_S - t_e)^2 \qquad , \quad j \geq 4 \quad (2)$$

enables the calculation of the satellite clock for epoch t_S but we also must take into account the beginning or end-of-week crossovers [1].

Among the error sources the largest one comes from the ionosphere as the GPS transmitted signals pass through. Having the pseudorange measurements in both L1 and L2 frequency we are able to decrease the ionospheric effects that can be further reduced during double differencing. The mathematical equation that gives the correction of the ionospheric effect to range measurement in L1 frequency in [1] is,

$$I_j^P = \frac{f_2^2}{f_2^2 - f_1^2} \cdot \left(P_j^{L1} - P_j^{L2}\right)$$

where f_1 is the frequency in L1 (1.575 GHz), f_2 is the frequency in L2 (1.227 GHz),

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TABLE I
WGS '84 PARAMETERES

Parameters and values	Description
$a = 6\,378137 m$	Semimajor axis of ellipsoid
$\overline{C}_{2,0} = 484{,}16685 \cdot 10^{-6}$	Normalized spherical harmonic
$\omega_E = 7\,292\,115 \cdot 10^{-11}\ rad$	Angular velocity of the earth
$\mu = 3\,986\,005 \cdot 10^{8}\ m^3 s^{-2}$	Earth's gravitational constant
$b = 6\,356\,752{,}31425\ m$	Semiminor axis of ellipsoid
$f = 3{,}35281066474 \cdot 10^{-3}$	Flattening of ellipsoid
$e^2 = 6{,}69437999013 \cdot 10^{-3}$	First numerical eccentricity
$e'^2 = 6{,}73949674226 \cdot 10^{-3}$	Second numerical eccentricity

and P_j^{L1}, P_j^{L2} are the pseudorange measurements in L1 and L2 frequency respectively.

The WGS '84 (World Geodetic System of 1984) coordinate reference frame is used by the GPS and the receiver coordinates are reported in that form. Its main parameters and values are given in Table I.

Both observation and navigation data were in Rinex format and were obtained by the 'OBEC Consulting Engineers Cooperative CORS Station' [6]. The initial conditions for the receiver position were obtained by the National Geodetic Survey (NGS) and shown in Table II.

TABLE II
INITIAL RECEIVER POSITION COORDINATES

ITRF00 POSITION (EPOCH 1997.0) Computed by NGS using OPUS on 03/18/02 w/ 10 days of data 01/27-02/06/02	
X = -2506736.064 m	latitude = 44 03 57.47564 N
Y = -3845597.058 m	longitude = 123 05 53.33297 W
Z = 4413438.953 m	ellipsoid height = 111.851 m

The initial values for the error covariance matrix are given by [5] :

$$P = \begin{bmatrix} 50^2(m^2) & 0 & 0 & 0 \\ 0 & 78^2(m^2) & 0 & 0 \\ 0 & 0 & 78^2(m^2) & 0 \\ 0 & 0 & 0 & 170^2(n\sec^2) \end{bmatrix}$$

For the variance of the system noise we assume a conservative choice of 10^{-5} describing the uncertainty of the model and for the measurement error variance R_k a choice of 35^2 (m^2) [8]. The time-correlated errors associated with pseudorange ε vary in a range between 0-15 m [5].

IV. RESULTS

The data processing program is written in Matlab and consists of three sessions:

1. Reading the observation file and manipulating its data.
2. Reading the navigation file and manipulating its data.
3. Estimating the receiver coordinates and clock bias using the extended Kalman filter algorithm.

The data obtained from 4 satellites (17th March 2004) have been elaborated with one measurement epoch at sampling rate of 5 seconds and for just a period of 45 seconds. The data from four different randomly selected dates were used so that we are able to verify the efficiency of the estimation process. The deviations of the values of the position estimates from the true coordinates are shown in Fig. 1-3. Fig. 4 shows the deviation of the receiver clock bias estimate from its mean value.

The extended Kalman filter processes the first 5 seconds period data for a number of 150 iterations until it converges and proceeds for the following set of data. Table III summarizes the results for the position estimation and shows the errors in each coordinate as also the estimated geometric distance from the true position.

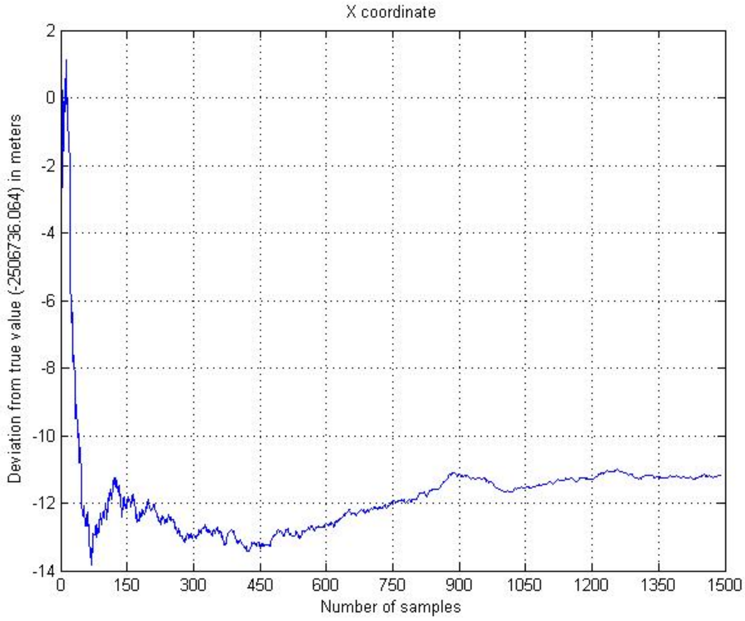

Fig. 1 X coordinate estimation error

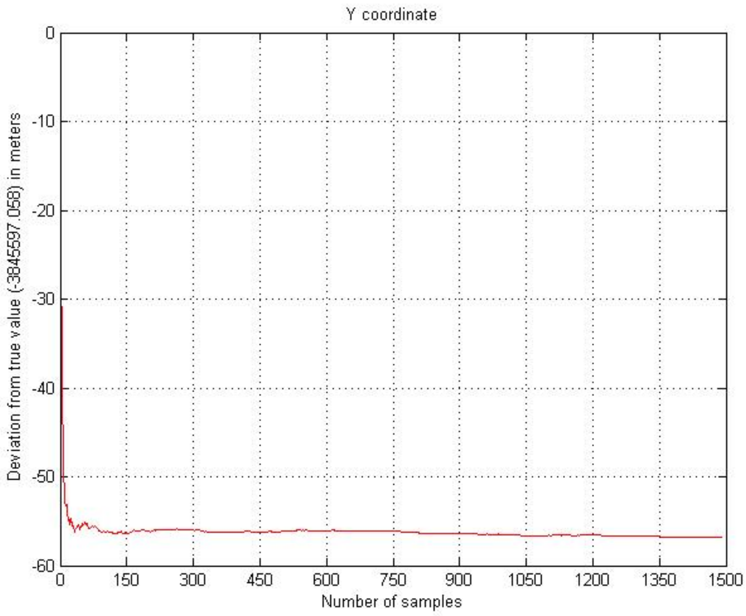

Fig. 2 Y coordinates estimation error

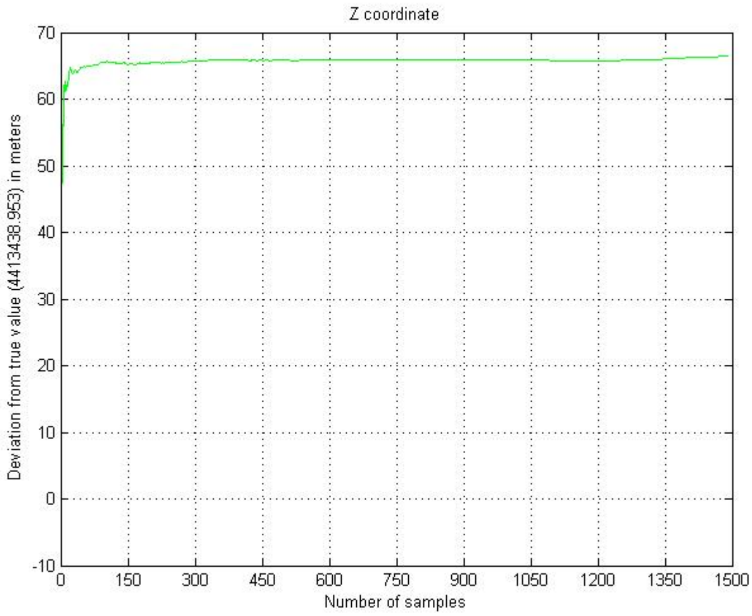

Fig. 3 Z coordinates estimation error

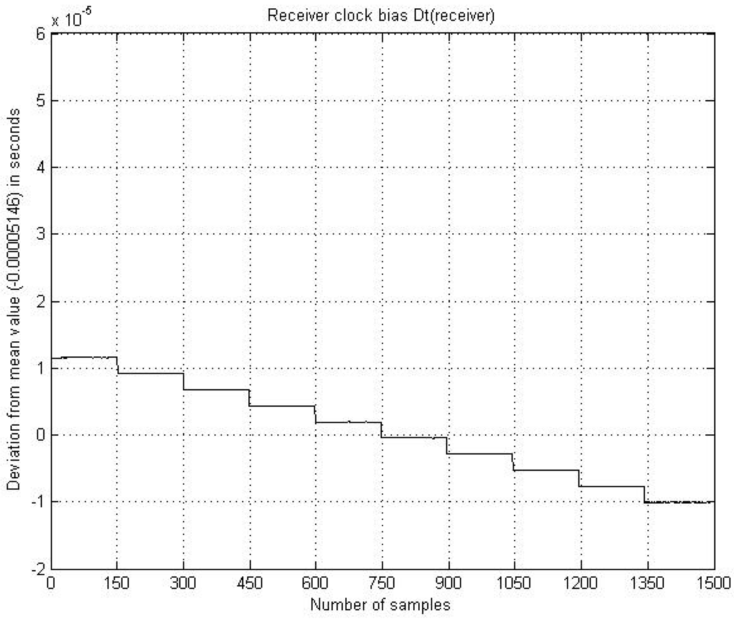

Fig. 4 Receiver clock bias deviation from mean value

TABLE III
ERROR OF MEAN VALUE FROM TRUE COORDINATES

Date	X (m)	Y (m)	Z (m)	Distance (m)
28/02/2004	50.037	13.959	136.427	145. 983
2/03/2004	-41.823	101. 805	-35.950	115. 784
16/03/2004	21.619	57.649	-65.788	90.105
17/03/2004	11.941	56.357	-65.835	87.481

Originally, the accuracy expected from C/A code pseudorange positioning with SA turned on was in the range of some 400m [2]. For civilian applications it is shown that the GPS system precision is increased to 100m [7]. Although we are processing less than a minute's duration data our results clearly show that we attain an average accuracy of 50-70 meters in each coordinate.

V. CONCLUSIONS

Single frequency GPS receivers are becoming the leading edge in GPS technology as they provide high precision at lower costs. In this paper, we provide an insight in estimating the GPS receiver position and clock bias using the single frequency L1 pseudorange measurements and taking into account the ionospheric effect using both L1 and L2 pseudoranges, although it can be computed only from the former one. Further improvement of the position estimates can be achieved by exploiting all of the 24-hour data. Also combining the troposheric correction given by [1] and the unmodeled ionospheric delays eliminating the atmospheric effect it is quite evident that a more appropriate measurement model would be generated. Future research will be directed to develop a fuzzy extended Kalman filter e.g. [11,12] and an Unscented Kalman filter [10] for the static single point positioning problem. Also further works can focus on how to improve the initial values of the filter and the covariance matrices R_k, Q_k.

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Ioannis Sarras (Student Member IEEE S'08) was born in Athens, Greece, in 1982. He graduated from the Automation Engineering Department of the Technological Education Institute (T.E.I.) of Piraeus in 2004 and received the Master of Research (M2R) in Automatic Control from the Université Paul Sabatier (Toulouse III), France, in 2006. Since October 2006, he has been a PhD candidate at the Laboratoire des Signaux et Systèmes (Université Paris-Sud XI), France, under the supervision of Dr. Romeo Ortega. His research interests are in the fields of estimation theory, nonlinear and geometric control with emphasis on mechanical systems.

George P. Syrcos received his M.Sc. in Systems Engineering in 1971 from Widener University and worked for the Boeing Company for 9 years as an Aircraft Simulations Engineer. After receiving his PhD in 1981 from Rutgers University he was appointed Assistant Professor at Rutgers University and at Wilkes University before returning to Greece, where he is presently a Professor at the Technological Educational Institute of Piraeus. He is a member of Eta Kappa Nu, Tau Beta Pi and the Technical Chamber of Greece.

Argiris Diamandis was born in Athens in 1972. He obtained his degree in Control Engineering from TEI of Piraeus, Greece and his MSc in Communications, Control and DSP (CCDSP) from Strathclyde University UK. He is a Research Assistant in the Technological Educational Institute of Piraeus and a senior R&D researcher in a private sector technology company in the field of machine vision and signal processing.

Anastasios I. Dounis received his B.Sc. degree in Physics, from the University of Patras, Greece, in 1983, his M.Sc. in Electronic Automation from the National and Kapodistrian University of Athens in 1988, and his Ph.D. degree from Department of Electronic and Computer Engineering of the Technical University of Crete (Chania), Greece, in 1993. From 1986 to 1990 he was a research assistant with the Institute of Informatics and Telecommunications of Research Center ''Democritos'', Athens. During 1993–1995 he was a part time Professor at the Department of Electronic Engineering, Hellenic Air Force Academy. Since 1994 Dr. Dounis is Adjunct Professor at the Department of Automation, Technological Educational Institute of Piraeus. His current research interests include fuzzy systems, fuzzy control and modelling, genetic algorithms, intelligent control, time series prediction, ambient intelligent environment and intelligent buildings. He is the author or co-author of over 40 technical papers, which have appeared in international journals, and conference proceedings. Dr. Dounis has served as a Reviewer for many prestigious international journals, including the IEEE Transactions on Fuzzy Systems, Journal of Uncertain Systems, Energy and Buildings, Energy: An International Journal, Engineering Structures, and IEEE Transactions on Aerospace & Electronic Systems. He is a Member of the IEEE Computational Intelligence Society, the IEEE Systems, Man, and Cybernetics Society, and the IEEE Control Systems Society. His biography is listed in Who's Who in the World and Marquis Who's Who in Science and Engineering.