

Periodicity for a semi-ratio-dependent predator-prey system with delays on time scales

Kejun Zhuang

Abstract—In this paper, the semi-ratio-dependent predator-prey system with nonmonotonic functional response on time scales is investigated. By using the coincidence degree theory, sufficient conditions for existence of periodic solutions are obtained.

Keywords—Semi-ratio-dependent, predator-prey system, coincidence degree, time scales.

I. INTRODUCTION

RECENTLY, many people have concentrated on the following semi-ratio-dependent predator-prey system with functional responses [1–3],

$$\begin{cases} \dot{x}_1(t) = x_1(t) \left[r_1(t) - a_{11}(t)x_1(t - \tau(t)) - f(x_1(t))x_2(t) \right], \\ \dot{x}_2(t) = x_2(t) \left[r_2(t) - \frac{a_{21}(t)x_2(t)}{x_1(t)} \right], \end{cases} \quad (1)$$

where x_1 and x_2 denotes the density of the prey and predator, respectively. $f(x_1)$ is the so-called predator functional response to prey, and the density of predator is in proportion to that of the prey. In [1], the simplified Monod-Haldane function of the form $f(x_1) = \frac{a_{12}(t)x_1}{m^2 + x_1^2}$ was considered. Moreover, time delay is usually important to the dynamics of differential equations, so we consider the following system

$$\begin{cases} \dot{x}_1(t) = x_1(t) \left[r_1(t) - a_{11}(t)x_1(t - \tau(t)) - \frac{a_{12}(t)x_2(t)}{m^2 + x_1^2(t)} \right], \\ \dot{x}_2(t) = x_2(t) \left[r_2(t) - \frac{a_{21}(t)x_2(t)}{x_1(t - \sigma(t))} \right], \end{cases} \quad (2)$$

if $\sigma(t) = 0$, then (2) was studied in [1]. Motivated by [4], we can obtain the following discrete analogy of (2), which is governed by difference equations with periodic coefficients,

$$\begin{cases} x_1(n+1) = x_1(n) \exp \left\{ r_1(n) - a_{11}(n)x_1(n - \tau(n)) - \frac{a_{12}(n)x_2(n)}{m^2 + x_1^2(n)} \right\}, \\ x_2(n+1) = x_2(n) \exp \left\{ r_2(n) - \frac{a_{21}(n)x_2(n)}{x_1(n - \sigma(n))} \right\}. \end{cases} \quad (3)$$

As we know, it is similar to explore the existence of periodic solutions for (2) and (3) in the approaches, the methods and the main results. So it is unnecessary to study the periodic solutions in separate ways. By using the theory of time scales, which was first proposed by Stefan Hilger[5], we can unify the existence of periodic solutions of population dynamics modelled by differential equations and difference equations. For this reason, we consider the following dynamics equations

on time scales,

$$\begin{cases} u_1^\Delta(t) = r_1(t) - a_{11}(t)e^{u_1(t - \tau(t))} - \frac{a_{12}(t)e^{u_2(t)}}{m^2 + e^{u_1^2(t)}}, \\ u_2^\Delta(t) = r_2(t) - \frac{a_{21}(t)e^{u_2(t)}}{e^{u_1(t - \sigma(t))}}, \end{cases} \quad (4)$$

where $r_1(t), r_2(t), a_{11}(t), a_{12}(t)$, and $a_{21}(t)$ are rd-continuous positive ω -periodic functions on time scales \mathbb{T} . Set $y_i(t) = e^{u_i(t)}$, $i = 1, 2$. If $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$, then (4) can be derived to (2) and (3) respectively.

The primary aim of this paper is to explore the existence of periodic solutions for dynamic equations on time scales. The approach is based on the coincidence degree theory, such as [6–8]. Moreover, with the help of new inequality on time scales, we can find the sharp priori bounds and improve existence criteria for periodic solutions.

The remainder of this paper is organized as follows. In the following section, some preliminary results about calculus on time scales and Continuation Theorem are stated. The existence of periodic solution for (4) is established in Section 3.

II. PRELIMINARIES

For convenience, we first present some basic definitions and lemmas about time scales and the continuation theorem of the coincidence degree theory; more details can be found in [9–10]. A time scale \mathbb{T} is an arbitrary nonempty closed subset of real numbers \mathbb{R} . Throughout this paper, we assume that the time scale \mathbb{T} is unbounded above and below, such as \mathbb{R} , \mathbb{Z} and $\bigcup_{k \in \mathbb{Z}} [2k, 2k+1]$. The following definitions and lemmas about time scales are from [9].

Definition 2.1. The forward jump operator $\sigma : \mathbb{T} \rightarrow \mathbb{T}$, the backward jump operator $\rho : \mathbb{T} \rightarrow \mathbb{T}$, and the graininess $\mu : \mathbb{T} \rightarrow \mathbb{R}^+ = [0, +\infty)$ are defined, respectively, by $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$, $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$, $\mu(t) = \sigma(t) - t$. If $\sigma(t) = t$, then t is called right-dense (otherwise: right-scattered), and if $\rho(t) = t$, then t is called left-dense (otherwise: left-scattered).

Definition 2.2. Assume $f : \mathbb{T} \rightarrow \mathbb{R}$ is a function and let $t \in \mathbb{T}$. Then we define $f^\Delta(t)$ to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s)| \leq \varepsilon |\sigma(t) - s| \quad \text{for all } s \in U.$$

In this case, $f^\Delta(t)$ is called the delta (or Hilger) derivative of f at t . Moreover, f is said to be delta or Hilger differentiable on \mathbb{T} if $f^\Delta(t)$ exists for all $t \in \mathbb{T}$. A function $F : \mathbb{T} \rightarrow \mathbb{R}$ is

Kejun Zhuang is with the School of Statistics and Applied Mathematics, Anhui University of Finance and Economics, Bengbu 233030, China, e-mail: zhkj123@163.com

called an antiderivative of $f : \mathbb{T} \rightarrow \mathbb{R}$ provided $F^\Delta(t) = f(t)$ for all $t \in \mathbb{T}$. Then we define

$$\int_r^s f(t) \Delta t = F(s) - F(r) \quad \text{for } r, s \in \mathbb{T}.$$

Definition 2.3. A function $f : \mathbb{T} \rightarrow \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in \mathbb{T} and its left-sided limits exist(finite) at left-dense points in \mathbb{T} . The set of rd-continuous functions $f : \mathbb{T} \rightarrow \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T})$.

Lemma 2.4. Every rd-continuous function has an antiderivative.

Lemma 2.5. If $a, b \in \mathbb{T}$, $\alpha, \beta \in \mathbb{R}$ and $f, g \in C_{rd}(\mathbb{T})$, then

- (a) $\int_a^b [\alpha f(t) + \beta g(t)] \Delta t = \alpha \int_a^b f(t) \Delta t + \beta \int_a^b g(t) \Delta t$;
- (b) if $f(t) \geq 0$ for all $a \leq t < b$, then $\int_a^b f(t) \Delta t \geq 0$;
- (c) if $|f(t)| \leq g(t)$ on $[a, b] := \{t \in \mathbb{T} : a \leq t < b\}$, then $|\int_a^b f(t) \Delta t| \leq \int_a^b g(t) \Delta t$.

Lemma 2.6. ([11]) Let $t_1, t_2 \in I_\omega$ and $t \in \mathbb{T}$. If $g : \mathbb{T} \rightarrow \mathbb{R} \in C_{rd}(\mathbb{T})$ is ω -periodic, then

$$g(t) \leq g(t_1) + \frac{1}{2} \int_k^{k+\omega} |g^\Delta(s)| \Delta s$$

and

$$g(t) \geq g(t_2) - \frac{1}{2} \int_k^{k+\omega} |g^\Delta(s)| \Delta s,$$

the constant factor $\frac{1}{2}$ is the best possible.

For simplicity, we use the following notations throughout this paper. Let \mathbb{T} be ω -periodic, that is $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$,

$$k = \min\{\mathbb{R}^+ \cap \mathbb{T}\}, \quad I_\omega = [k, k + \omega] \cap \mathbb{T}, \quad g^L = \inf_{t \in \mathbb{T}} g(t),$$

$$g^M = \sup_{t \in \mathbb{T}} g(t), \quad \bar{g} = \frac{1}{\omega} \int_{I_\omega} g(s) \Delta s = \frac{1}{\omega} \int_k^{k+\omega} g(s) \Delta s,$$

where $g \in C_{rd}(\mathbb{T})$ is an ω -periodic real function, i.e., $g(t + \omega) = g(t)$ for all $t \in \mathbb{T}$.

Now, we introduce some concepts and a useful result from [10].

Let X, Z be normed vector spaces, $L : \text{Dom } L \subset X \rightarrow Z$ be a linear mapping, $N : X \rightarrow Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \ker L = \text{codim Im } L < +\infty$ and $\text{Im } L$ is closed in Z . If L is a Fredholm mapping of index zero and there exist continuous projections $P : X \rightarrow X$ and $Q : Z \rightarrow Z$ such that $\text{Im } P = \ker L$, $\text{Im } L = \ker Q = \text{Im}(I - Q)$, then it follows that $L|_{\text{Dom } L \cap \ker P} : (I - P)X \rightarrow \text{Im } L$ is invertible. We denote the inverse of that map by K_P . If Ω is an open bounded subset of X , the mapping N will be called L -compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \rightarrow X$ is compact. Since $\text{Im } Q$ is isomorphic to $\ker L$, there exists an isomorphism $J : \text{Im } Q \rightarrow \ker L$.

Next, we state the Mawhin's continuation theorem, which is a main tool in the proof of our theorem.

Lemma 2.7. (Continuation Theorem) Let L be a Fredholm mapping of index zero and N be L -compact on $\bar{\Omega}$. Suppose

- (a) for each $\lambda \in (0, 1)$, every solution u of $Lu = \lambda Nu$ is such that $u \notin \partial\Omega$;
- (b) $QN u \neq 0$ for each $u \in \partial\Omega \cap \ker L$ and the Brouwer degree $\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0$.

Then the operator equation $Lu = Nu$ has at least one solution lying in $\text{Dom } L \cap \bar{\Omega}$.

III. EXISTENCE OF PERIODIC SOLUTIONS

Theorem 3.1. If the following assumption holds,

$$\bar{r}_1 m^2 - \bar{a}_{12} e^{M_2} > 0,$$

where $M_2 = \ln \frac{\bar{r}_2 \bar{r}_1}{\bar{a}_{21} \bar{a}_{11}} + \omega(\bar{r}_1 + \bar{r}_2)$, then (4) has at least one ω -periodic solution.

Proof: Let $X = Z = \{(u_1, u_2)^T \in C(\mathbb{T}, \mathbb{R}^2) : u_i(t + \omega) = u_i(t), i = 1, 2, \forall t \in \mathbb{T}\}$, $\|(u_1, u_2)^T\| = \sum_{i=1}^2 \max_{t \in I_\omega} |u_i(t)|$, $(u_1, u_2)^T \in X$ (or in Z).

Then X and Z are both Banach spaces when they are endowed with the above norm $\|\cdot\|$.

Let

$$N \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} r_1(t) - a_{11}(t)e^{u_1(t-\tau(t))} - \frac{a_{12}(t)e^{u_2(t)}}{m^2 + e^{u_1^2(t)}} \\ r_2(t) - \frac{a_{21}(t)e^{u_2(t)}}{e^{u_1(t-\sigma(t))}} \end{bmatrix},$$

$$L \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1^\Delta \\ u_2^\Delta \end{bmatrix},$$

$$P \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = Q \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega} \int_k^{k+\omega} u_1(t) \Delta t \\ \frac{1}{\omega} \int_k^{k+\omega} u_2(t) \Delta t \end{bmatrix}.$$

Obviously, $\ker L = \{(u_1, u_2)^T \in X : (u_1(t), u_2(t))^T = (h_1, h_2)^T \in \mathbb{R}^2, t \in \mathbb{T}\}$, $\text{Im } L = \{(u_1, u_2)^T \in Z : \bar{u}_1 = \bar{u}_2 = 0, t \in \mathbb{T}\}$, $\dim \ker L = 2 = \text{codim Im } L$. Since $\text{Im } L$ is closed in Z , then L is a Fredholm mapping of index zero. It is easy to show that P and Q are continuous projections such that $\text{Im } P = \ker L$ and $\text{Im } L = \ker Q = \text{Im}(I - Q)$. Furthermore, the generalized inverse (of L) $K_P : \text{Im } L \rightarrow \ker P \cap \text{Dom } L$ exists and is given by

$$K_P \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \int_k^t u_1(s) \Delta s - \frac{1}{\omega} \int_k^{k+\omega} \int_k^t u_1(s) \Delta s \Delta t \\ \int_k^t u_2(s) \Delta s - \frac{1}{\omega} \int_k^{k+\omega} \int_k^t u_2(s) \Delta s \Delta t \end{bmatrix}.$$

Thus

$$QN \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega} \int_k^{k+\omega} \left(r_1(t) - a_{11}(t)e^{u_1(t-\tau(t))} - \frac{a_{12}(t)e^{u_2(t)}}{m^2 + e^{u_1^2(t)}} \right) \Delta t \\ \frac{1}{\omega} \int_k^{k+\omega} \left(r_2(t) - \frac{a_{21}(t)e^{u_2(t)}}{e^{u_1(t-\sigma(t))}} \right) \Delta t \end{bmatrix},$$

and

$$K_P(I - Q)N \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \int_k^t u_1(s) \Delta s - \frac{1}{\omega} \int_k^{k+\omega} \int_k^t u_1(s) \Delta s \Delta t - \left(t - k - \frac{1}{\omega} \int_k^{k+\omega} (t - k) \Delta t \right) \bar{u}_1 \\ \int_k^t u_2(s) \Delta s - \frac{1}{\omega} \int_k^{k+\omega} \int_k^t u_2(s) \Delta s \Delta t - \left(t - k - \frac{1}{\omega} \int_k^{k+\omega} (t - k) \Delta t \right) \bar{u}_2 \end{bmatrix}.$$

Clearly, QN and $K_P(I - Q)N$ are continuous. According to Arzela-Ascoli theorem, it is not difficulty to show that $K_P(I - Q)N(\bar{\Omega})$ is compact for any open bounded set $\Omega \subset X$ and $QN(\bar{\Omega})$ is bounded. Thus, N is L -compact on $\bar{\Omega}$.

Now, we shall search an appropriate open bounded subset Ω for the application of the continuation theorem, Lemma 2.7.

For the operator equation $Lu = \lambda Nu$, where $\lambda \in (0, 1)$, we have so we have

$$\begin{cases} u_1^\Delta(t) = \lambda \left(r_1(t) - a_{11}(t)e^{u_1(t-\tau(t))} - \frac{a_{12}(t)e^{u_2(t)}}{m^2 + e^{u_1^2(t)}} \right), \\ u_2^\Delta(t) = \lambda \left(r_2(t) - \frac{a_{21}(t)e^{u_2(t)}}{e^{u_1(t-\sigma(t))}} \right). \end{cases} \quad (5)$$

Assume that $(u_1, u_2)^T \in X$ is a solution of (5) for a certain $\lambda \in (0, 1)$. Integrating (5) on both sides from k to $k + \omega$, we obtain

$$\begin{cases} \bar{r}_1\omega = \int_k^{k+\omega} a_{11}(t)e^{u_1(t-\tau(t))} \Delta t + \int_k^{k+\omega} \frac{a_{12}(t)e^{u_2(t)}}{m^2 + e^{u_1^2(t)}} \Delta t, \\ \bar{r}_2\omega = \int_k^{k+\omega} \frac{a_{21}(t)e^{u_2(t)}}{e^{u_1(t-\sigma(t))}} \Delta t. \end{cases} \quad (6)$$

Since $(u_1, u_2)^T \in X$, there exist $\xi_i, \eta_i \in [k, k + \omega]$, $i = 1, 2$, such that

$$u_i(\xi_i) = \min_{t \in [k, k+\omega]} \{u_i(t)\}, u_i(\eta_i) = \max_{t \in [k, k+\omega]} \{u_i(t)\}. \quad (7)$$

From (5) and (6), we have

$$\begin{aligned} \int_k^{k+\omega} |u_1^\Delta(t)| \Delta t &< \bar{r}_1\omega + \int_k^{k+\omega} a_{11}(t)e^{u_1(t-\tau(t))} \Delta t \\ &\quad + \int_k^{k+\omega} \frac{a_{12}(t)e^{u_2(t)}}{m^2 + e^{u_1^2(t)}} \Delta t = 2\bar{r}_1\omega, \\ \int_k^{k+\omega} |u_2^\Delta(t)| \Delta t &< \bar{r}_2\omega + \int_k^{k+\omega} \frac{a_{21}(t)e^{u_2(t)}}{e^{u_1(t-\sigma(t))}} \Delta t = 2\bar{r}_2\omega. \end{aligned}$$

From the first equation of (6) and (7), we have

$$\bar{r}_1\omega > \bar{a}_{11}\omega e^{u_1(\xi_1)},$$

and

$$u_1(\xi_1) < \ln \frac{\bar{r}_1}{\bar{a}_{11}} := l_1,$$

thus,

$$u_1(t) \leq u_1(\xi_1) + \frac{1}{2} \int_k^{k+\omega} |u_1^\Delta(t)| \Delta t < \ln \frac{\bar{r}_1}{\bar{a}_{11}} + \bar{r}_1\omega := M_1.$$

On the other hand, from the second equation of (6) and (7), we have

$$\bar{r}_2\omega \geq \bar{a}_{21}\omega \frac{e^{u_2(\xi_2)}}{e^{M_1}},$$

and

$$u_2(\xi_2) \leq \ln \frac{\bar{r}_2 e^{M_1}}{\bar{a}_{21}} := l_2,$$

so,

$$u_2(t) \leq u_2(\xi_2) + \frac{1}{2} \int_k^{k+\omega} |u_2^\Delta(t)| \Delta t \leq \ln \frac{\bar{r}_2 e^{M_1}}{\bar{a}_{21}} + \bar{r}_2\omega := M_2.$$

By the first equation of (6) and (7),

$$\bar{r}_1\omega \leq \bar{a}_{11}\omega e^{u_1(\eta_1)} + \bar{a}_{12}\omega \frac{e^{M_2}}{m^2},$$

and

$$u_1(\eta_1) \geq \ln \frac{\bar{r}_1 m^2 - \bar{a}_{12} e^{M_2}}{\bar{a}_{11} m^2} := L_1,$$

$$\begin{aligned} u_1(t) &\geq u_1(\eta_1) - \frac{1}{2} \int_k^{k+\omega} |u_1^\Delta(t)| \Delta t \\ &\geq \ln \frac{\bar{r}_1 m^2 - \bar{a}_{12} e^{M_2}}{\bar{a}_{11} m^2} - \bar{r}_1\omega \\ &:= M_3. \end{aligned}$$

From the second equation of (6) and (7), we have

$$\bar{r}_2\omega \leq \bar{a}_{21}\omega e^{u_2(\eta_2) - M_3},$$

and

$$u_2(\eta_2) \geq \ln \frac{\bar{r}_2 e^{M_3}}{\bar{a}_{21}} := L_2,$$

thus,

$$u_2(t) \geq u_2(\eta_2) - \frac{1}{2} \int_k^{k+\omega} |u_2^\Delta(t)| \Delta t \geq \ln \frac{\bar{r}_2 e^{M_3}}{\bar{a}_{21}} - \bar{r}_2\omega := M_4.$$

So, we have

$$\begin{aligned} \max_{t \in [k, k+\omega]} |u_1(t)| &\leq \max\{|M_1|, |M_3|\} := R_1, \\ \max_{t \in [k, k+\omega]} |u_2(t)| &\leq \max\{|M_2|, |M_4|\} := R_2. \end{aligned}$$

Clearly, R_1 and R_2 are independent of λ . Let $R = R_1 + R_2 + R_0$, where R_0 is taken sufficiently large such that $R_0 \geq |l_1| + |l_2| + |L_1| + |L_2|$. Now, we consider the algebraic equations:

$$\begin{cases} \bar{r}_1 - \bar{a}_{11}e^x - \frac{\bar{a}_{12}e^y}{m^2 + e^{x^2}} = 0, \\ \bar{r}_2 - \bar{a}_{21}e^{y-x} = 0, \end{cases} \quad (8)$$

every solution $(x^*, y^*)^T$ of (8) satisfies $\|(x^*, y^*)^T\| < R$. Now, we define $\Omega = \{(u_1(t), u_2(t))^T \in X, \|(u_1(t), u_2(t))^T\| < R\}$. Then it is clear that Ω verifies the requirement (a) of Lemma 2.7. If $(u_1(t), u_2(t))^T \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^2$, then $(u_1(t), u_2(t))^T$ is a constant vector in \mathbb{R}^2 with $\|(u_1(t), u_2(t))^T\| = |u_1| + |u_2| = R$, so we have

$$QN \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

By direct computation, we can obtain $\deg(JQN, \Omega \cap \ker L, 0) = 1 \neq 0$. By now, we have verified that Ω fulfills all requirements of Lemma 2.7; therefore, (4) has at least one ω -periodic solution in $\text{Dom } L \cap \bar{\Omega}$. The proof is complete. ■

Remark 3.2. If $\mathbb{T} = \mathbb{R}$, then (2) is the special case of (4). So our result is more general than that of [1]. Further, If $\mathbb{T} = \mathbb{Z}$, then the existence of periodic solution for system (3) can be obtained.

Remark 3.3. By Theorem 3.1, we know that (4) has at least one periodic solution with the same period as the parameters under certain condition. Besides, time delays do not change the periodicity of the dynamic equations.

ACKNOWLEDGMENT

This work was supported by Anhui Province Natural Science Foundation (No.090416222) and Natural Science Research Project of colleges and universities in Anhui Province (No.KJ2009B076Z).

REFERENCES

- [1] Xiaohua Ding, Chun Lu and Mingzhu Liu, Periodic solutions for a semi-ratio-dependent predator-prey system with nonmonotonic functional response and time delay, *Nonl. Anal: RWA*, 9(2008), 762–775.
- [2] Haifeng Huo, Periodic solutions for a semi-ratio-dependent predator-prey system with functional responses, *Applied Mathematics Letters*, 18(2005), 313–320.
- [3] Qian Wang, Meng Fan and Ke Wang, Dynamics of a class of nonautonomous semi-ratio-dependent predator-prey systems with functional responses, *J. Math. Anal. Appl.*, 278(2003), 443–451.
- [4] Pingzhu Liu and K. Gopalsamy, Global stability and chaos in a population model with piecewise constant arguments, *Applied Mathematics and Computation*, 101(1999), 63–88.
- [5] Stefan Hilger, Analysis on measure chains—a unified approach to continuous and discrete calculus, *Results Math.*, 18(1990), 18–56.
- [6] Martin Bohner, Meng Fan and Jiming Zhang, Existence of periodic solutions in predator-prey and competition dynamic systems, *Nonl. Anal: RWA*, 7(2006), 1193–1204.
- [7] Martin Bohner, Meng Fan and Jiming Zhang, Periodicity of scalar dynamic equations and applications to population models, *J. Math. Anal. Appl.*, 330(2007), 1–9.
- [8] Jie Liu, Yongkun Li and Lili Zhao, On a periodic predator-prey system with time delays on time scales, *Comm Nonlinear Sci Numer Simulat*, 14(2009), 3432–3438.
- [9] Martin Bohner and Allan Peterson, *Dynamic Equations on Time Scales: An Introduction with Applications*, Boston: Birkhäuser, 2001.
- [10] R. E. Gaines and J. L. Coincidence Degree and Nonlinear Differential Equations, *Lecture Notes in Mathematics*, Berlin: Springer-Verlag, 1977.
- [11] Bingbing Zhang and Meng Fan, A remark on the application of coincidence degree to periodicity of dynamic equations on time scales, *J. Northeast Normal University(Natural Science Edition)*, 39(2007), 1–3.[in Chinese]