# An Efficient Method for Load–Flow Solution of Radial Distribution Networks

Smarajit Ghosh , Karma Sonam Sherpa

*Abstract*—This paper reports a new and accurate method for load–flow solution of radial distribution networks with minimum data preparation. The node and branch numbering need not to be sequential like other available methods. The proposed method does not need sending–node, receiving–node and branch numbers if these are sequential. The proposed method uses the simple equation to compute the voltage magnitude and has the capability to handle composite load modelling. The proposed method uses the set of nodes of feeder, lateral(s) and sub lateral(s). The effectiveness of the proposed method is compared with other methods using two examples. The detailed load–flow results for different kind of load–modellings are also presented.

*Keywords*—Load–flow, Feeder, Lateral, Power, Voltage, Composite, Exponential

## I INTRODUCTION

THE exact electrical performance and power flows of the system operating under steady state is required in efficient way known load–flow study that provides the real and reactive power losses of the system and voltages at different nodes of the system. With the growing market in the present time, effective planning can only be assured with the help of efficient load–flow study. The distribution network is radial in nature having high R/X ratio whereas the transmission system is loop in nature having high X/R ratio. Therefore, the variables for the load–flow analysis of distribution systems are different from that of transmission systems. The distribution networks are known as ill–conditioned. The conventional Gauss Seidel (GS) and Newton Raphson (NR) method does not converge for the distribution networks. A number of efficient load–flow methods for transmission systems are available in literature. A few methods had been reported in literature for load–flow analysis of distribution systems. The analysis of distribution systems is an important area of activity as distribution systems is the final link between a bulk power system and consumers [1–3].

Smarajit Ghosh is with the Thapar University, Department of Electrical & Instrumentation Engineering, Patiala, Punjab–147004, India
e-mail:smarajitg@hotmail.com
K.S.Sherpa is with Sikkim Manipal Institute of Technology under Sikkim Manipal University, Department of Electrical & Electronics Engineering, Maitar, Rangpo, Esat Sikkim, India.  e-mail: sonam_sp@yahoo.co.in

The methods proposed in [4,5] were very time consuming and increased the complexity. Kersting and Mendive [6] and Kersting [7] proposed a load–flow technique for solving radial distribution networks by updating voltages and currents using the backward and forward sweeps with the help of ladder–network theory. Stevens *et al.* [8] showed that the method proposed in [6,7] became fastest but could not converge in five out of twelve cases studied. Shirmohammadi *et al.* [9] proposed a method for solving radial distribution networks with the help of direct voltage application of Kirchoff's laws and presented a branch–numbering scheme to enhance numerical performance of the solution method. They also extended their method for solving the weakly meshed distribution networks. Their method needs a rigorous data preparation. Baran and Wu [10] developed the load–flow solution of radial distribution networks by iterative solution of three fundamental equations representing the real power, reactive power and voltage magnitude. Renato [11] proposed one method for obtaining load–flow solution of radial distribution networks computing the electrical equivalent for each node summing all the loads of the network fed through the node including losses and then starting from the source node, voltage of each receiving–end node was computed. Chiang [12] presented three different algorithms for solving radial distribution networks based on the method of Baran and Wu [10]. Goswami and Basu [13] proposed an approximate method for solving radial and meshed distribution networks where any node in the network could not be the junction of more than three branches i.e., one incoming and two outgoing. They had used sequential branch and node numbering scheme. Jasmon and Lee [14] developed a load–flow method for obtaining the load–flow solution of radial distribution networks using the three fundamental equations representing the real power, reactive power and voltage magnitude that had been proposed by Baran and Wu [10]. Das *et al.* [15] proposed a load–flow method using power convergence with the help of coding at the lateral and sub lateral nodes. For large system that increased complexity of computation. Their method worked only for sequential branch and node numbering scheme. They had calculated voltage of each receiving–end node using forward sweep. They had taken the initial guess of zero initial power loss. Rahaman *et al.* [16] proposed a method for the improved load–flow solution of radial distribution networks. They had proposed a voltage equation of the order

of four. Ghosh and Das [17] presented a load–flow method for solving radial distribution networks based on the technique with nodes beyond branches using voltage convergence. They had considered flat voltage start. They had shown proof of convergence and also shown that incorporation of charging admittances reduces losses and improves voltage profile. The main draw back of this method was that it stores nodes beyond each branch. This method calculated current for each branch by adding load currents of nodes beyond the respective branch. Jamali *et al.* [18] presented a load–flow technique based on sequential branch numbering scheme to design distribution network by considering committed loads. Aravindhababu *et al.* [19] had shown a simple and efficient branch-to-node matrix-based power flow (BNPF) for radial distribution systems and this method was unsuitable for extension to optimal power flow for which the NR method seems to be more appropriate. In that method any presence of sub laterals complicates the matrix formation. Mekhamer *et al.* [20] developed a method for load–flow solution of radial distribution networks using terminal conditions. Afsari *et al.* [21] proposed a load–flow method based on estimation of node voltage and assuming the loads of the nodes of lateral and their sub lateral are concentrated at the originating node of the feeder. They had tried to reduce the computation time only. But the computation becomes very complex when the number of laterals and sublaterals increases. Ranjan *et al.* [22] proposed a new load–flow technique using power convergence characteristic. They had calculated voltage of each node using forward sweep by the same voltage expression available in reference [15]. They had calculated the total power flow of each branch that is fed to the receiving–end node of that branch. Their method also needed the storage of nodes beyond each branch. They also claimed that their algorithm could easily accommodate the composite load modeling if composition of load was known. The main disadvantage of this method was that their method needed a repetitive search for connection of receiving–end node of each branch with other nodes. In their method, they claimed that the proposed method worked for arbitrary node numbering but remained silent regarding the branch numbering scheme. Chakraborty and Das [23] had stated that the power convergence has the capability to handle composite load modeling. Ranjan *et al.* [24] had used the voltage convergence to handle the different composition of load for the same example used in refernce [23]. All the proposed methods need branch number, sending–end node and receiving–end node. The methods proposed in [13,15] needed sequential numbering scheme. In the all the proposed methods, the examples used were with sequential numbering scheme.

The main aim of the authors is to reduce the data preparation and to assure computation for any type of numbering scheme for node and branch. If the nodes and branch numbers are sequential, the proposed method needs only the starting node of feeder, each of lateral and each of sub lateral only. The proposed method needs only the set of nodes and branch numbers of feeder, each of laterals and each of sub

laterals only when node and branch numbers are not sequential. The proposed method computes branch power flow most efficiently and does not need to store nodes beyond each branch. The voltage of each node is calculated by using a simple algebraic equation. Although the present method is based on the forward sweep, it computes efficient load–flow of any complicated radial distribution networks very efficiently even when branch and node numbering scheme are not sequential. The proposed method needs minimum data preparation compared to other methods. Two examples (33–node and 69–node radial distribution networks) with constant power (CP), constant current (CI), constant impedance (CZ), composite and exponential load modellings for each of these examples are considered. The proposed method is compared with other existing methods [15,17,22] . The initial voltage of all nodes is taken $1+j0$ and initial power loss of all branches are also taken zero.

## II. ASSUMPTIONS

It is assumed that three-phase radial distribution networks are balanced and represented by their single-line diagrams and charging capacitances are neglected at the distribution voltage levels.

## III. SOLUTION METHODOLOGY

A single line diagram of a radial distribution network is shown in Fig. 1 with sequential numbering.

In Fig. 1, the node and branch numbering scheme have been shown sequential. From Fig. 1, set of nodes of feeder, lateral and sub lateral are FN={1,2,3,4,5,6}, LN={3,7,8} and SLN={7,9,10}respectively. In Fig. 1 the set of branch number of feeder are FB = {1,2,3,4,5}, LB={6,7} and SLB = {8,9} respectively.
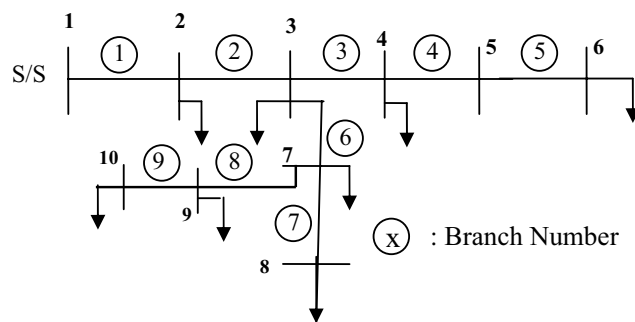


Fig. 1 Single-line diagram of a radial distribution network

Fig. 2 shows when the node and branch numbering scheme are not sequential. From Fig. 2, set of nodes of feeder, lateral and sub lateral are FN={1,6,4,8,10,2}, LN={4,9,3} and SLN={9,7,5} respectively. In Fig. 1 the set of branch number of feeder are FB = {1,7,3,9,5}, LB={6,2} and SLB = {8,4} respectively.
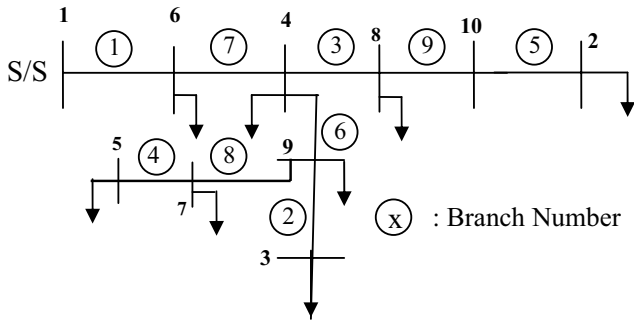
Fig. 2 without sequential numbering scheme

From Fig. 1 and Fig. 2, the sub lateral has two branches, the lateral has two branches and the feeder has five branches. Let the feeder is denoted by 1, lateral by 2 and sub lateral by 3 in Fig. 1 and Fig. 2.

Here the two dimensional array FN denotes the node of feeder, each lateral and each sub lateral where the first number of the array indicates feeder, lateral and sub lateral. At first feeder is kept, then lateral and sub lateral. The second number denotes the order of the node of the set. From Fig. 1, the nodes of feeder, lateral and sub lateral are shown below.

$FN(1,1) = 1$, $FN(1,2) = 2$, $FN(1,3) = 3$, $FN(1,4) = 4$, $FN(1,5) = 5$ and $FN(1,6) = 6$

$FN(2,1) = 3$, $FN(2,2) = 7$ and $F(2,3) = 8$ and $FN(3,1) = 7$, $FN(3,2) = 9$ and $F(3,3) = 10$.

From Fig. 1, the branches of feeder, lateral and sub lateral are shown below.

$FB(1,1) = 1$, $FB(1,2) = 2$, $FB(1,3) = 3$, $FB(1,4) = 4$ and $FB(1,5) = 5$

$FB(2,1) = 6$ and $FB(2,2) = 7$ and $FB(3,1) = 8$ and $FB(3,2) = 9$.

From Fig. 2, the nodes of feeder, lateral and sub lateral are shown below.

$FN(1,1) = 1$, $FN(1,2) = 6$, $FN(1,3) = 4$, $FN(1,4) = 8$, $FN(1,5) = 10$ and $FN(1,6) = 2$

$FN(2,1) = 4$, $FN(2,2) = 9$ and $F(2,3) = 3$ and $FN(3,1) = 9$, $FN(3,2) = 7$ and $F(3,3) = 5$.

From Fig. 2, the branches of feeder, lateral and sub lateral shown below.

$FB(1,1) = 1$, $FB(1,2) = 7$, $FB(1,3) = 3$, $FB(1,4) = 9$ and $FB(1,5) = 5$

$FB(2,1) = 6$ and $FB(2,2) = 2$ and $FB(3,1) = 8$ and $FB(3,2) = 4$.

Let $jj = FB(i,j)$, $m2 = FN(i,j+1)$ and $m1 = FN(i,j)$. We have

$$V(m2) = V(m1) - I(jj)Z(jj) \qquad (1)$$

Let $V(m2) = |V(m2)| \angle \delta_2$

$V(m1) = |V(m1)| \angle \delta_1$

$Z(jj) = |Z(jj)| \angle \varphi = R(jj) + jX(jj)$ and

$I(jj) = |I|(jj) \angle -\theta$

Voltage of node m2 is expressed by

$$|V(m2)| = |V(m1)| - \frac{\left[ P_s^2(jj) + Q_s^2(jj) \right]^{\frac{1}{2}} |Z(jj)|}{|V(m1)|} \qquad (2)$$

where $P_s(jj)$ and $Q_s(jj)$ are the real and reactive powers coming out from the node m1. The detailed derivation has been shown in Appendix−A. Voltage of node m2 can also be calculated using the following expression also:

$$|V(m2)| = \frac{|V(m1)| \pm \sqrt{|V(m1)|^2 - 4\sqrt{\{P_r^2(jj) + Q_r^2(jj)\}} |Z(jj)|}}{2} \qquad (3)$$

where $P_r(jj) = P_s(jj) - LP(jj)$ and $Q_r(jj) = Q_s(jj) - LQ(jj)$ are the real and reactive power fed through the node m2.

Equation (2) is used to calculate $|V(m2)|$ due to its simplicity.

The current through the branch−jj is expressed by

$$|I(jj)| = \frac{|V(m1)| - |V(m2)|}{|Z(jj)|} \qquad (4)$$

The real and reactive power loss of branch−jj is expressed by

$$LP(jj) = |I(jj)|^2 R(jj) \qquad (5)$$

and $LQ(jj) = |I(jj)|^2 X(jj) \qquad (6)$

$P_s(jj)$ = Sum of real power load of all nodes after the branch−jj plus the real power loss of all the branches after the branch−jj including the branch−jj also.

$Q_s(jj)$ = Sum of reactive power load of all the nodes after the branch−jj plus the reactive power loss of all the branches after the branch−jj including the branch−jj also.

To discuss the calculation of $P_s(jj)$ and $Q_s(jj)$, $P_s(jj)$ and $Q_s(jj)$ for sub lateral(s), lateral(s) and feeder are calculated at first with an assumption that they are separated.

For the sub lateral:

$$\left. \begin{array}{l} P_s[FB(3,2)] = PL[FN(3,3)] + LP[FB(3,2)] \\ P_s[FB(3,1)] = PL[FN(3,2)] + LP[FB(3,1)] + P_s[FB(3,2)] \end{array} \right\} \quad (7)$$

$$\left. \begin{array}{l} P_s[FB(2,2)] = PL[FN(2,3)] + LP[FB(2,2)] \\ P_s[FB(2,1)] = PL[FN(2,2)] + LP[FB(2,1)] + P_s[FB(2,2)] \end{array} \right\} \quad (8)$$

$$\left. \begin{array}{l} P_s[FB(1,5)] = PL[FN(1,6)] + LP[FB(1,5)] \\ P_s[FB(1,4)] = PL[FN(1,5)] + LP[FB(1,4)] + P_s[FB(1,5)] \\ P_s[FB(1,3)] = PL[FN(1,4)] + LP[FB(1,3)] + P_s[FB(1,4)] \\ P_s[FB(1,2)] = PL[FN(1,3)] + LP[FB(1,2)] + P_s[FB(1,3)] \\ P_s[FB(1,1)] = PL[FN(1,2)] + LP[FB(1,1)] + P_s[FB(1,2)] \end{array} \right\} \quad (9)$$

From (7), (8) and (9), we can conclude the following:

For the end branch

$$P_s[FB(i,j)] = PL[FN(i,j+1)] + LP[FB(i,j)] \qquad (10)$$

and for other branches,

$$P_s[FB(i,j)] = PL[FN(i,j+1)] + LP[FB(i,j)] + P_s[FB(i,j+1)] \qquad (11)$$

Equations (10) and (11) shows generalized expressions for the computation of $P_s$'s through the feeder, lateral and sub lateral when they are separated. Similarly, the following are the generalized expressions for $Q_s$'s:

For the end branch

$$Q_s[FB(i,j)] = QL[FN(i,j+1)] + LQ[FB(i,j)] \qquad (12)$$

and for other branches,

$$Q_s[FB(i,j)]=QL[FN(i,j+1)]+LQ[FB(i,j)]+Q_s[FB(i,j+1)] \quad (13)$$

Now from Fig. 1 and Fig. 2, we have the following:

Sub lateral is connected to lateral at the node F(2,2). Therefore, power flow through the branch FB(2,1) becomes

$$P_s[FB(2,1)] = PL[FN(2,2)] + LP[FB(2,1)]$$
$$+ P_s[FB(2,2)] + P_s[FB(3,1)] \qquad (14)$$

and $Q_s[FB(2,1)] = QL[FN(2,2)] + LQ[FB(2,1)]$
$$+ Q_s[FB(2,2)] + Q_s[FB(3,1)] \qquad (15)$$

The lateral is connected to feeder at the node F(1,3). Therefore, power flow through the branch FB(1,2) becomes

$$P_s[FB(1,2)] = PL[FN(1,3)] + LP[FB(1,2)]$$
$$+ P_s[FB(1,3)] + P_s[FB(1,1)] \qquad (16)$$

and $Q_s[FB(1,2)] = QL[FN(1,3)] + LQ[FB(1,2)]$
$$+ Q_s[FB(1,3)] + Q_s[FB(1,1)] \qquad (17)$$

From the above discussion, it can be concluded that the common nodes of among the sub lateral(s) and lateral(s) as well as that of feeder and lateral(s) must be marked at first. If FN(i,j) be the node of lateral which is the source node of the sub lateral also or be the node of feeder which is the source node of the lateral also, the branch number FB(i,j−1) is required to be stored.

The proposed logic checks the common nodes of lateral(s) and sub lateral(s) [ first node of the sub lateral(s)] and also stores the branch number. If the node FN(i,j) of the lateral and first node FN(x,1) of the sub lateral are identical, the branch FB(i,j−1) of the lateral to be stored in the memory say the variable mm[TN−1] where TN is the total number denoting the sum of numbers of feeder, lateral(s) and sub lateral(s) and the sub lateral number is also stored in the array mn[TN−1]. Here TN−1 shows the total memory size of the array. Similarly, the common nodes of lateral(s) and feeder are found out and the branch number of the feeder corresponding to the common node of feeder and lateral are stored in mm[TN−1] and simultaneously lateral number is stored in mn[TN−].

The branches of lateral(s) and feeder(s) are checked with the branches stored in the array mm[TN−1]. If any branch number of lateral and feeder matches with any element of mm[TN−1], say the branch number of FB(i,j) matched with mm[2], the $P_s$ and $Q_s$ for the branch FB(i,j) will be

$$P_s[FB(i,j)] = PL[FN(i,j+1)] + LP[FB(i,j)]$$
$$+ P_s[FB(i,j+1)] + P_s[FB(mn[2],1)] \qquad (18)$$

and $Q_s[FB(i,j)] = QL[FN(i,j+1)] + LQ[FB(i,j)]$
$$+ Q_s[FB(i,j+1)] + Q_s[FB(mn[2],1)] \qquad (19)$$

where mn[2] is the number of lateral or sub lateral depending of the value of i.

From above discussion, it is clear that the proposed method does not depend upon the node and branch numbering. To make the computation of $P_s$ and $Q_s$ faster, the logic used in the proposed method is described below:

Step 1 : Get the number of Feeder(A), lateral(s) (B) and sub lateral(s) (C).

Step 2 : TN = A + B + C

Step 3 : Read total number of nodes of feeder, each

lateral and sub lateral respectively i.e., N(i) for i = 1,2,…..,TN.

Step 4 : Get the status of numbering scheme.

Step 5 : If it is sequential, ask for the starting node of feeder, each lateral and sub lateral respectively. Go to Step 7.

Step 6 : If it is not sequential, read the set of nodes as well as branches of feeder, each lateral and sub lateral respectively.

Step 7 : Find the common nodes of sub lateral(s) and lateral(s) i.e., FN(i,1) for i = TN to TN−C+1 from FN(i,j) for j = 1,2,…,N(i) and i = TN−C to TN−C−B. Store them in mm(i) for i = 1,2,..,C and store the branch of lateral FB(i,j−1) corresponding to the node FN(i,j) in mn(i) for i=1,2,..,C.

Step 8 : Find the common nodes of lateral(s) and Feeder i.e., FN(i,1) for i = TN−C to TN−C−B+1 from FN(1,j) for j = 1,2,…,N(1). Store them in mm(i) for i = C+1,..,C+B and the branch of feeder FB(i,j−1) corresponding to the node FN(i,j) in mn(i) for i = C+1,..,C+B.

Step 9 : Calculate $P_s[FB(i,j)]$ and $Q_s[FB(i,j)]$ for j = N(i)−1 ,…,2,1 and for i = TN to TN−C+1 using (10) or (11) and (12) or (13) respectively.

Step 10 : Calculate $P_s[FB(i,j)]$ and $Q_s[FB(i,j)]$ for j = N(i)−1 ,…,2,1 and for i = TN−C to TN−C−B+1 using (18) and (19) respectively with a check of FB(i,j) for j = N(i)−1 ,…,2,1 and for i = TN−C to TN−C−B+1 with mn(k) for k =1,2,..,C.

Step 11 : Calculate $P_s[FB(1,j)]$ and $Q_s[FB(1,j)]$ for j = N(i)−1 ,…,2,1 using (18) and (19) respectively with a check of FB(1,j) for j = N(i)−1 ,…,2,1 with mn(k) for k =C+1,…,C+B.

## IV. LOAD MODELLING

A balanced load that can be represented either as constant power, constant current, constant impedance or as an exponential load is considered here. The general expression of load is shown below.

$$P(m2) = P_n [a_0 + a_1V(m2) + a_2V^2(m2) + a_3V^{el}(m2)] \qquad (20)$$
$$Q(m2)= Q_n[b_0 + b_1V(m2) + b_2V^2(m2) + b_3V^{el}(m2)] \qquad (21)$$

where, $P_n$ and $Q_n$ are nominal real and reactive power respectively and V(m2) is the voltage at node m2.

For all the loads, ( 20) and (21) are modeled as

$$a_0 + a_1 + a_2 + a_3 = 1.0 \qquad (22)$$
$$b_0 + b_1 + b_2 + b_3 = 1.0 \qquad (23)$$

For constant power (CP) load $a_0 = b_0= 1$ and $a_i = b_i = 0$ for i = 1, 2, 3. For constant current (CI) load $a_1 = b_1= 1$ and $a_i = b_i = 0$ for i = 0, 2, 3. For constant impedance (CZ) load $a_2 = b_2= 1$ and $a_i = b_i = 0$ for i = 0, 1, 3. Composite load modelling

is combination of CP, CI and CZ. For exponential load $a_3 = b_3 = 1$ and $a_i = b_i = 0$ for $i = 0, 1, 2$ and e1 and e2 are 1.38 and 3.22 respectively [23].

## V. ALGORITHM FOR COMPUTATION OF LOAD–FLOW

To calculate the node voltages and branch currents and the total system loss, a initial guess of zero real and reactive power loss is assumed. Also flat voltage start is used. The convergence criteria is such that if $\text{Max}|V_{old}[FN(i,j)] - V_{New}[FN(i,j)]| < \varepsilon$, for $i = 1,2,..,TN$ and $j = 1,2,....,N(i)$=total number of nodes of FN(i). The following are the steps for load flow calculation:

Step 1 : Get the number of Feeder(A), lateral(s) (B) and sub lateral(s) (C).

Step 2 : TN = A + B + C

Step 3 : Read the total number of nodes N(i) of feeder, lateral(s) and sub lateral(s) for $i = 1,2,...,TN$

Step 4 : Read the nodes and branch numbers of feeder, lateral(s) and sub lateral(s) i.e., FN(i,j) for $j = 1,2,...,N(i)$ and $i = 1,2,....,TN$ if these are not sequential..

Step 5 : Read real and reactive power load at each node i.e., PL[FN(i,j)] and QL[FN(i,j)] for $j = 2,3,..,N(j)$ and $i = 1,2,..,TN$.

Step 6 : Initialize PL[FN(1,1)] = 0.0 and QL[FN(1,1)] = 0.0

Step 7 : Read the branches of feeder, lateral(s) and sub lateral(s) i.e., FB(i,j) for $j = 1,2,...,N(i) - 1$ and $i = 1,2,....,TN$.

Step 8 : Read resistance and reactance of each branch i.e., R[FB(i,j)] and X[FB(i,j)] for $j = 2,3,..,N(j) -1$ and $I = 1,2,..,TN$.

Step 9 : Read base kV and base MVA, Total number of iteration (ITMAX), $\varepsilon$ (0.00001)

Step 10 : Compute the per unit values of PL[FN(i,j)] and QL[FN(i,j)] for $j = 2,3,..,N(j)$ and $i = 1,2,..,TN$ as well as R[FB(i,j)] and X[FB(i,j)] for $j = 1,2,3,..,N(j) -1$ and $i = 1,2,..,TN$.

Step 11 : Set PL1[FN(i,j)] = PL[FN(i,j)] and QL1[FN(i,j)] = QL[FN(i,j)] for $j = 2,3,..,N(j)$ and $i = 1,2,..,TN$

Step 12 : Set LP[FB(i,j)] = 0.0 and LQ[FB(i,j)] = 0.0 for all $j = 1,2,...,N(i) -1$ and $i = 1,2,....,TN$.

Step 13 : Set V[FN(i,j)] = 1.0 + j0.0 for $j = 1,2,...,N(i)$ and $i = 1,2,....,TN$ and set V1[FN(i,j)] = V[FN(I,j)] for $j = 1,2,...,N(i)$ and $i = 1,2,....,TN$.

Step 14 : Use the Step7 to Step 11 (Art 3.0) to calculate the branch currents of each feeder, lateral(s) and sub lateral(s) respectively.

Step 15 : Set IT = 1

Step 16 : Set PL[FN(i,j)] = PL1[FN(i,j)] and QL[FN(i,j)] = QL1[FN(i,j)] for $j = 2,3,..,N(j)$ and $i = 1,2,..,TN$

Step 17 : Use proper load modeling using (20) and (21).

Step 18 : Compute voltage |V[FN(I,j)]| using (2) for $j = 2,3,..,N(j)$ and $i = 1,2,..,TN$.

Step 19 : Compute $|\Delta V[FN(i,j)]|$ = $|V1[FN(i,j)]| - |V[FN(i,j)]|$ for $j = 2,3,..,N(j)$ and $i = 1,2,..,TN$.

Step 20 : Compute current |I[FB(i,j)]| using (4) for $j = 1,2,3,..,N(j)-1$ and $i = 1,2,..,TN$.

Step 21 : Set |V1[FN(i,j)]| = |V[FN(i,j)]| for $j = 1,2,3,..,N(j)$ and $I = 1,2,..,TN$.

Step 21 : Compute LP[FB(i,j)] and LQ[FB(i,j)] for all $j = 1,2,...,N(i)-1$ and $i = 1,2,....,TN$ using (5) and (6) respectively.

Step 22 : Find $\Delta V_{max}$ from $|\Delta V[FN(i,j)]|$ for $j = 2,3,..,N(j)$ and $i = 1,2,..,TN$.

Step 23 : If $\Delta V_{min} \leq 0.00001$ go to Step 26 else go to Step 24.

Step 24 : IT = IT + 1

Step 25 : If IT $\leq$ ITMAX go to Step 16 else write "NOT CONVERGED" and go to Step 27.

Step 26 : Write " SOLUTION HAS CONVERGED" and display the results : Total Real and Reactive Power Losses , Voltages of each node, minimum value of voltage and its node number and total real and reactive power load for CP, CI, CZ, Composite and Exponential Load Modelling.

Step 27 : Stop

## VI. EXAMPLES

To demonstrate the effectiveness of the proposed method, the following two examples are considered here:

The first example is 33–node radial distribution network (nodes have been renumbered with Substation as node 1) shown in Fig. 3. Data for this system are available in [25]. Real and reactive power loss for CP, CI, CZ, Composite and Exponential load modeling as well as the minimum voltage and its node number is shown in Table 1. Base values for this system are 12.66 kV and 100 MVA respectively.
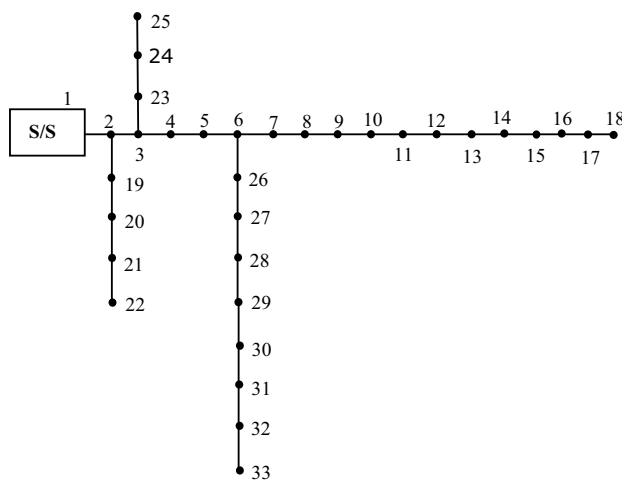


Fig. 3 33 Node Radial Distribution Network [25]

The second example is 69–node radial distribution network (nodes have been renumbered with Substation as node 1). Data for this system are available in [10]. Real and reactive power loss for CP, CI, CZ, Composite and Exponential load modeling as well as the minimum voltage and its node number is shown in Table 1. Base values for this system are 12.66 kV and 100 MVA respectively.
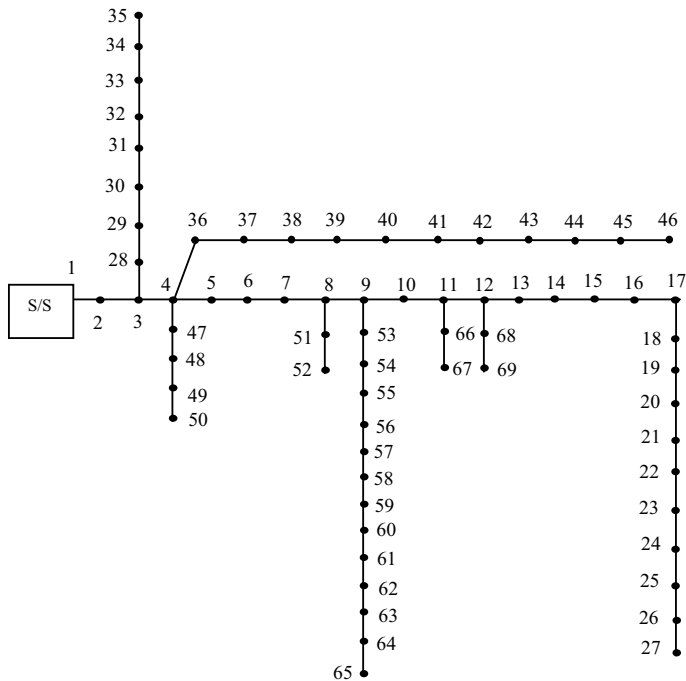


Fig. 4   69 node radial distribution network[10]

In all cases Composite Load = 40%CP + 30%CI + 30% CZ has been considered. Comparison of CPU time of the proposed method with the methods [15,17,22] is shown in Table 2.

TABLE I
REAL POWER LOSS, REACTIVE POWER LOSS, MINIMUM VOLTAGE FOR CP, CI, CZ, COMPOSITE AND EXPONENTIAL LOAD MODELLING FOR 33–NODE AND 69–NODE RESPECTIVELY

| Minimum Voltage | Type of Load | Total Load | | Power Loss | | Minimum Voltage (p.u.) |
|---|---|---|---|---|---|---|
| | | Real (kW) | Reactive (kVAr) | Real (kW) | Reactive (kVAr) | |
| 33–node radial distribution network [25] | CP | 3715.00 | 2300.00 | 202.30 | 135.020 | $V_{18} = 0.909924$ |
| | CI | 3534.84 | 2175.25 | 176.20 | 117.305 | $V_{18} = 0.916587$ |
| | CZ | 3366.20 | 2058.92 | 154.67 | 102.651 | $V_{18} = 0.922519$ |
| | Composite | 3559.37 | 2192.43 | 178.71 | 119.031 | $V_{18} = 0.915873$ |
| | Exponential | 3469.44 | 1927.52 | 155.43 | 103.172 | $V_{18} = 0.921396$ |
| 69–node radial distribution network [10] | CP | 3802.80 | 2693.07 | 225.00 | 102.095 | $V_{65} = 0.906755$ |
| | CI | 3622.08 | 2564.97 | 191.23 | 87.632 | $V_{65} = 0.914548$ |
| | CZ | 3455.58 | 2446.82 | 164.09 | 76.023 | $V_{65} = 0.921351$ |
| | Composite | 3647.60 | 2583.10 | 194.47 | 89.003 | $V_{65} = 0.913749$ |
| | Exponential | 3557.30 | 2314.36 | 165.87 | 76.779 | $V_{65} = 0.920887$ |

The comparison of relative CPU Time of the proposed method with the other existing methods [15,17,22] for constant power load modelling has been shown in Table II. All simulation works have been carried out in Celeron Processor 1GHz.

TABLE II
COMPARISON OF RELATIVE CPU TIMEOF THE PROPOSED METHOD WITH OTHER EXISTING  METHODS [15,17,22] FOR CONSTANT POWER LOAD  MODELING

| Methods \ Examples | Example 3 | Example 4 |
|---|---|---|
| | CPU Time | CPU Time |
| Proposed method | 1.00 | 1.00 |
| D.Das et al. [15] | 1.90 | 2.23 |
| S.Ghosh and D.Das [17] | 1.41 | 1.82 |
| Ranjan and D.Das [22] | 1.59 | 1.94 |

VII CONCLUSION

An efficient method for load–flow solution of radial distribution network has been proposed in this paper. The proposed method reduces the data preparation. The proposed method simply needs starting nodes of feeder, lateral(s) and sub lateral(s) and no data of branch numbers for sequential numbering scheme. If the node and branch numbers are not sequential, only node numbers and branch numbers of each feeder, lateral(s) and sub lateral(s) are required. Therefore, the proposed method consumes less computer memory. The proposed method uses the simple voltage equation. The proposed method takes the zero initial loss for computation of voltage of each node and considers flat voltage start to incorporate voltage convergence. The proposed method overcomes the shortfalls of the methods reported in [15,17,22]. Effectiveness of the proposed method has been demonstrated by two examples (33–node and 69–node radial distribution networks) with constant power load, constant current load, constant impedance load, composite load and exponential load for each of these examples. The efficiency of the proposed method in terms of CPU time has been checked by comparing it with the other existing methods [15,17,22].

From the proposed voltage equation a suitable stability index can also be formed.

## APPENDIX

Let $FB(i,j) = jj$ , $FN(i,j) = m1$ and $FN(i,j+1) = m2$. Therefore, we have

$$V(m2) = V(m1) - I(jj)Z(jj) \qquad (A1)$$

Let $V(m2) = |V(m2)| \angle \delta_2$

$\quad V(m1) = |V(m1)| \angle \delta_1$

$\quad Z(jj) = |Z(jj)| \angle \varphi = R(jj) + jX(jj)$ and

$\quad I(jj) = |I|(jj) \angle -\theta$

Therefore, (A1) becomes

$\therefore |V(m2)| \, (\cos\delta_2 + j\sin\delta_2)$

$\quad = |V(m1)| \, (\cos\delta_1 + j\sin\delta_1) - |I| \, (\cos\theta + j\sin\theta) \, \{R(jj) + jX(jj)\}$

i.e., $|V(m2)|\cos\delta_2 + j|V(m2)|\sin\delta_2$

$\quad = |V(m1)| \, \cos\delta_1 \, - |I(jj)| \{R(jj)\cos\theta + X(jj)\sin\theta \}$

$\quad + j|V(m1)| \, \sin\delta_1 \, - |I(jj)| \{(X(jj)\cos\theta - R(jj)\sin\theta )\}$

$\therefore |V(m2)| \, \cos\delta_2$

$\quad = |V(m1)| \, \cos\delta_1 \, - |I(jj)| \{R(jj)\cos\theta + X(jj)\sin\theta \} \qquad (A2)$

and $\dfrac{|V(m2)| \, \sin\delta_2}{}$

$\quad = |V(m1)| \, \sin\delta_1 \, - |I(jj)| \{X(jj)\cos\theta - R(jj)\sin\theta \} \qquad (A3)$

From (A2) and (A3), we have

$\therefore |V(m2)|^2 = |V(m1)|^2 \, - 2|V(m1)||I(jj)|\cos\delta_1\{R(jj)\cos\theta + X(jj)\sin\theta \}$

$\quad + |I(jj)|^2 \{R^2(jj) + X^2(jj)\}$

$\quad - 2|V(m1)| \, |I(jj)| \sin\delta_1 \{X(jj)\cos\,\theta - R(jj)\sin\,\theta \}$

$= |V(m1)|^2 \, - 2|V(m1)||I(jj)| \{R(jj)(\cos\delta_1\cos\theta - \sin\delta_1\sin\theta )$

$\quad + X(jj)(\cos\delta_1\sin\theta + \sin\delta_1\cos\theta )\}$

$\quad + |I(jj)|^2 \{R^2(jj) + X^2(jj) \}$

$= |V(m1)|^2 \, - 2|V(m1)||I(jj)| \{R(jj)(\cos(\delta_1 + \theta) + X(jj)\sin(\delta_1 + \theta)\}$

$\quad + |I(jj)|^2 |Z(jj)|^2$

$= |V(m1)|^2$

$\quad - 2|V(m1)||I(jj)||Z(jj)| \{\cos\varphi \, \cos(\delta_1 + \theta) + \sin\varphi \, \sin(\delta_1 + \theta)\}$

$\quad + |I(jj)|^2 |Z(jj)|^2$

$= |V(m1)|^2 \, - 2|V(m1)||I(jj)||Z(jj)| \cos(\varphi - \delta_1 - \theta)$

$\quad + |I(jj)|^2 |Z(jj)|^2$

Since, $\phi - \delta_1 - \theta$ is very very small and hence

$$\cos(\phi - \delta_1 - \theta) \approx 1$$

Therefore,

$\therefore |V(m2)|^2 = |V(m1)|^2 \, - 2|V(m1)||I(jj)|Z(jj)$

$\quad + |I(jj)|^2 |Z(jj)|^2$

i.e., $|V(m2)|^2 = \left[ |V(m1)| \, - |I(jj)||Z(jj)| \right]^2$

i.e., $|V(m2)| = |V(m1)| \, - |I(jj)||Z(jj)| \qquad (A4)$

Again, $|I(jj)| = \dfrac{\left[ P_r^2(jj) + Q_r^2(jj) \right]^{\frac{1}{2}}}{|V(m2)|} \qquad (A5)$

and also $|I(jj)| = \dfrac{\left[ P_s^2(jj) + Q_s^2(jj) \right]^{\frac{1}{2}}}{|V(m1)|} \qquad (A6)$

where $P_r(jj) = P_s(jj) - LP(jj)$ and $Q_r(jj) = Q_s(jj) - LQ(jj)$ are the real and reactive power fed through the node m2.

Using (A6), (A4) can be written as

$$|V(m2)| = |V(m1)| \, - \dfrac{\left[ P_s^2(jj) + Q_s^2(jj) \right]^{\frac{1}{2}} |Z(jj)|}{|V(m1)|} \qquad (A7)$$

Using (A5), (A4) can be written as

$$|V(m2)| = |V(m1)| \, - \dfrac{\left[ P_r^2(jj) + Q_r^2(jj) \right]^{\frac{1}{2}} |Z(jj)|}{|V(m2)|} \qquad (A8)$$

i.e., $|V(m2)|^2 = |V(m1)||V(m2)| \, - \left[ P_r^2(jj) + Q_r^2(jj) \right]^{\frac{1}{2}} |Z(jj)|$

i.e., $|V(m2)|^2 \, - |V(m1)||V(m2)| \, + \left[ P_r^2(jj) + Q_r^2(jj) \right]^{\frac{1}{2}} |Z(jj)| = 0$

i.e.,

$$|V(m2)| = \dfrac{|V(m1)| \pm \sqrt{|V(m1)|^2 \, - 4\sqrt{\{P_r^2(jj) + Q_r^2(jj)\}} |Z(jj)|}}{2} \qquad (A9)$$

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**Smarajit Ghosh** serving as a Professor in Thapar University, Department of Electrical and Instrumentation Engineering. He did his B.Tech., M.Tech. in Electrical Machines and Power Systems from Calcutta University in 1994 and 1996 respectively. Finally, he did his Ph.D. from Indian Institute of Technology, Khapargpur, India in 2000. His research areas include load–flow study, network reconfigurations, optimum capacitor allocation, application of soft computing in Elctrical Power Distribution Systems.

**Karma Sonam Sherpa** serving as a Reader in Sikkim Manipal Institute of Technology, Department of Electrical and Electronics Engineering under Sikkim Manipal University. He did his B.E. from Malviya Regional Engineering College, Jaipur in Electrical Engineering and M.Tech. in Power Electronics and Drives from Indian Institute of Technology, Kharagpur, India. Currently he is doing Ph.D. in Sikkim Manipal University.