

Development of Maximum Entropy Method for Prediction of Droplet-size Distribution in Primary Breakup Region of Spray

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Abstract—Droplet size distributions in the cold spray of a fuel are important in observed combustion behavior. Specification of droplet size and velocity distributions in the immediate downstream of injectors is also essential as boundary conditions for advanced computational fluid dynamics (CFD) and two-phase spray transport calculations. This paper describes the development of a new model to be incorporated into maximum entropy principle (MEP) formalism for prediction of droplet size distribution in droplet formation region. The MEP approach can predict the most likely droplet size and velocity distributions under a set of constraints expressing the available information related to the distribution.

In this article, by considering the mechanisms of turbulence generation inside the nozzle and wave growth on jet surface, it is attempted to provide a logical framework coupling the flow inside the nozzle to the resulting atomization process. The purpose of this paper is to describe the formulation of this new model and to incorporate it into the maximum entropy principle (MEP) by coupling sub-models together using source terms of momentum and energy. Comparison between the model prediction and experimental data for a gas turbine swirling nozzle and an annular spray indicate good agreement between model and experiment.

Keywords—Droplet, instability, Size Distribution, Turbulence, Maximum Entropy

I. INTRODUCTION

SPECIFICATION of droplet size and velocity distributions in the immediate downstream of injectors is essential as boundary conditions for advanced computational fluid dynamics (CFD) and two-phase spray transport calculations [1,2]. Classic models to predict Sauter Mean Diameter (SMD) distribution and velocity of the droplets were derived mainly from experimental data. In this procedure, a distribution curve is obtained by fitted upon different data from various nozzle operating conditions. This procedure is the main basis for distributions such as the Rosin-Rambler, Nukiyama-Tanasawa and Log-Kernel distributions, etc. [3,4].

Several studies attempt to derive a more general droplet size and velocity distributions based on statistical approaches. Since the mid-1980s, the Maximum Entropy Principle (MEP) method has gained popularity in atomization and spray field to predict droplet size and velocity distribution and has obtained

reasonable success. The MEP approach can predict the most likely droplet size and velocity distributions under a set of constraints expressing the available information related to the distribution. The application of MEP to spray modeling was pioneered by Sellens and Brzustowski [5] and Li and Tankin [6]. This approach assumes that in addition to conservation of mass, momentum and energy, the droplet size distribution function satisfies a maximum entropy principle. The MEP approach suggests that the most probable size distribution can be obtained under conservation principles while system entropy is maximized. Li and Tankin [6] used a single constraint combining liquid-gas surface energy and kinetic energy of the system, whereas Sellens et al. used separate constraint for each mode of energy [5, 7, 8]. In practical sprays such as air-blast and pressure swirl sprays, the surface energy of the liquid always increases as a result of atomization. This work is further extended by Dumouchelet [9] and Sirignano and Mehring [10] to obtain the parameters needed to fit their results for practical sprays.

Most previous works considered the MEP model independent from turbulence generation and unstable wave growth. Whereas, it is well established that the forces acting on a liquid gas interface including surface tension, pressure, inertia force, centrifugal force and viscous force result in the growth of disturbances and instability of liquid sheet, which eventually breaks up into ligaments [11, 12]. Also, the effect of the liquid turbulence level on the mass stripping of the drops and on the product drop size is represented by the initial kinetic energy. The initial turbulent kinetic energy should be estimated according to the jet internal turbulence originates from the strong shear stress along the nozzle wall.

The instability of liquid jets and sheets has received much attention since the classical studies of Rayleigh [13]. For authoritative reviews of liquid sheet and jet instability and breakup, readers are referred to review by Sirignano and Mehring [14] and a recent monograph by Lin [15]. Mitra and Li [16] also used the MEP to obtain droplet size distributions of liquid sheets and used instability analysis, which provides information for prior distribution for the droplet sizes corresponding to the unstable wave growth. Besides, the maximum entropy principle they used was based on Bayesian entropy. However, the MEP model in this paper is based on Shannon entropy and does not need prior distribution as an initial condition.

Considering the nozzle exit turbulence conditions of diesel sprays, Huh et al. [17] proposed a modeling approach taking

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into account turbulence in the atomization process.

In this paper a linear instability theory is used to determine the most unstable waves on the bulk liquid before its breakup, and the resulting jet breakup length and droplet size are assumed to relate to the maximum wave growth rate and its corresponding wavelength, respectively. Then, the stage of droplet formation after the liquid bulk breakup is modeled based on the maximum entropy principle (MEP). The MEP provides formulation that predicts the atomization process while satisfying constraint equations based on conservations of mass, momentum and energy. The two sub-models are coupled together using momentum source term and mass mean diameter of droplets.

Annular nozzle with hollow cone spray that characterized by previous literatures [18, 19] was selected as a case study. Comparison between the model prediction and available experimental data for an annular and a gas turbine nozzle indicates good agreement between the two.

II. LINEAR INSTABILITY ANALYSIS OF ANNULAR LIQUID SHEET

The stability model considers a swirling inviscid annular liquid sheet subject swirling airstreams. Gas phases are assumed to be inviscid and incompressible. Liao et al. (2001) [20] have shown that the difference in growth rate between an annular viscous sheet and an inviscid sheet becomes negligible for Reynolds number greater than about 100.

The basic flow velocities for liquid, inner gas and outer gas are assumed to be $(U_1, 0, A_1/r)$, $(U_i, 0, \Omega r)$, $(U_o, 0, A_o/r)$ respectively. A_o, A_1 (m^2/s) are Vortex Strength and Ω ($1/s$) is Angular velocity. Inner gas is considered as a forced vortex, as it is restricted by liquid jet' sheet. The governing equations for inviscid annular fluid flows are the continuity and Navier–Stokes equations in the cylindrical coordinate system. The linearized equations for the liquid phase are written in vector form as

$$\nabla \cdot \vec{v} = 0 \quad (1)$$

$$\left(\frac{\partial}{\partial t} + \frac{A}{r^2} \frac{\partial}{\partial \theta} \right) \vec{v} = 0 \quad (2)$$

The disturbances are assumed to have the forms

$$(u, v, w, p') = (\hat{u}(r), \hat{v}(r), \hat{w}, \hat{p}) e^{i(kz - \omega t)} \quad (3)$$

Where $\hat{}$ indicates the disturbance amplitude which is a function of r only. For the temporal analysis, the wave number k and n are real while frequency ω is complex. The imaginary part of $\omega = \omega_r + i\omega_i$ reflects the growth rate of the disturbance. The displacement disturbances at the inner and outer interfaces are

$$\eta_j(x, \theta, t) = \hat{\eta}_j e^{i(kz - \omega t)} \quad (4)$$

Considering the linearized disturbed equations for the inner and outer air and the kinematic and dynamic boundary conditions at the liquid interface according literatures [17, 21], the pressure disturbances inside the annular liquid sheet and inner and outer gas are obtained.

The dispersion equation is obtained by substituting the pressure disturbances inside the liquid and the gas phases into the dynamic boundary conditions. In order to determine the effect of the various forces, properties of fluids and other geometric parameters, the non-dimension form of final equations are made. The fourth order dispersion equation is obtained and solved numerically using the secant method. [21]

$$a_1 \bar{\omega}^4 + a_2 \bar{\omega}^3 + a_3 \bar{\omega}^2 + a_4 \bar{\omega} + a_5 = 0 \quad (5)$$

Substituting $\bar{\omega}$ and its corresponding most unstable wave number into Eq. (6, 7) breakup length L_b , initial drop diameter d_D are obtained [22]:

$$L_b = \frac{12 \cdot R_b}{\bar{\omega}} \quad (6)$$

$$d_D = \sqrt{\frac{16 \cdot h_s \cdot R_b}{K}} \quad (7)$$

As a validation of results, both analytical [11] and experimental data [23] of previous works are used in the same liquid jet and inner and outer gas specifications. To show the accuracy of the analysis, figure 1 presents the comparison of the wave dispersion equation results for linear instability analysis (Eq. 21) with the experimental result of Bruce [24]. It shows the capability of linear analysis to predict instability criterion and agrees with previous literatures [13]. However, It is important to mention that linear theory can predict breakup of an annular sheet for small level initial disturbances and the nonlinear model is necessary to accurately determine the breakup length for high initial disturbances [11].

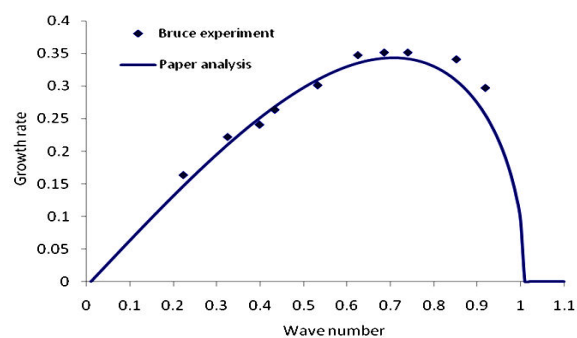


Fig. 1 Growth rates versus wave number of liquid jet
 $Re = 169, d_n = 0.02 mm, \rho_l = 1022 g/l, \mu_l = 2.62 \times 10^{-3} Pa, \sigma = 0.052$

III. PREDICTION OF DROPLET DISTRIBUTION

The deterministic sub-model, as discussed so far, is valid up to the breakup of liquid sheet. After the sheet breakup, the entire process becomes random and highly nonlinear, with the formation of droplets of different diameters and velocities. Therefore, the droplet formation process deals with the stochastic sub-model, where a probability density function (PDF) is used to describe the distribution of droplets in sprays. To extract governing equations and to determine size and velocity distribution for particles, a control volume is considered from the outlet of the injector to the droplet formation location. Its length is equal to breakup length of spray that was discussed in previous sub model.

The liquid mass, momentum, and energy must be conserved during the atomization process. Regarding the formulation of entropy maximization, the conservation equation can be stated

in terms of the joint probability density function: P_{ij} , which is the probability of finding a droplet with volume V_i and velocity \mathbf{u}_j . Hence, the mass, momentum and energy conservation equation can be restated as:

$$\sum_i \sum_j p_{ij} V_i \rho \dot{n} = \dot{m}_o + S_m \quad (8)$$

$$\sum_i \sum_j p_{ij} V_i \rho \dot{n} u_j = \dot{J}_o + S_{mu} \quad (9)$$

$$\sum_i \sum_j p_{ij} \dot{n} (V_i \rho u_j^2 + 2\sigma) = \dot{E}_o + S_e \quad (10)$$

In these equations, \dot{n} is the droplet generation rate in the spray. \dot{m}_o , \dot{J}_o , \dot{E}_o are mass flow rate, momentum and energy, which enter the control volume from injector outlet. S_m , S_{mu} and S_e are the source terms for mass, momentum and energy equations, respectively.

In addition to the three above mentioned equations, according to the probability concept, total summation of probabilities should be equal to unity:

$$\sum_i \sum_j p_{ij} = 1 \quad (11)$$

As mentioned before, there is an infinite number of probability distributions function (P_{ij}) which satisfies equations (8) through (11); therefore, the most appropriate distribution is the one in which Shannon entropy is maximized [27].

$$S = -K \sum_i \sum_j p_{ij} \ln p_{ij} \quad (12)$$

K is the Boltzmann's constant. It is also feasible to convert analytical domain from volume and velocity of droplets to their diameter and velocity. Hence, the formulation can be written according to the probability of finding droplets whose diameters are between \bar{D}_{n-1} and \bar{D}_n and whose velocities are between \bar{u}_{m-1} and \bar{u}_m [6]. Equations (8) to (11) can be non-dimensional and restated in integral forms within the analytical domains of velocity and diameter of the droplet in the form of equation (13) [25]. Using Lagrangian multiplier method, the probability of finding the droplets while entropy (equation 12) is maximized can be obtained. The non-dimensional and integral form of probability function is presented in equation (14), where the set of λ_i is a collection of arbitrary Lagrange multipliers which must be evaluated for each particular solution. Hence, regarding above mentioned statement, to obtain Lagrange coefficient (λ_i) in probability function (f), it is necessary to solve the following normalized set of equations [25].

$$\begin{cases} \int_{\bar{D}_{\min}}^{\bar{D}_{\max}} \int_{\bar{u}_{\min}}^{\bar{u}_{\max}} f \bar{D}^3 d\bar{u} d\bar{D} = 1 + \bar{S}_m \\ \int_{\bar{D}_{\min}}^{\bar{D}_{\max}} \int_{\bar{u}_{\min}}^{\bar{u}_{\max}} f \bar{D}^3 \bar{u} d\bar{u} d\bar{D} = 1 + \bar{S}_{mu} \\ \int_{\bar{D}_{\min}}^{\bar{D}_{\max}} \int_{\bar{u}_{\min}}^{\bar{u}_{\max}} f \left(\frac{\bar{D}^3 \bar{u}^2}{H} + \frac{B \bar{D}^2}{H} \right) d\bar{u} d\bar{D} = 1 \\ \int_{\bar{D}_{\min}}^{\bar{D}_{\max}} \int_{\bar{u}_{\min}}^{\bar{u}_{\max}} f d\bar{u} d\bar{D} = 1 \end{cases} \quad (13)$$

$$f = 3\bar{D}^2 \exp[-\lambda_0 - \lambda_1 \bar{D}^3 - \lambda_2 \bar{D}^3] \quad (14)$$

In these equations diameter, velocity and dimensionless source terms can be described as:

$$We = \frac{\rho \bar{u}_o^2 D_{30}}{\sigma}, \quad B = \frac{12}{We}, \quad \bar{D}_i = D_i / D_m, \quad \bar{u}_j = u_j / \bar{u}_o, \\ \bar{S}_m = S_m / \dot{m}_o, \quad \bar{S}_{mu} = S_{mu} / \dot{J}_o, \quad \bar{S}_e = S_e / \dot{E}_o$$

H is the shape factor for the velocity profile and if the shear layer inside the nozzle is neglected, the outlet velocity profile will be uniform and the shape factor (H) is equal to 1 [8]. As it can be seen from the equations, the solution domains are changed from \bar{D}_{\min} to \bar{D}_{\max} and from \bar{u}_{\min} to \bar{u}_{\max} .

In this work, the source terms of mass and energy are set to zero indicating that the evaporation and heat transfer during spraying process have been neglected. If there is any energy conversion within the control volume, it is not considered as a source term. Within the control volume, there is a momentum exchange between the liquid flow and the gas. This momentum transformation should be considered as a momentum source term to account for the drag force on liquid

body.

To obtain this function, it is imperative to determine Lagrange multipliers λ_i in equations (14) which can be computed from solving the equation set (13) simultaneously. To solve this set of equations, the Newton-Raphson method is used. At first, some initial value for the $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ was assumed. Then, using these values and the Newton-Raphson procedure, a new value for λ_0 and then $\lambda_1, \lambda_2, \lambda_3$ was computed and this procedure continued until the final answer was obtained.

To solve these equations, it is noted that functions in equation set (13) and their derivatives are integral functions. Therefore, double integral functions should be solved numerically for all iterations. Another important point is that integral functions and the terms in these integrals are exponential; hence, if the selection of an initial guessed of λ_i turns out to be close to the answer, the value starts to converge to the answer immediately. To solve the governing equations, analytical domains for non dimensional diameter and velocity are considered from 0 to 3.

IV. INCLUSION OF TURBULENCE EFFECT

As indicated in the Introduction section, turbulence developed inside the liquid remained dominant and became a main contributor in the spray development [26]. This study follows the method of [17] to estimate turbulence generation inside the nozzle. The jet internal turbulence originates from the strong shear stress along the nozzle wall and possible cavitation effects and is a main reason of initial surface perturbations on jet [27].

The initial turbulent kinetic energy can be approximated as:

$$k_t = \frac{U_0^2}{8L/D_{noz}} \left[\frac{1}{C_d^2} - 1 \right] \quad (15)$$

The velocity U_0 is the liquid mean velocity at the nozzle exit; the nozzle has length L and diameter D_{noz} . The discharge coefficient, the loss coefficient due to the nozzle entrance sharpness and the downstream-to-upstream contraction area ratio of the injection nozzle are represented by C_d , K_c and S respectively. Detailed derivations of these turbulence scales can be found in [17].

V. COUPLING TWO SUB-MODELS

Previous MEP models of predicting droplet size distributions [28] had utilized instability analysis to provide information for prior distribution of the droplets' size. The novel approach used in this paper does not require prior distribution as an initial condition for MEP modeling. In the current approach, the linear instability theory, with enhanced turbulence effect, is used to determine the maximum wave growth rate and consequently the jet breakup length, L_b , as well as the mass mean droplets' size, D_m . Energy source term is another parameter that makes a connection between main

MEP model and the turbulence sub-model. The initial turbulent kinetic energy entering the control volume was used to estimate energy source term as seen in Eqs. (18). The control volume extends from the nozzle exit to the sheet breakup region. The momentum source term is obtained by considering the drag force acting on the liquid sheet due to the relative motion of the gas phase over the breakup length. Also, the length of liquid sheet can be estimated using the breakup length according to wave analysis sub model. The analysis is carried out assuming that the flat plate of liquid sheet is fixed and the gas phase velocity above the liquid-sheet boundary layer is taken as mean liquid velocity at the nozzle outlet.

The drag force on the liquid sheet can be written as [29]:

$$F = \frac{1}{2} \rho_g (U_g - U_l)^2 \ell \quad (16)$$

$$A = \pi d_{noz} L_b$$

C_f is the drag coefficient for flow over a flat liquid plate with length L_b and contact area of A , which has different values for laminar and turbulent flows. The drag force is equal to the amount of momentum transferred from the surrounding gas medium to the liquid sheet per unit of time. Therefore, the momentum source term is obtained as:

$$\bar{S}_{mu} = \frac{F}{\dot{J}_m} = \frac{F}{\rho_l U_l^2 A_{crv}} \quad (17)$$

Considering a laminar boundary layer flow passing on a flat plate, C_f can be computed; so the momentum source term can be evaluated as shown later.

Using the turbulence generation model the energy source term of MEP model can be calculated. If there is any energy conversion within the control volume, it is not considered as a source term. Also the heat and energy exchange during spraying process was neglected. So, the liquid jet turbulence generated inside the nozzle is the only source of energy entering to the control volume and initial turbulent kinetic energy from Eqs. (15) can be used to estimate energy source term for MEP model (Eq. 18).

$$\bar{S}_e = \frac{k_t}{\dot{E}_0} = \quad (18)$$

VI. RESULTS AND DISCUSSION

To assess maximum entropy principle for the determination of PDF in conjunction with instability analysis, the procedure is evaluated for two different sprays; a spray resulting from the conical hollow nozzle [30] and a spray from an industrial gas turbine [16]. The spray characteristics are presented in Table 1 and 2. A linear instability analysis was done to determine the dominant wave characteristic that causes the sheet breakup. Figure 2 shows the wave growth rates for different wave numbers. According to the figure, the amount of maximum growth rate and its corresponding wave number was achieved (Table II).

TABLE I
SPRAY CHARACTERISTIC [30]

Liquid Density	998.2 (Kg/m ³)
Surface Tension	0.0736 (N/m)
Ambient Pressure	1 atm
Gas Density	1.22 (Kg/m ³)
Flow rate	2.809*10 ⁻³ (Kg/s)
Liquid Average Velocity	40.8 (m/s)

TABLE II
COMPUTED DRAG FORCE AND SOURCE TERM FOR MOMENTUM EQUATIONS

Weber Number W_e	Reynolds Number Re	Drag Force F (N)
311	18200	1.953*10 ⁻³
Non-dimensional Momentum source term \bar{S}_{mu}	Maximum Growth Rate $\bar{\omega}$	Wave Number \bar{k}
-0.01702	1.222	15

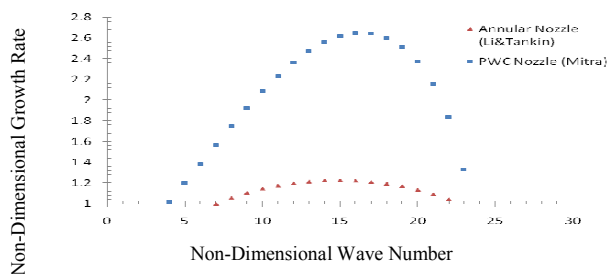


Fig. 2 Wave growth rates for different wave numbers for the annular nozzle of Li & Tankin and gas turbine nozzle of Mitra

In Figure III, the measured and computed probability distributions of size for the annular nozzle are demonstrated. This function is acquired from the integration of velocity-size probability distribution function over the velocity interval. As can be seen from the figure, agreement between theoretical and experimental results is satisfactory.

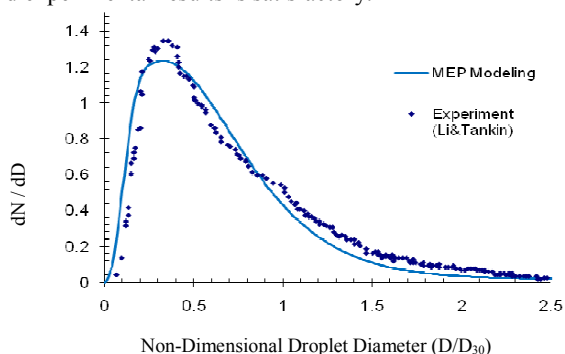


Fig. 3 Comparison of theoretical (solid line) and experimental (dashed line) [30] droplet size distribution

Next, the present model is compared with the test results of an actual gas turbine nozzle (PWC nozzle) provided by Kim et al. [16]. The PWC nozzle produces an annular liquid sheet at

the nozzle exit, with air flow both inside and outside of the liquid sheet. The nozzle exit conditions and source terms required were provided in Table 3. According to the instability analysis and the diagram shown in Figure 2, the amount of maximum growth rate and corresponding wave number was successfully obtained (Table III).

TABLE III
ESTIMATES OF NOZZLE CONSTANT AND SOURCE TERMS OF PWC NOZZLE [16]

UL(m/s)	4.2	Drag Coefficient (CD)	0.0124
Ug(m/s)	42	D30(micron)	41
Breakup Length (mm)	4.4	Smu	0.036
Maximum Growth Rate	2.64	Wave Number	16

In Figure 4 dimensional drop size distribution from measurement and MEP model have been shown for two different positions from the nozzle exit. Therefore, these data have been used to investigate the droplet size distribution downstream of breakup position using a comparison of results for two different positions, 5mm and 10mm from the nozzle exit. As seen in Figure 4, at higher distance from breakup position the population of droplets for different droplet diameters is more homogenous and has less maximum value. The same result has been attained from modeling using modified MEP. With an increase in momentum source term that is the result of moving through down-stream of the spray, the peak of size distribution decrease and moves to bigger drop sizes. On the other hand, one can see that the size distribution of droplets tends to be broader as going far from nozzle exit. Comparison between experimental and theoretical results shows the agreement between the two. The goal of comparing is more showing the trends rather than competitive exact data. It is observed that the theoretical distribution in 10mm downstream direction predicts slightly greater values for big droplets compared to the experimental distribution. The over-prediction may be understandable because the present measurements are made at a location slightly downstream of the breakup region, where the turbulence effect which causes faster breakup might be significant; whereas, the MEP model could not see this effect.

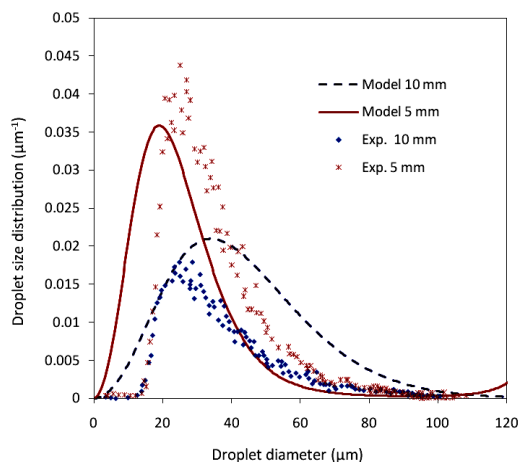


Fig. 4 Droplet size distribution of PWC nozzle for two different positions in downstream direction. Experiment result [16, 31] Reg= 11420 , Lb=4 mm, D30=45 Micron

It is observed that the droplet size distribution over-predicts for smaller droplet diameters, even though other experiments for droplet size distribution show non-zero value for small droplets. Thus, there is the possibility that lacking any data for small droplets is a limitation of this experiment. This problem has also been reported by Dumochel [9]. It is also observed that the theoretical distribution predicts slightly greater values for small droplets compared to the experimental distribution. It may be understandable because of presence of turbulence effect inside the liquid jet. In case of low speed liquid jet, that causes the formation of bigger eddies and consequently bigger initial droplets [27]. As the presented MEP model doesn't consider turbulence effect inside the jet, it over-estimates small droplets. However, for droplet diameter greater than 30 micron, the theoretical prediction matches better with the experimental distribution. The above comparisons show that the present model can predict initial drop let size and velocity distributions reasonably well for sprays produced by two nozzles of considerably different geometries.

VII. CONCLUSION

In the present paper, the random process of distributing the diameter and velocity of the droplets in the primary breakup region was modeled implementing maximum entropy principle (MEP). Also the deterministic aspect was done, which involves the determination of the liquid bulk breakup length and the mass mean diameter by means of hydrodynamic instability theory and unstable wave development on the liquid jet surface. Two sub-models are coupled together by the momentum source term and mass mean diameter of droplets.

This approach is applicable for predicting the size and velocity distribution of droplets in the systems in which thermodynamics equilibrium prevails. However, the process of spray formation is irreversible and not adiabatic, and there is always interaction between atomized liquid and surrounding gas. Therefore, establishing a harmony between the results of modeling using MEP and experimental data is difficult. Although, simplified assumptions were used to solve the equations, the results demonstrated a satisfactory conformity with the experiments. This revealed the model ability to account for the effects of processes that occur in the spray control volume. Since the functions and their derivatives in the governing equations are in the integral form and functions in the integral are exponentials, the solution is sensitive to the initial guess λ_i .

A precise estimation of the source terms is very important, so as to acquire exact results, estimating the drag forced on droplets through the gas flow field should be considered. Comparisons of the present model predictions with the experimental measurements have been carried out. It is observed that a satisfactory agreement is achieved between the predicted droplet size distributions and experimental measurements for two different sprays with and without high speed surrounding gas. Therefore, the present model may be

applied to obtain the initial droplet size and velocity distributions for sprays.

NOMENCLATURE

$A_{l,o}$	Vortex Strength (m^2/s or $1/s$)
A_{cross}	Jet cross section area
A	Droplet cross section area
g	Gas-to-liquid density ratio
h	Ratio of inner and outer radius
h_s	Liquid sheet thickness
I_n	n th order modified Bessel function of first kind
K_n	n th order modified Bessel function of second kind
$k = 1/\lambda$	Axial wave number ($1/m$)
n	Circumferential wave number (Rad)
P	Mean pressure (N/m^2)
p'	Disturbance pressure (N/m^2)
R_a	Inner diameter of liquid sheet (m)
R_b	Outer diameter of liquid sheet (m)
L_b	Breakup length
d_{nozz}	Nozzle diameter
d_L	ligament diameter
d_{Drop}	Droplet diameter
U	Mean axial velocity (m/s)
u	Disturbance axial velocity (m/s)
v	Disturbance radial velocity (m/s)
W	Mean tangential velocity (m/s)
We	Weber number ($\rho_l U^2 R_b / \sigma$)
w	Disturbance tangential velocity (m/s)
η	Displacement disturbance (m)
σ	Surface tension (kg/s^2)
ω	Temporal growth rate ($1/s$)
p_i	probability of occurrence of state i
k	Boltzmann constant
N	normalized cumulative droplet number
\dot{n}	Total number of droplets being produced per unit time
D_{30}	Mass mean diameter
V_m	Mean volume of droplet
V_i	Volume of i th droplet
S_m	dimensionless mass source term
S_{mu}	dimensionless momentum source
S_e	Energy source term
We	Weber number
\dot{m}_o	mass flow rate get into the C.V.
\dot{J}_o	Momentum flow rate get into the C.V.
\dot{E}_o	Energy flow rate get into the C.V.
λ_i	Lagrange coefficient
C_f	Drag coefficient over the liquid sheet
C_D	Drag coefficient on a droplet
\bar{u}_o	Mean velocity of jet in nozzle outlet
U_m	Droplets mean velocity
u	Droplet velocity
H	Shape factor

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