

Distributed Generator Placement and Sizing in Unbalanced Radial Distribution System

J. B. V. Subrahmanyam, C. Radhakrishna

Abstract—To minimize power losses, it is important to determine the location and size of local generators to be placed in unbalanced power distribution systems. On account of some inherent features of unbalanced distribution systems, such as radial structure, large number of nodes, a wide range of X/R ratios, the conventional techniques developed for the transmission systems generally fail on the determination of optimum size and location of distributed generators (DGs). This paper presents a simple method for investigating the problem of contemporaneously choosing best location and size of DG in three-phase unbalanced radial distribution system (URDS) for power loss minimization and to improve the voltage profile of the system. Best location of the DG is determined by using voltage index analysis and size of DG is computed by variational technique algorithm according to available standard size of DGs. This paper presents the results of simulations for 25-bus and IEEE 37-bus Unbalanced Radial Distribution system.

Keywords—Distributed generator, unbalanced radial distribution system, voltage index analysis, variational algorithm.

I. INTRODUCTION

ONE of the most important motivations for the studies on the integration of distributed resources to the grid is the exploitation of the renewable resources such as; hydro, wind, solar, geothermal, biomass and ocean energy, which are naturally scattered around the country and also are smaller in size. Accordingly, these resources can only be tapped through integration to the distribution system by means of distributed generation. Although there is no consensus on the exact definition of distributed generation (DG), there are some significant attempts, in the literature [1,2], to define the concept. Meanwhile DG, which generally consists of various types of renewable resources, can best be defined as electric power generation within distribution networks or on the customer side of the system [1, 2], in general. This definition is preferred in this paper.

DG affects the flow of power and voltage conditions on the system equipment. These impacts may manifest themselves either positively or negatively depending on the distribution system operating conditions and the DG characteristics. Positive impacts are generally called 'system support benefits', and include voltage support, power loss reduction,

transmission and distribution capacity release, improved utility system reliability and power quality. On account of achieving above benefits, the DG must be reliable, dispatchable, of the proper size and at the proper locations [3, 4].

Energy cost of renewable-based distributed generation when compared to the conventional generating plants is generally high because the social and environmental benefits could not be included in the cost account. Accordingly, most of the studies to determine the optimum location and size of DG could not consider the generation cost, directly. Although one of the most important benefits of DG is reduction of the line losses, it is important to determine the size and the location of local generation to be placed in distribution systems. For the minimization of system losses, there have been number of studies to define the optimum location of DG. The various approaches on the optimum DG placement for minimum power losses can be listed as the classical approach: second-order algorithm method [5], the meta-heuristic approaches [6–8]: genetic algorithm and Hereford Ranch algorithm [6], fuzzy-GA method [7], tabu search [8], and the analytical approaches [9–13].

In the analytical studies [9–11], optimal place of the DGs are determined exclusively for the various distributed load profiles such as; uniformly, increasingly, centrally in radial systems to minimize the total losses. Additionally, in [12], optimal size and place of DG is obtained and analyzed by considering the effects of static load models. These analytical studies are generally based on phasor current injection method which has unrealistic assumptions such as; uniformly, increasingly, centrally distributed load profiles. These assumptions may cause erroneous solution for the real systems.

In [13] the optimal size and location of DG is calculated based on exact loss formula and compared with successive load flows and loss sensitivity methods. The method is computationally less demanding for radial and networked systems, however, it requires the calculation of the bus impedance matrix, Z bus, the inverse of the bus admittance matrix, Y bus. It should be noted that due to the size, complexity and specific characteristics of distribution networks, the method could not be directly applied to distribution systems. It fails to meet the requirements in robustness aspects in the distribution system environments [14]. Although the heuristic methods are intuitive, easy to understand and simple to implement as compared to analytical and numerical programming methods, the results produced by heuristic algorithms are not guaranteed to be optimal [15].

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While most works have been done on DG placement in radial balanced distribution systems, very little research has been done on DG placement and sizing in unbalanced radial distribution systems. Cheng and Shirmohammadi [16] proposed PV nodes for three phase unbalanced system. Noel Schulz and Sarika Khushalani [17] developed three phase unbalanced power flow algorithm with the choice of modeling DG (single and multiple) as PQ or PV node. Chen et al. [18] proposed different mathematical modeling of DG and transformers.

In this paper, voltage sensitive nodes are first identified by penetrating DG with 20% of the total feeder loading, at each node and the node with least voltage index after placing DG will be picked as the best location for the DG placement. The sizing method is formulated as a constrained optimization problem adapted from a reactive power compensation sizing algorithm. Variational Algorithm is used to find the optimal size of the DG with the standard size of the DGs. The objective function formulated is to minimize the system losses satisfying the voltage and power limits.

II. DISTRIBUTION SYSTEM COMPONENT MODELING

In a three phase unbalanced load flow of distribution system the following each individual system component is mathematically represented by models that approximate their physical behaviour. Network components include the distribution lines, shunt capacitors, cogeneration and transformers. In the power flow calculation, components are modeled by their equivalent circuits in terms of inductance, capacitance, resistance and injected current.

A. Distribution Line

In general, the voltage at p^{th} bus V_p , and at q^{th} bus, V_q , are related by

$$V_q^{abc} = V_p^{abc} - I_{pq}^{abc} Y_{pq}^{abc} \quad (1)$$

Where

Y_{pq}^{abc} is the series impedance matrix represented by

$$Y_{pq}^{abc} = \begin{bmatrix} Y_{pq}^{aa} & Y_{pq}^{ab} & Y_{pq}^{ac} \\ Y_{pq}^{ba} & Y_{pq}^{bb} & Y_{pq}^{bc} \\ Y_{pq}^{ca} & Y_{pq}^{cb} & Y_{pq}^{cc} \end{bmatrix} \quad (2)$$

Shunt capacitance is also taken into consideration and the current injections at bus number k can be expressed in terms of variables in the above.

B. Shunt Capacitors

Shunt capacitors, which act as sources of reactive power, are often placed at strategic locations throughout distribution networks. Shunt capacitors are represented by their equivalent injected currents. Let Q_q^{a0} , Q_q^{b0} and Q_q^{c0} be the rated value of shunt capacitors at phase a, b and c at bus q . The injected currents are

$$\begin{bmatrix} IC_q^a \\ IC_q^b \\ IC_q^c \end{bmatrix} = \begin{bmatrix} -\frac{jQ_q^{a0}}{V_a^*} \\ -\frac{jQ_q^{b0}}{V_b^*} \\ -\frac{jQ_q^{c0}}{V_c^*} \end{bmatrix} \quad (3)$$

C. Transformers

Copper loss and core loss (which is a function of the voltage on the secondary side of the transformer), winding connections, the phase shifting between primary and secondary windings and the off-nominal tapping are incorporated into transformer models. Transformer core loss functions are represented in per unit on the system. Several different types of transformer connections should be considered.

D. Load Model

Load models are represented by the equivalent injected currents, IL_q^a , IL_q^b and IL_q^c . The voltage dependency of loads is considered, which is a combination of constant power, constant current and constant impedance models. In addition, both ground and ungrounded loads are considered.

E. Loss Calculation

For distribution line, the power loss in the pq branch of the actual feeder can be written as:

$$\begin{bmatrix} LS_{pq}^a \\ LS_{pq}^b \\ LS_{pq}^c \end{bmatrix} = \begin{bmatrix} LP_{pq}^a + jLQ_{pq}^a \\ LP_{pq}^b + jLQ_{pq}^b \\ LP_{pq}^c + jLQ_{pq}^c \end{bmatrix} = \begin{bmatrix} V_p^a \cdot (I_{pq}^a)^* - V_q^a \cdot (I_{qp}^a)^* \\ V_p^b \cdot (I_{pq}^b)^* - V_q^b \cdot (I_{qp}^b)^* \\ V_p^c \cdot (I_{pq}^c)^* - V_q^c \cdot (I_{qp}^c)^* \end{bmatrix} \quad (4)$$

F. Unbalance Three phase load flow

The load flow studies for general unbalanced radial distribution networks takes account of the following features:

- Sparse matrix techniques and data storage techniques are incorporated.
- Accurate modeling of network components (including shunt capacitance, series admittance, shunt capacitors, transformers, voltage dependent load models) has been considered.
- No divergent cases have been encountered
- Only a few iterations are required for each power flow study.

The unbalanced load flow solutions consist of voltage magnitude and angle at each bus and each phase can be exactly calculated because detailed network component modeling is considered. Since the determination of capacitor placement and size relies upon the unbalanced load flow solution, this portion of study is essential for the overall study.

III. MATHEMATICAL FORMULATION FOR DG MODEL

The DG can be treated as PV or PQ model in the unbalanced distribution system. The PV model represents a DG which delivers power at a specific terminal voltage; while PQ model DG delivers power irrespective of the node voltage. The latter DG model representation is adopted in this paper. Such source is modeled as a negative load delivering real and reactive power to the unbalanced distribution system.

The objective function of the present work is to,

$$\text{Minimize } f = \sum_{pq=1}^{nb} P_{loss\ pq}^{abc} \quad (5)$$

subject to

Voltage constraint

Voltage magnitude at each node must lie with their permissible ranges to maintain power quality.

$$V_q^{\min} \leq V_q \leq V_q^{\max} \quad (6)$$

Current constraint

Current magnitude of each branch (feeder, laterals, and switches) must lie with their permissible ranges.

$$I_{pq} \leq I_{pq}^{\max} \quad (7)$$

Power source limit constraint

The total loads of a certain partial network can not exceed the capacity limit of the corresponding power source.

$$P_{pq} \leq P_{pq}^{\max} \quad (8)$$

$$Q_{pq} \leq Q_{pq}^{\max} \quad (9)$$

Distributed Generator losses constraint

The distributed generator active losses of the network can not exceed the total active power line losses of the network.

$$\sum_{pq} P_{lossG} \leq \sum_{pq} P_{loss} \quad (10)$$

Generator losses constraint

Limit on total power generated by DG subject to a penetration level of 20% (i.e. it must not exceed 20% of feeder load).

$$\sum_{q=1}^n P_{Gq} \leq 0.2 \sum_{q=1}^n P_{Lq} \quad (11)$$

$$\sum_{q=1}^n Q_{Gq} \leq 0.2 \sum_{q=1}^n Q_{Lq} \quad (12)$$

A. Location of DG Placement

The location of DG is chosen as the one that gives the best voltage profile. This could be done by injecting the 20% of the DG at each node calculating the voltage index by using eqn. (13). The best location is selected as the node with least voltage index.

Voltage index (v_{index}) is defined as

$$V_{index} = \sqrt{\frac{\sum (1 - V_q)^2}{n}} \quad (13)$$

Where V_q is voltage at q^{th} node and

n is the number of nodes.

The voltage index is calculated for unbalanced analysis with 20% DG penetration. Though per-phase analysis gives better voltage indices as compared to unbalanced analysis, they may not be representing the actual conditions.

To study the effect of placement of DG on isolated power systems, the same sample system is assumed to be isolated. The location of node at which generator is placed is varied from 2 to 25(except source node) for the sample 25 node test feeder whose single line diagram is shown in fig. 1. The voltage index for different locations is shown in fig. 2 for best location of DG.

B. DG Size

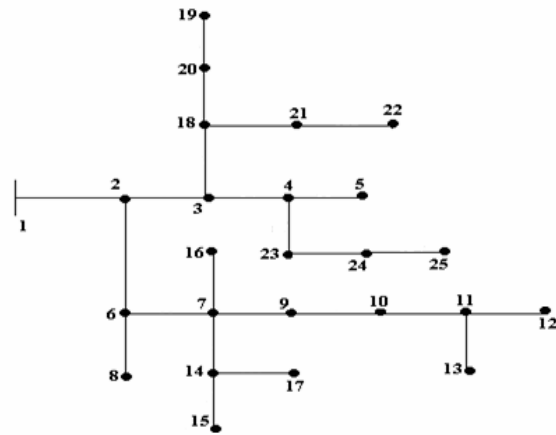


Fig. 1. Single line diagram of 25-bus URDS

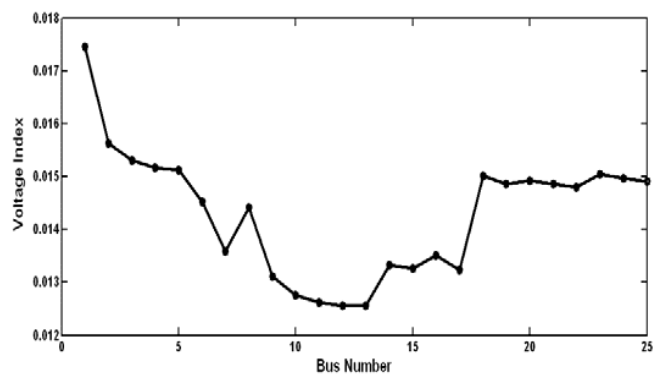


Fig. 2 Voltage Index for 25-bus URDS

In this section determination of the amount of standard DG that can be added at the identified nodes without loss increase and operational constraints violation is presented. Given information is on the available distributed generation and

assuming no expected load growth in the region of interest. Variational algorithm is used to deal with this optimization problem.

IV. ALGORITHM FOR DG PLACEMENT AND SIZING

- Step 1: Run the base case UBRD load flow.
 Step 2: Find the voltage index at each node using eqn. (13) by penetrating the 20% DG value at respective node and rank the sensitivities of all nodes in Ascending order to form priority list.
 Step 3: Select the bus with lowest priority and place DG at that bus.
 Step 4: Change the size of DG in "small" steps and calculate power loss for each by running UBRD load flow.
 Step 5: Store the size of DG that gives minimum loss.
 Step 6: Compare the loss with the previous solution. If loss is less than previous solution, store this new solution and discard previous solution.
 Step 7: Repeat Step 4 to Step 6 for all buses in the priority list.
 Step 8: End

V. SIMULATION RESULTS AND ANALYSIS

A. Case Study I: 25-bus URDS

The proposed algorithm is tested on 4.16 kV, 25-bus unbalanced radial distribution system as shown in Fig. 1. The line and load data are given in Appendix A.

The minimum voltages are highlighted. From table 1 It has been observed that the minimum voltages in phases A, B & C are improved from 0.9284, 0.9284, 0.9366 p.u (without DG) to 0.9504, 0.9511, 0.9557 p.u (with DG) respectively. Hence improvement has observed in the minimum voltage of the system in all phases after DG placement. It has also observed that the optimal placement of the DG at 13 bus with the Variational algorithm sizing of the 215 kW at 0.85 lagging power factor. Table 2 shows the summary of test results before and after DG placement. The total active power losses for the 25 bus URDS before and after DG placement is given in table.1.

B. Case Study II: 37-bus IEEE URDS

The problem of placement and sizing of the capacitors banks has been solved for the unbalanced IEEE 37-bus test system shown in Fig. 4 [19] where, obviously, original voltage regulator has been removed; the IEEE 37-bus test system is an actual distribution system located in California. Three-wire delta operating at a nominal voltage level of the test network is 4.8 kV. All line segments are underground. All line segments are underground. All loads are spot loads and consist of constant PQ and loading is very unbalanced.

TABLE 1
VOLTAGE PROFILE FOR IEEE 25-BUS URDS

Bus No	Before DG placement			After DG placement		
	Va (p.u)	Vb (p.u)	Vc (p.u)	Va (p.u)	Vb (p.u)	Vc (p.u)
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.9702	0.9711	0.9755	0.9758	0.9765	0.9799
3	0.9632	0.9644	0.9698	0.9689	0.9699	0.9744
4	0.9598	0.9613	0.9674	0.9654	0.9667	0.9719
5	0.9587	0.9603	0.9664	0.9644	0.9657	0.9710
6	0.9550	0.9559	0.9615	0.9660	0.9666	0.9708
7	0.9419	0.9428	0.9492	0.9584	0.9589	0.9634
8	0.9529	0.9538	0.9596	0.9639	0.9646	0.9689
9	0.9359	0.9367	0.9438	0.9576	0.9580	0.9627
10	0.9315	0.9319	0.9395	0.9585	0.9584	0.9632
11	0.9294	0.9296	0.9376	0.9595	0.9592	0.9641
12	0.9284	0.9284	0.9366	0.9504	0.9511	0.9557
13	0.9287	0.9287	0.9368	0.9627	0.9622	0.9671
14	0.9359	0.9370	0.9434	0.9525	0.9532	0.9577
15	0.9338	0.9349	0.9414	0.9504	0.9511	0.9557
16	0.9408	0.9418	0.9483	0.9573	0.9579	0.9625
17	0.9347	0.9360	0.9420	0.9513	0.9522	0.9563
18	0.9573	0.9586	0.9643	0.9630	0.9641	0.9689
19	0.9524	0.9544	0.9600	0.9581	0.9599	0.9645
20	0.9548	0.9563	0.9620	0.9605	0.9618	0.9666
21	0.9537	0.9549	0.9605	0.9594	0.9604	0.9651
22	0.9518	0.9525	0.9585	0.9575	0.9580	0.9631
23	0.9565	0.9584	0.9648	0.9622	0.9638	0.9693
24	0.9544	0.9565	0.9631	0.9601	0.9620	0.9677
25	0.9520	0.9547	0.9612	0.9577	0.9602	0.9657

TABLE II
SUMMARY OF TEST RESULTS FOR 25 BUS URDS

Description	Before DG Placement			After DG Placement		
	Phase a	Phase b	Phase c	Phase a	Phase b	Phase c
Distributed generator placed node and size of DG	13	-	-	215 kW at 0.85 p.f	215 kW at 0.85 p.f	215 kW at 0.85 p.f
Minimum Voltage	0.9284	0.9284	0.9366	0.9504	0.9511	0.9557
Voltage regulation (%)	7.16	7.16	6.34	4.96	4.89	4.43
Improvement of Voltage regulation (%)	-	-	-	30.72	31.70	30.12
Active Power Loss (kW)	52.82	55.44	41.86	32.01	33.39	25.04
Total Active Power Loss reduction (%)	-	-	-	39.39	39.77	40.18
Reactive Power Loss (kVAr)	58.32	53.29	55.69	37.08	33.66	35.40
Total Reactive Power Loss reduction (%)	-	-	-	36.42	36.83	36.43
Total Demand (kW)	1126.12	1138.74	1125.16	1105.31	1116.69	1108.34
Total Released Demand (kW)	-	-	-	20.81	22.05	16.82
Total Reactive Power Demand (kVAr)	850.32	854.29	855.69	829.08	834.66	835.4
Total Released Reactive Power Demand (kVAr)	-	-	-	21.24	19.63	20.29
Total Feeder Capacity (kVA)	1411.09	1423.57	1413.57	1381.69	1394.14	1387.91
Total Released Feeder Capacity (kVA)	-	-	-	29.4	29.43	25.65

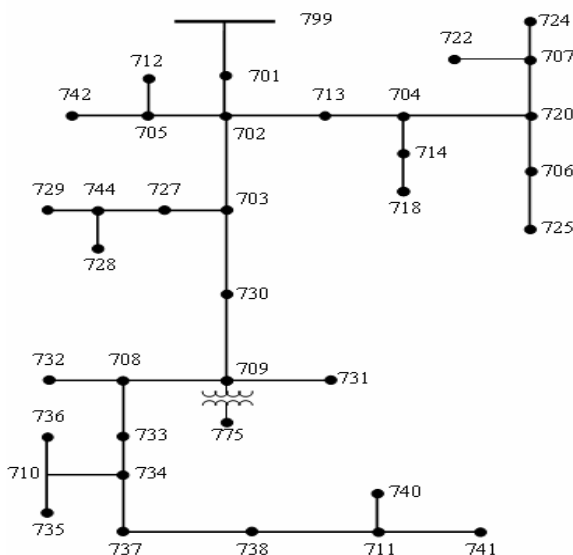


Fig. 4 Single line diagram of 37-bus IEEE URDS

From the voltage sensitive analysis, it is observed that node 722 is the best location for the DG. Voltage index graph for IEEE 37-bus is shown in fig. 5. From the graph it has been observed that node 722 shows the minimum voltage index which is the best location for DG placement.

The minimum voltages are highlighted. From table 3 It has been observed that the minimum voltages in phases a, b & c are improved from 0.9418, 0.9559, 0.9425 p.u. (without DG) to 0.9814, 0.9863, 0.9534 p.u. (with DG) respectively. Hence, there is an improvement in the minimum voltage when compared with the before DG placement and after DG placement. Table 4 shows the summary of test results before

and after DG placement.

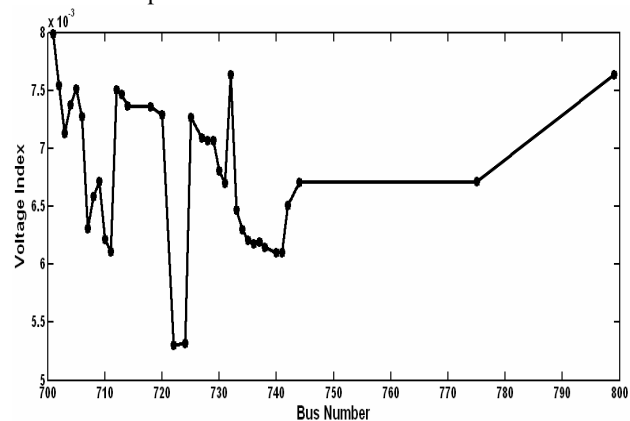


Fig. 5 Voltage Index for 37-bus IEEE URDS

VI. CONCLUSIONS

This paper presents a methodology for solving the best location and sizing of DG problem in unbalanced radial distribution systems through Voltage Index Analysis and Variational Algorithm with the standard sizes of DG. The effectiveness of the algorithm has been demonstrated and tested. The sizing of DG has been obtained with an objective function of reducing power losses. The proposed methodology was successfully applied to IEEE 25 node and IEEE 37 node URDS test feeders. The obtained solution has been successful in reducing total active power losses in 25 node URDS and in 37 node URDS. Thus the proposed method has been observed as efficient for solving DG placement and sizing in unbalanced radial distribution systems.

TABLE III
VOLTAGE PROFILE FOR IEEE 37BUS URDS

Bus No.	Before DG Placement			After DG Placement		
	Va (p.u)	Vb (p.u)	Vc (p.u)	Va (p.u)	Vb (p.u)	Vc (p.u)
799	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
701	0.9878	0.9894	0.9812	0.9987	0.9967	0.9871
702	0.9808	0.9825	0.9722	1.0000	0.9957	0.9813
703	0.9720	0.9779	0.9652	0.9976	0.9954	0.9763
730	0.9658	0.9736	0.9593	0.9968	0.9948	0.9715
709	0.9636	0.9720	0.9581	0.9965	0.9945	0.9705
708	0.9600	0.9705	0.9559	0.9956	0.9949	0.9690
733	0.9558	0.9715	0.9535	0.9923	0.9962	0.9661
734	0.9505	0.9727	0.9490	0.9886	0.9980	0.9608
737	0.9437	0.9746	0.9471	0.9828	1.0000	0.9584
738	0.9418	0.9754	0.9454	0.9814	1.0009	0.9565
711	0.9421	0.9758	0.9432	0.9820	1.0015	0.9542
741	0.9422	0.9760	0.9425	0.9822	1.0017	0.9534
713	0.9805	0.9797	0.9703	1.0023	0.9949	0.9803
704	0.9797	0.9752	0.9694	1.0053	0.9935	0.9807
720	0.9816	0.9685	0.9672	1.0122	0.9910	0.9810
706	0.9819	0.9674	0.9674	1.0125	0.9901	0.9810
725	0.9820	0.9667	0.9675	1.0127	0.9894	0.9811
705	0.9811	0.9807	0.9704	1.0007	0.9941	0.9792
742	0.9812	0.9791	0.9707	1.0009	0.9925	0.9794
727	0.9703	0.9777	0.9642	0.9964	0.9954	0.9751
744	0.9690	0.9776	0.9641	0.9955	0.9954	0.9747
729	0.9682	0.9777	0.9642	0.9949	0.9956	0.9747
775	0.9636	0.9720	0.9581	0.9965	0.9945	0.9705
731	0.9640	0.9699	0.9584	0.9971	0.9926	0.9707
732	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
710	0.9510	0.9717	0.9464	0.9897	0.9973	0.9578
735	0.9511	0.9719	0.9453	0.9900	0.9974	0.9566
740	0.9422	0.9760	0.9421	0.9822	1.0017	0.9530
714	0.9792	0.9752	0.9695	1.0049	0.9935	0.9807
718	0.9764	0.9756	0.9699	1.0025	0.9940	0.9808
707	0.9832	0.9580	0.9675	1.0219	0.9881	0.9873
722	0.9833	0.9569	0.9675	1.0230	0.9880	0.9881
724	0.9834	0.9559	0.9678	1.0226	0.9863	0.9873
728	0.9686	0.9772	0.9637	0.9952	0.9951	0.9743
736	0.9514	0.9683	0.9469	0.9910	0.9943	0.9576
712	0.9813	0.9809	0.9691	1.0011	0.9943	0.9779

TABLE IV
SUMMARY OF TEST RESULTS FOR 37 BUS IEEE URDS

Description		Before DG Placement			After DG Placement		
		Phase a	Phase b	Phase c	Phase a	Phase b	Phase c
Distributed generator placed bus and size of DG	722	-	-	-	160 kW At 0.85 p.f	160 kW at 0.85 p.f	160 kW at 0.85 p.f
Minimum Voltage		0.9418	0.9559	0.9425	0.9814	0.9863	0.9534
Voltage regulation (%)		5.03	4.12	5.55	1.87	1.37	4.70
Improvement of Voltage regulation (%)		-	-	-	62.82	66.74	15.45
Active Power Loss (kW)		31.56	23.67	30.44	25.13	2.91	23.16
Total Active Power Loss reduction (%)		-	-	-	20.37	87.70	23.91
Reactive Power Loss (kVAr)		24.01	22.32	29.19	22.76	2.91	23.16
Total Reactive Power Loss reduction (%)		-	-	-	5.20	87.52	20.16
Total Demand (kW)		885.56	789.67	1163.4	879.13	768.91	1156.12
Total Released Demand (kW)		-	-	-	6.43	20.76	7.28
Total Reactive Power Demand (kVAr)		442.01	397.32	521.81	440.76	376.91	515.78
Total Released Reactive Power Demand (kVAr)		-	-	-	1.25	20.41	6.03
Total Feeder Capacity (kVA)		989.74	883.99	1275.1	983.43	856.31	1265.95
Total Released Feeder Capacity (kVA)		-	-	-	6.31	27.68	9.15

APPENDIX

TABLE A1
Load data and line connectivity of 25-bus unbalanced radial distribution system

Branch	Sending End	Receiving End	Conductor type	Length, ft	Receiving end load in kVA		
					A phase	B phase	C phase
1	1	2	1	1000	0	0	0
2	2	3	1	500	35 + j25	40 + j30	45 + j32
3	2	6	2	500	40 + j30	45 + j32	35 + j25
4	3	4	1	500	50 + j40	60 + j45	50 + j35
5	3	18	2	500	40 + j30	40 + j30	40 + j30
6	4	5	2	500	40 + j30	40 + j30	40 + j30
7	4	23	2	400	60 + j45	50 + j40	50 + j35
8	6	7	2	500	0	0	0
9	6	8	2	1000	40 + j30	40 + j30	40 + j30
10	7	9	2	500	60 + j45	50 + j40	50 + j35
11	7	14	2	500	50 + j35	50 + j40	60 + j45
12	7	16	2	500	40 + j30	40 + j30	40 + j30
13	9	10	2	500	35 + j25	40 + j30	45 + j32
14	10	11	2	300	45 + j32	35 + j25	40 + j30
15	11	12	3	200	50 + j35	60 + j45	50 + j40
16	11	13	3	200	35 + j25	45 + j32	40 + j30
17	14	15	2	300	133.3 + j100	133.3 + j100	133.3 + j100
18	14	17	3	300	40 + j30	35 + j25	45 + j32
19	18	20	2	500	35 + j25	40 + j30	45 + j32
20	18	21	3	400	40 + j30	35 + j25	45 + j32
21	20	19	3	400	60 + j45	50 + j35	50 + j40
22	21	22	3	400	50 + j35	60 + j45	50 + j40
23	23	24	2	400	35 + j25	45 + j32	40 + j30
24	24	25	3	400	60 + j45	50 + j30	50 + j35

Type	Impedance in ohms/mile			
	a	b	c	
1	a	0.3686 + j0.6852	0.0169 + j0.1515	0.0155 + j0.1098
	b	0.0169 + j0.1515	0.3757 + j0.6715	0.0188 + j0.2072
	c	0.0155 + j0.1098	0.0188 + j0.2072	0.3723 + j0.6782
2	a	0.9775 + j0.8717	0.0167 + j0.1697	0.0152 + j0.1264
	b	0.0167 + j0.1697	0.9844 + j0.8654	0.0186 + j0.2275
	c	0.0152 + j0.1264	0.0186 + j0.2275	0.9810 + j0.8648
3	a	1.9280 + j1.4194	0.0161 + j0.1183	0.0161 + j0.1183
	b	0.0161 + j0.1183	1.9308 + j1.4215	0.0161 + j0.1183
	c	0.0161 + j0.1183	0.0161 + j0.1183	1.9337 + j1.4236

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