Reliable Capacitated Facility Location Problem Considering Maximal Covering

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Abstract—This paper provides a framework in order to incorporate reliability issue as a sign of disruption in distribution systems and partial covering theory as a response to limitation in coverage radios and economical preferences, simultaneously into the traditional literatures of capacitated facility location problems. As a result we develop a bi-objective model based on the discrete scenarios for expected cost minimization and demands coverage maximization through a three echelon supply chain network by facilitating multi-capacity levels for provider side layers and imposing gradual coverage function for distribution centers (DCs). Additionally, in spite of objectives aggregation for solving the model through LINGO software, a branch of LP-Metric method called Min-Max approach is proposed and different aspects of corresponds model will be explored.

I. INTRODUCTION

ACTUALLY, the most important and difficult issue in supply chain (SC) structure is making strategic decision about the location of facilities related to different layers. Since such a decision belongs to long term planning and it isn't preferable and cost-effective to change over a short or even middle-age period of time, all efforts must be done to take into account the different aspects of SC uncertainties [1].

This study covers two fields of uncertainty simultaneously which are resulted due to the disruption in distribution systems & economical preferences and coverage limitations. In fact in basic facility location problems the default goal is to minimize both fixed cost of opening facilities and transportation costs while supplier layers are truly functioning and 100% of customers demands are supposed to be covered. But in reality provider-side facilities because of weather conditions, labor sabotage, getting into conflict with subcontractors of distribution duties, natural disasters and etc are subject to failure. It is also proven that considering probability of facilities failure could change the topology of SC structure and leads to block future cost of unreliable facilities. Moreover, enforcing the traditional models to cover all demands of customers could be unpractical and unreasonable.

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Because in real world there is coverage radios limitation of distributors and besides that sometimes covering less than total demands with cheaper costs can be more profitable than all demands coverage with higher costs. So in the proposed model optimum Pareto solutions are consist of set of plants and DCs with their correspond level of capacity and distribution patterns for goods delivery from plants through customer zones that are adopted with respect to cost minimization and coverage maximization in different scenarios.

However, to solve such a bi-objective model through commercial software, an aggregation approach for objectives required to be applied. Therefore in this paper we utilize from a new branch of LP-Metric called Min-Max method.

The rest of this study in order to cover the following discussion is designed as follows. In section 2 the literature review of related studies is presented. Next in section 3 the problem definition (Assumptions, Notations and Model) is described then in section 4 solution procedure is proposed. Consequently, section 5 provides computational results. Finally, conclusions and recommendations for future studies will be explored in section 6.

II. LITERATURE REVIEW

Literature on area of supply chain management (SCM) extended by gradual involvement of Operation Research science and consequently incorporation of facility location problems into the SCM studies [2]. The first event of this mixture resulted to configuration of basis facilities location model with fixed cost called UFLP which concerned to minimize fixed & transportation costs in two echelon SC structure [3]. After it, researchers have tried to omit assumptions of correspond basic model (such as deterministic demand, single good, two echelon structure of SC, total capacity usage of transportation facilities, single capacity level of facilities un-capacitated feature of facilities, 100% coverage of demands, truly functioning of all facilities and etc) in order to configure more adaptive structure with real world [4].

In this spite, Snyder & Daskin (2005) have focused their study in order to investigate disruption in distribution systems and degree of cost incurred to whole structure of SC because of not considering probability of DCs failure [5]. They developed a two echelon un-capacitated bi-objective model by making a trade-off between basic objective function and expected transportation cost (by means of backup coverage idea in congestion systems applied in [6]) in times of considering probability of DCs failure. They called their model as Reliable UFLP (RUFLP) and solved it by Lagrangian Relaxation through weighting method which its

computational results indicated that by little consideration to fact of uncertainties in supplying layers, large future losses could be hedged. Shen et al. (2010) by gathering two goals of [5] into the single objective function utilized from scenario programming instead of backup coverage idea in order to compute expected transportation cost and solve it via Sample Average Approximation (SAA) model [7]. Gade & Pohl (2009) extended the model formulation of [5] by imposing capacitated constraint for DCs [8].

Also as a new development of covering models a partial coverage type called as Generalized Hierarchical Covering Location Problem (G-HCLP) is utilized by Lee & Lee (2010) who uses gradual coverage function instead of static ones for modeling the DCs radios coverage limitation [9]. Besides that Berman et al. (2010) proposed idea about direct relationship between capacity of facilities and their coverage radios which could be practical in reality especially in multi capacities level problems (such as [10]) [11].

Finally, to best of our knowledge, it should be noted that current study by involving RCFLP and new development of partial covering into the basic models has presented a novel model formulation.

III. PROBLEM MODELING

In this section besides addressing the nomenclature, problem assumption and model formulation are described.

A. Notation

The notations given in nomenclature are required for the purpose of this paper.

1. Indices

I: index set of customers

J: index set of distributor centers

K: index set of suppliers

S: index set of all possible states of distributor centers working and failure except for states of all DCs failure at once. In fact it totally includes $\sum_{z=0}^{J-1} \binom{J}{z}$ number of possible states (suppose at s=1 all DCs work properly).

F_s: Index set of failed DC at state s which is consist of $n(F_s)$ elements

F_s': Complement set of F_s which is consist of $n(F_s')$ elements

R: index set of capacity levels for potential distributor centers

H: index set of capacity levels for potential suppliers

2. Parameters

a_i: Customer demand of zone i

b_j^r: Capacity of DC j at capacity level r

q: Failing probability of each proper DC

p_s: Probability of each state occurrence which is equal to $q^{n(F_s)}(1-q)^{n(F_s')}$, but according to the 4th problem assumption in s=1, $p_s = 1$.

$\emptyset^r(d_{ij})$: Partial degree of coverage demand point i provided by distributor center j at capacity level r

$o_{ij}^r = \begin{cases} 1 & if \ \emptyset^r(d_{ij}) \neq 0 \\ 0 & if \ \emptyset^r(d_{ij}) = 0 \end{cases}$

F_j^r: Fixed cost of opening DC j at capacity level r

G_k^h: Fixed cost of opening plant k at capacity level h

e_k^h: Capacity of plant k at capacity level h

C_{1ij}: Unit cost of transporting commodities from DC j to customer zone i

C_{2jk}: Unit cost of transporting commodities from plant k to DC j

$Q_{s,j} = \begin{cases} 1 & if \ j \in F_s' \\ 0 & otherwise \end{cases}$

f_1^*: Optimal value of 1st objective function when the 2nd one is ignored

f_2^*: Optimal value of 2nd objective function when the 1st one is ignored

γ_1: Constant weighting factor of 1st objective function

γ_2: Constant weighting factor of 2nd objective function

3. Decision Variables

X_{ij}^s: Percent amount of satisfying customer zone i demand by DC j at state s

Y_{jk}^{rs}: Percent amount of supplying DC j by plant k at state s and capacity level r

$U_j^r = \begin{cases} 1 & if \ a \ DC \ at \ capacity \ level \ r \ is \ opened \ in \ location \ j \\ 0 & otherwise \end{cases}$

$V_k^h = \begin{cases} 1 & if \ a \ plant \ at \ capacity \ level \ h \ is \ opened \ in \ location \ k \\ 0 & otherwise \end{cases}$

B. Assumption

- There is a direct relationship between capacity level and coverage ratio served by DCs.
- The input parameters are deterministic.
- Multiple capacities are allowed for plants & DCs.
- All DCs work properly at the beginning of running model.
- DCs have uniform failure probabilities.
- Plants can supply DCs without any coverage constraint.

C. Model Formulation

Here a bi-objective model addressee to optimize total supply cost in certain environment, expected failure cost and demands coverage simultaneously. In the rest of the paper, the proposed model will be called as MRCFLP (Maximal Reliable Capacitate Facility Location Problem) and formulated as bellows.

$$\text{Min } f_1 = \sum_{j \in J} \sum_{r \in R} F_j^r U_j^r + \sum_{k \in K} \sum_{h \in H} G_k^h V_k^h$$
$$+ \sum_{s \in S} p_s(\sum_{i \in I} \sum_{j \in J} C_{1ij} a_i X_{ij}^s + \sum_{r \in R} \sum_{j \in J} \sum_{k \in K} C_{2jk} b_j^r Y_{jk}^{rs}) \quad (1)$$

$$\text{Max } f_2 = \frac{1}{\sum_{i \in I} a_i} (\sum_{i \in I} \sum_{j \in J} \sum_{s \in S} p_s a_i X_{ij}^s) \quad (2)$$

Subject to:

$$\sum_{j \in J} X_{ij}^s \leq 1 \qquad \forall i \in I \ \& \ s \in S \quad (3)$$

$$X_{ij}^s \leq \sum_{r \in R} \emptyset^r(d_{ij}) U_j^r Q_{s,j} \quad \forall i \in I \ \& \ j \in J \ \& \ s \in S \quad (4)$$

$$\sum_{r \in R} U_j^r \leq \sum_{i \in I} \sum_{r \in R} o_{ij}^r \quad \forall j \in J \quad (5)$$

$$\sum_{i \in I} a_i X_{ij}^s \leq \sum_{k \in K} \sum_{r \in R} b_j^r Y_{jk}^{rs} \quad \forall j \in J \ \& \ s \in S \quad (6)$$

$$\sum_{k \in K} Y_{jk}^{rs} \leq U_j^r \quad \forall j \in J, r \in R \ \& \ s \in S \quad (7)$$

$$\sum_{j \in J} \sum_{r \in R} b_j^r Y_{jk}^{rs} \leq \sum_{h \in H} e_k^h v_k^h \quad \forall k \in K \,\&\, s \in S \quad (8)$$

$$\sum_{r \in R} U_j^r \leq 1 \quad \forall j \in J \quad (9)$$

$$\sum_{h \in H} V_k^h \leq 1 \quad \forall k \in k \quad (10)$$

$$X_{ij}^s \geq 0 \quad \forall i \in I \,\&\, j \in J \,\&\, s \in S \quad (11)$$

$$U_j^r \in \{0,1\} \quad \forall j \in J \,\&\, r \in R \quad (12)$$

$$Y_{jk}^r \geq 0 \quad \forall j \in J \,\&\, k \in K \,\&\, r \in R \quad (13)$$

$$V_k^h \in \{0,1\} \quad \forall k \in K \,\&\, h \in H \quad (14)$$

In MRCFLP, 1st objective function (1) wants to minimize both fixed and expected transportation costs. 2nd one (2) is interested to maximize demands coverage in all possible states. Constraint (3) prohibits from servicing a customer zone more than its demand in any state. Constraint (4) states that the fraction of customer i demand satisfied by DC j at state s must be less than coverage ratio of zone i provided by (j). Also it implies that is necessary for j to be opened at level r and not be failed at state s in advance. Constraint (5) ensures that a DC can be opened at site j if it be able to partially cover at least one customer. Constraint (6) monitors that a DC output doesn't pass its inventory supplied from all potential plants. Constraint (7) imply two fact concurrently: firstly prevents from allocating potential plants to DC j since it hasn't been opening and secondly adjust an opened inventory DC at last as equal as its capacity. Constraint (8) for a plant k blocks extra potential DCs supplying which overflows plant k capacity. Constraint (9) and (10) avoid model in establishing DCs and plants with more than one capacity level in any index set of J and K respectively. Finally, Constraints (11) through (14) determine type of variables.

IV. SOLUTION PROCEDURE

Generally, aggregation methods based on necessity of whether to identify initial information of Decision Maker (DM) are categorized into the two groups. Indeed, LP-Metric method is one of the remarkable approaches that try to reach the optimal Pareto-front by laying different weighting factors among objective functions without any dependency to initial DM's preferences [12].

However, one of the big challenges in LP-Metric is the determination of P value. In 1991 it was proved by assigning the $P = \infty$ will change the formulation to a Min-Max approach while in terms of feasible values of γ_i efficient solutions of correspond model will be resulted. Therefore following model will reformulated as bellows.

$$\text{Min } \alpha \quad (15)$$

Subject to:

$$\alpha \geq \gamma_1 \left[\frac{f_1 - f_1^*}{f_1^{\wedge} - f_1^*} \right] \quad (16)$$

$$\alpha \geq \gamma_2 \left[\frac{f_2^* - f_2}{f_2^* - f_2^{\wedge}} \right] \quad (17)$$

$$\gamma_1 + \gamma_2 = 1 \quad (18)$$

$$\sum_{j \in J} X_{ij}^s \leq 1 \qquad \forall i \in I \,\&\, s \in S \quad (19)$$

$$X_{ij}^s \leq \sum_{r \in R} \emptyset^r(d_{ij}) U_j^r Q_{s,j} \quad \forall i \in I \,\&\, j \in J \,\&\, s \in S \quad (20)$$

$$\sum_{r \in R} U_j^r \leq \sum_{i \in I} \sum_{r \in R} o_{ij}^r \quad \forall j \in J \quad (21)$$

$$\sum_{i \in I} a_i X_{ij}^s \leq \sum_{k \in K} \sum_{r \in R} b_j^r Y_{jk}^{rs} \quad \forall j \in J \,\&\, s \in S \quad (22)$$

$$\sum_{k \in K} Y_{jk}^{rs} \leq U_j^r \quad \forall j \in J, r \in R \,\&\, s \in S \quad (23)$$

$$\sum_{j \in J} \sum_{r \in R} b_j^r Y_{jk}^{rs} \leq \sum_{h \in H} e_k^h v_k^h \quad \forall k \in K \,\&\, s \in S \quad (24)$$

$$\sum_{r \in R} U_j^r \leq 1 \quad \forall j \in J \quad (25)$$

$$\sum_{h \in H} V_k^h \leq 1 \quad \forall k \in k \quad (26)$$

$$X_{ij}^s \geq 0 \quad \forall i \in I \,\&\, j \in J \,\&\, s \in S \quad (27)$$

$$U_j^r \in \{0,1\} \quad \forall j \in J \,\&\, r \in R \quad (28)$$

$$Y_{jk}^r \geq 0 \quad \forall j \in J \,\&\, k \in K \,\&\, r \in R \quad (29)$$

$$V_k^h \in \{0,1\} \quad \forall k \in K \,\&\, h \in H \quad (30)$$

V. COMPUTATIONAL RESULTS

In this section in order to evaluate the performance of LINGO software against small, medium and size problems, three suppositional Dataset was generated which the Ds1 is consist of 10 customers, 4 DCs and 2 plants, Ds2 includes 27 customers, 8 DCs and 3 plants and Ds3 is made up of 60 customers, 11 DCs and 6 plants. Then, based on the two Dataset and different values of (γ_i), value of first and second objective functions, vector of opened plants (OP) and DCs (OD) at their capacity levels and correspond solution time were tested.

TABLE I
LINGO RESULTS FOR THREE DATASET

	γ		j	OP	OD	Time (s)
Ds1	1	0	0	[2,0]	[3,3,3,0]	14
	0.5	3756910	1.58	[3,0]	[3,3,3,0]	19
	0	5163332	1.73	[3,3]	[3,3,3,3]	15
Ds2	1	0	0	[0,0]	[0,0,0,0,0,0,0,0]	74
	0.5	6959825	1.77	[3,0]	[3,3,3,0,3,0,3,3]	25203
	0	14305400	1.82	[3,3]	[3,3,3,3,3,3,3,3]	5443
Ds3	1					
	0.5			Ran out of memory		
	0					

As it is seen from Table I, by increasing the size of the problem solution time exponentially augments. This event is basically due to the increment in the number of possible failure sates belong to DCs. For instance in Ds3 there are $2^{11} - 1 = 2046$ states which disables LINGO to even run its internal procedure.

VI. CONCLUSION

In this paper by incorporation of a new development of maximal covering into the 3 echelons RCFLP, a novel model was presented. Also in order to solve correspond bi-objective model, we utilized from a new branch of LP-Metric aggregation method. In the proposed solution procedure by adding three constraint and parameters added to main body of model the Min-Max approach was resulted. Moreover, computational results indicated that the increase in size of the problem intentionally makes the solution intractable which strongly suggests the use of heuristic methodologies. Besides that for future studies it's worthwhile to include multi statue for DCs functioning situation instead of binary state represented in this paper.

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