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Design A Three-dimensional pursuit guidance law with Feedback Linearization Method

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***Abstract*—**In this paper, we will implement three-dimensional pursuit guidance law with feedback linearization control method and study the effects of parameters. First, we introduce guidance laws and equations of motion of a missile. Pursuit guidance law is our highlight. We apply feedback linearization control method to obtain the accelerations to implement pursuit guidance law. The solution makes warhead direction follow with line-of-sight. Final, the simulation results show that the exact solution derived in this paper is correct and some factors e.g. control gain, time delay, are important to implement pursuit guidance law.

***Keywords*—**pursuit guidance law, feedback linearization

I. Introduction

THE pursuit guidance law has been both well studied and successfully implemented in many flight vehicles. In this paper, we attempt to combine the philosophies of feedback linearization control and the nonlinear engagement kinematics to implement pursuit guidance law.

At this time have to speak of the guidance law. Guidance law is a calculation formula, its function is through missile's core computer to calculate the relative position in space between target and missile, and then computing a kind of method make missile approach to target. Guidance law contain rather many types, as we all know of them are Pursuit guidance law, Parallel approach logarithm and ratio guidance law. Guidance law plays an important role in process of missile attacking object. Hence the research papers are also considerable variety, such as paper [1] [2] [3] are talking about the calculation and application for ratio guidance law. The paper [4] talks about pursuit guidance law and ratio guidance law at the same time. However take pursuit guidance law for example, all papers assume missile attitude angle has been the same of visual angle to carry out research in past. Among them did not explain how in the equation of motion of the missile carry out pursuit Guidance law with acceleration. Another in carrying out process, the time delay and the effect of tracking order quality are no discussing. Therefore, this paper will implement non-linear pursuit guidance law with feedback linearization control method, and through the method to explore the capability and the stability to track the angle of sight.

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II. Missile Guidance Law Introduce

Missile guidance law is the core calculation formula in guidance system, and its quality will have a direct impact of ability of missile hit the target. In the modern Aerial warfare, fighter planes have excellent performance and have the ability to make high-G action to avoid missile attacks, so for air-to-air tactical missiles, the design of guidance law has become very important to hit target. In this paper is following adoption of pursuit guidance law [5]. So called pursuit guidance law is a guidance approach of missile attacks target which speed vector always point to a goal. The requirement for this method is the visual angle that between missile and objective equal to zero. Therefore, the guided relation equation for pursuit guidance law is

$$\eta = 0 \tag{1}$$

Missile and object related equations of motion as follows and the location of motion as figure1. Use letters for table footnotes (see Table I).

$$\begin{aligned} \dot{r} &= V_T \cos \eta_T - V \\ r\frac{dq}{dt} &= -V_T \sin \eta_T \\ q &= \sigma_T + \eta_T \end{aligned} \tag{2}$$

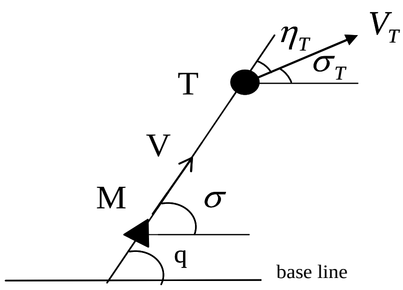

Fig. 1 Location of motion between missile and object

III. Missile Motion Equation

The principle of pursuit guidance law is the same as hunting

TABLE I

Symbol	Description
r	The distance between missile and target, when missile hit target, r =0.
V_T , V	The speed of target and missile.
q	The angle between goal line and baseline, called the goal line azimuth (the brief name is goal-line angle). Counter-clockwise rotates from baseline to goal line, at this time q is positive.
σ , σ_T	σ is the angle between speed vector and baseline of missile, called missile trajectory angle. σ_T is the angle between speed vector and baseline of target, called target heading angle. Respect takes missile and object location as origin point, counter-clockwise rotates from the speed vector to speed vector, at this time σ , σ_T are positive.
η , η_T	η is the angle between speed vector and goal line on missile, called pre-angle of speed vector of missile. η_T is the angle between speed vector and goal line of target, called pre-angle of speed vector of target. Respect takes the missile and the object location as origin point, counter-clockwise rotates from the speed vector to goal line, at this time η , η_T are positive.

dog grasp rabbit, as long as the warhead towards the goal forever, it will be able to pursue the goal ultimately. Therefore, in this study will focus on derivation of the guidance law and explore the hidden meaning.

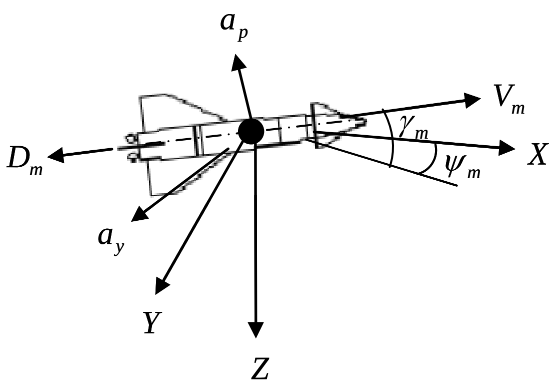

Fig. 2 Flight motion of missile

First of all, we define missile's equation of motion in space, the equation descript detail as follows:

$$\dot{V}_m = \frac{1}{m_m}\left(T_m - D_m\right) - g\sin\gamma_m \tag{3}$$

$$\dot{a}_p = \frac{\left(a_{pc} - a_p\right)}{\tau} \tag{4}$$

$$\dot{a}_y = \frac{\left(a_{yc} - a_y\right)}{\tau} \tag{5}$$

$$\dot{\gamma}_m = \frac{\left(a_p - g\cos\gamma_m\right)}{V_m} \tag{6}$$

$$\dot{\Psi}_m = \frac{a_y}{V_m\cos\gamma_m} \tag{7}$$

$$\dot{x}_m = V_m\cos\gamma_m\sin\Psi_m \tag{8}$$

$$\dot{y}_m = V_m\cos\gamma_m\cos\Psi_m \tag{9}$$

$$\dot{z}_m = V_m\sin\gamma_m \tag{10}$$

Symbols are described as follows table footnotes (see Table II).

TABLE II

Symbol	Description
γ_m	Path angle, the angle between missile's speed vector and horizontal plane.
Ψ_m	heading angle, the angle between missile's speed vector project on horizontal plane and the X-axis.
X_m, Y_m, Z_m	Missile inertial coordinates of the center of mass.
a_p	Acceleration along the direction of pitch.
a_{pc}	Acceleration command symbol along the direction of pitch.
a_y	Acceleration along the direction of Yaw.
a_{yc}	Acceleration command symbol along the direction of Yaw.
τ	Missile time constant.
T_m	Missile's thrust, different missiles have different values.
D_m	Missile flight resistance.

Through the equation of motion will be able to accurately predict missile's dynamic in space, then coupled with the conversion of coordinates to calculate the location of the missile in space, and then calculate the relative position between aircraft and missile.

IV. Implement Pursuit Guidance Law With Feedback Linearization Control Method

When define missile's equation of motion, and then we explore the relationship between pursuit guidance law and equation of motion of missile. The pursuit guidance law emphasizes the visual angle between missile and target equal to zero. Therefore, from figure 1 we can know if visual angle is zero, so γ_m and ψ_m exactly the same as ϕ and θ. Therefore, equation (8) (9) (10) equal to speed vector of X, Y, Z axis, No matter what kind of flight movements of target, the missile speed vector should always point to the target. Therefore, as long as solve the equation $\dot{X}_m$, $\dot{Y}_m$ and $\dot{Z}_m$, we can find missile's coordinates X_m , Y_m and Z_m in space. From the equation (8) (9) (10) can know, for solve $\dot{X}_m$, $\dot{Y}_m$ and $\dot{Z}_m$ have to obtain γ_m and ψ_m , but solving γ_m and ψ_m , first have to solve a_p and a_y . a_p is Acceleration along the direction of Pitch, in other word is longitudinal acceleration. a_y is Acceleration along the direction of Yaw, in other word is lateral acceleration. But from equation (4), (5), for solve a_p and a_y have to through a dynamic time delay model. That is to say, when the missile issue command and change direction to pursue the goal that can through change a_p and a_y values to complete the task, but its value will not equal to order instantaneous, must trough dynamic time delay model of missile, then the values similar. Therefore, first we defined a_{pc} and a_{yc} , a_{pc} is acceleration command symbol along the direction of pitch, a_{yc} is acceleration command symbol along the direction of Yaw. Through these two commands values can let γ_m and ψ_m close to ϕ and θ, in figure 3.

From equation (6) can derived

$$\dot{\Psi}_m = \frac{a_y}{V_m \cos\gamma_m} \tag{11}$$

The term of $g\cos\gamma_m$ will make the equation (11) become a nonlinear equation and make system unstable. So we combine with feedback linearization control method to make equation (11) into a linear differential equation and ensure the system stable.

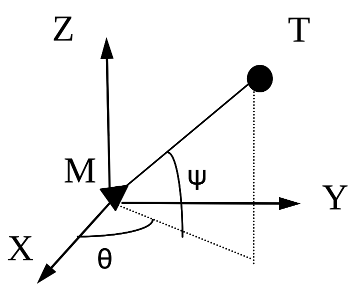

Fig. 3 Direction and location in space

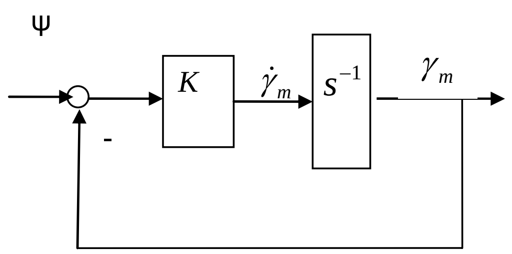

Fig. 4 Block diagram of feedback system

We can see from figure 4, in order to make γ_m tracking ϕ, have to transform equation (11) into (12)as follows

$$\dot{\gamma}_m + K\gamma_m = K\phi \tag{12}$$

So must be the elimination of $g\cos\gamma_m$. We assume as follows

$$a_p = a_{pc} = V_m(K\phi - K\gamma_m) + g\cos\gamma_m \tag{13}$$

Make equation (13) into (11), through calculation can get equation (12). By the same theorem assume equation (7) as follows

$$a_y = a_{yc} = KV_m \cos\gamma_m(\theta - \Psi_m) \tag{14}$$

Then transform equation (7) into (15) as follows

$$\dot{\Psi}_m + K\Psi_m = K\theta \tag{15}$$

Through the formula (12) and (15), γ_m and ψ_m respectively will enable close to ϕ and θ, that is the angle between target and missile. Among them, K value is feedback constant, by the constant modulate will be able to change the speed of γ_m and ψ_m respectively close to ϕ and θ. The impact of the modulation will be discussed in simulation.

V. Simulation Results

Equations have many parameters, by the modulation of these parameters will have different results, and also cause different effects. Therefore, we will discuss the effects from these parameters.

A. Simulation results of missile pursue two kinds of objectives.

Assuming both cases, first is missile pursue target that fly with straight line trajectory. Second, missile pursue target that fly with changing trajectory. First of all, to define the initial values of missile and target, initial values assume as follows table footnotes (see Table III and Table IV).

TABLE III
MISSILE AND TARGET INITIAL VALUES

	Location in X axis (ft)	Location in Y axis (ft)	Location in Z axis (ft)	Initial speed(ft/sec)
missile	0	0	0	200
target	0	0	500	100

TABLE IV
TIME CONSTANT

	integral-step Δt	X xais time constant τ_x	Y xais time constant τ_y	Z xais time constant τ_z
missile	0.01sec	0.1sec	0.1sec	0.1sec

Figure 5 is missile pursue target that fly with straight line trajectory, we can see that missile and target in the same XY plane, at this time there is no different value of ϕ. Thus missile fast approach to the value of θ when attack target at beginning. Therefore, we can see using pursuit guidance law method to pursue target, the warhead must always move forward towards the target.

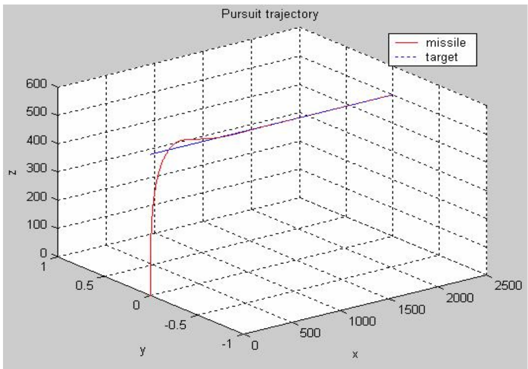

Fig. 5 Missile pursue fixed trajectory-straight line target

When we are confident that missile can pursue target normally, then simulate missile pursue target that fly with changing trajectory as figure 6.

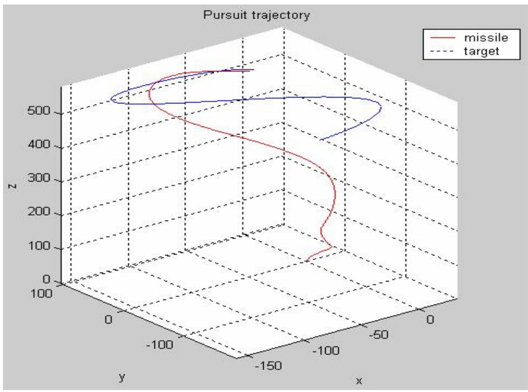

Fig. 6 Missile pursue trajectory changes target

From figure 6, we can see that pursuit guidance law method also applies to trajectory-change target. This means that missile could be changed its attack angle due to target at any time. So warhead can move forward towards the target and pursue the goal ultimately. From above two cases could prove the value of a_{pc} and a_{yc} correctly, and by the two command symbol value can make γ_m and ψ_m respectively close to ϕ and θ, then achieve the purpose of pursuit guidance law method.

B. Impact of integral-step on missile

First of all discuss the impact of integral-step on missile pursue target. So-called the integral-step is the interval time of computer program calculate guidance law every time. When the integration interval time increase, that means the next step of computing interval time also increase. Because of computing interval time increase, at this time missile will deviate the normal pursuit track. What is the cause of the phenomenon? When the missile is close to the tail of target, because of interval time increase caused missile has exceeded the normal flight path at program issued next action, at this time missile have to spend more time to lead position return to normal trajectory. So under the same condition, if the integral-step increase will cause spent more time to pursue target. From table V compare the impact of different integral-step interval time.

TABLE V
MISSILE FLY TIME WITH DIFFERENT INTEGRAL-STEP

Integral-step Δt	0.01sec	0.1sec
Target fly with straight line trajectory	25.78sec	27sec

In addition to the integral-step interval time affect the outcome of the missile pursuit, three time constants of axis will also affect pursuit result, because time constant represents delay time, so the value is the smaller the better, that is to say the performance of missile is more intelligent. When the program issue yaw or pitch order, which can quickly respond to missile movements, if take the same object that fly with straight line as a goal, and the time constant will be replaced by table VI, the results shown in figure 7.

TABLE VI
TIME CONSTANT

	integral-step Δt	X xais time constant τ_x	Y xais time constant τ_y	Z xais time constant τ_z
missile	0.01sec	1sec	1sec	1sec

From figure 7, we can see that increase time constants of three axis will decrease the performance of missile, the guidance law ideal trajectory that calculated by computer program can not be combined with the missile movements. Missile has been adjusted γ_m in order to be able to approach ϕ. Therefore, missile not be able to keep up with the ideal pursuit trajectory, the phenomenon are forming on figure 7. So we can understand the impact of time constants on missile pursue target.

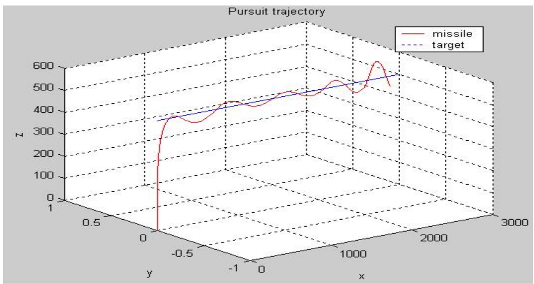

Fig. 7 The trajectory of Missile pursue target that fly in straight line with different time constants

C. Impact of K value on missile pursuit

In the equation (12) and (15), we can find the speed of γ_m and ψ_m respectively close to ϕ and θ depend on value k, which is feedback constant. In order to compare the impact of value k on missile guidance, we modulate value k. If missile pursue trajectory-change target, set initial condition value k=10 and the results obtained as figure 8 left, and then lower again value K=1, the result as figure 8 right. After comparing the two figures can find that the K value is high, the decline in value θ is faster. This is because the greater feedback constant will enable γ_m and ψ_m become small in feedback loop, when missile pursue target γ_m and ψ_m fast approach to ϕ and θ. That is program command can quickly pass to missile, reduce angle of turn and pursue the target with minimum time.

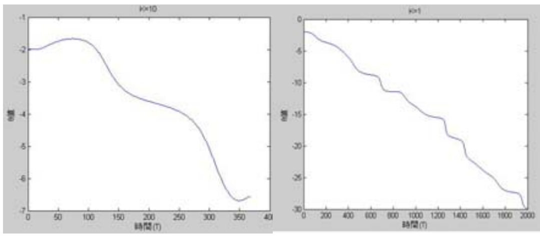

Fig. 8 k=10 and k=1 θ value versus time

D. The impact of speed on missile pursuit

We observe the impact of speed on missile pursuit by modulate missile speed, assuming missile pursue trajectory-change target and is speed up to 300 and 400 ft / sec, the conditions are shown in table VII.

TABLE VII
MISSILE FLY TIME WITH DIFFERENT SPEED

Speed(ft/sec)	200	300	400
Target fly with changing trajectory	3.33sec	2.28sec	1.66sec

From table VII, we can see that missile speed more quickly, pursuing target of shorter time, but it should be noted that missile must have its speed limit, in order to prevent speed too fast to over-pursuit, then must spend more time to turn head to go on pursue target.

E. The impact of error on missile pursuit

In reality, missile's sensors and actuators subject to instruments precision and create error. Among them, sensor error will cause the error of ϕ and θ value, The error due to actuator will cause the error of γ_m and ψ_m value in missile, Therefore, the following will explore the impact of error on system simulation. First of all explore the impact of the error due to actuator that cause the error of γ_m and ψ_m value, assuming the value of error random generate, from simulation result can find if the error in the 20 degrees outside, the missile can no longer pursue the target. But if error values random choose less than 20 degrees, although the missile can pursue target but the trajectory already in irregular status as shown in figure 9. The main reason for this situation is the error degree too large, although the missile can be guided to pursue the target, but can not be the best state to complete task.

Assuming again the error degree less than 5 degrees, the tracking trajectory as shown in figure 10, and the tracking trajectory has been similar to no error ones. Therefore, this can determine if the actuator error can be controlled less than 5 degrees which does not affect missile pursue the target.

Next we will discuss the error of ϕ value and θ value caused by sensor error, assuming the value of error random generate, from simulation result know the allowed error degrees of ϕ value and θ value are greater than γ_m and ψ_m ones. The reason is the error of observation angle on target can be reduced through the action of actuator to achieve the goal of pursuit. The missile pursue target trajectory with sensor error less than 15 degrees shown as figure 11. From above, we can know, the impact of components precision error on missile pursue target will change with the size of error angle, but if error angle in allowing range, missile continues to pursue target.

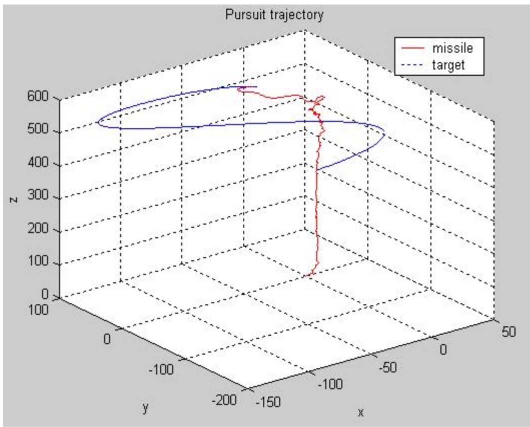

Fig. 9 Error value of and less than 20 degrees trajectory

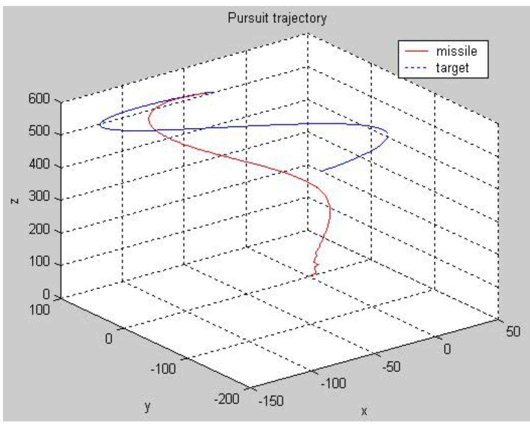

Fig. 10 Error value of and less than 5 degrees trajectory

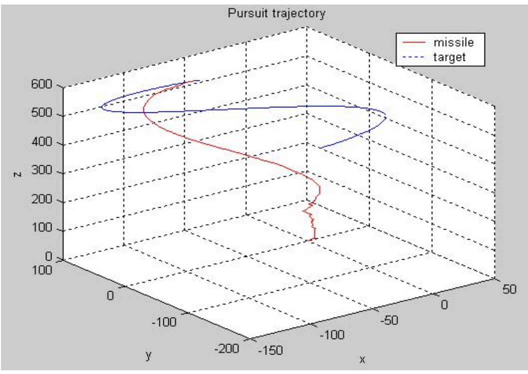

Fig. 11 Error value of and less than 15 degrees trajectory

F. Impact of acceleration on missile pursuit

Pursuit guidance law on the use of the most critical problem is when the missile more close to the target, the acceleration will be required more and more. From the data in figure 12 we can see, missile more close to the object, the acceleration will be required more and more, but also in the normal permissive scope. In terms of other simulation conditions, maybe as a result of acceleration too large to exceed the range of missile can bear. Therefore, we will limit the value of g. If missile acceleration exceeds the set value of g, program will calculate with the maximum set value of g. Through the restriction will enable simulation closer to reality.

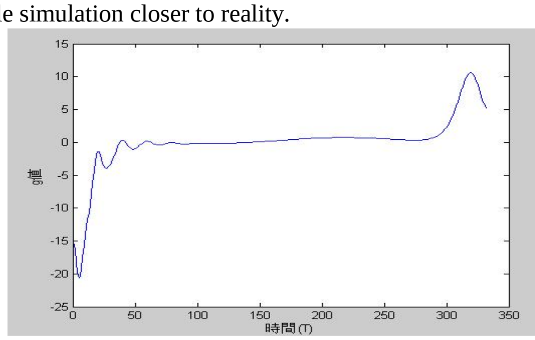

Fig. 12 Missile acceleration

VI. Conclusion

In this paper, we implement pursuit guidance law with feedback linearization control method, and converted into linear control problem cleverly. Its track sight angle capability and stability can explore by linear control theory methods. Therefore, through simulations can be fully aware of the properties of pursuit guidance law, through modulate parameters to get the best effect of attack that is the greatest achievement for this study. In the future we will use the results of this study combined air combat theory to simulate two fighters against with missile as weapon. In modern Aerial warfare, aircraft have been equipped with missiles. Therefore, taking missile as weapon in fighter against simulation will be a follow-up development point.

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