

# Microwave Imaging by Application of Information Theory Criteria in MUSIC Algorithm

M. Pourahmadi

**Abstract**—The performance of time-reversal MUSIC algorithm will be dramatically degraded in presence of strong noise and multiple scattering (i.e. when scatterers are close to each other). This is due to error in determining the number of scatterers. The present paper provides a new approach to alleviate such a problem using an information theoretic criterion referred as minimum description length (MDL). The merits of the novel approach are confirmed by the numerical examples. The results indicate the time-reversal MUSIC yields accurate estimate of the target locations with considerable noise and multiple scattering in the received signals.

**Keywords**—Microwave imaging, Time reversal, MUSIC algorithm, Minimum Description Length (MDL).

## I. INTRODUCTION

THE time reversal approach [1] has attracted increasing interest recently with broad applications, including underwater acoustics, radar, detection of defects in metals and communications. It obtains the location and reflectivity of point scatterers (targets) from the measurements of the wave emitted and received by arrays of transducers. Based on physics of wave propagation in a reciprocal medium, the backpropagation version of time reversal traces back the wave to the origin of the waves and obtains the location of the scatterers or wave sources [1]. Exploiting the orthogonality of the scatterer and noise subspaces, the multiple signal classification (MUSIC) algorithm improves significantly the resolution of the time reversal, as it is named time-reversal super-resolution imaging [2].

The presence of noise and multiple scattering between targets degrade the resolution of the backpropagation algorithm, whereas this is not the case for the MUSIC [3]. However, the noise and multiple scattering make difficulty and ambiguity in estimating the number of targets, what is required in the time-reversal music before running the imaging procedure. This paper introduces an estimation method for the number of scatterers according to minimum description length (MDL), an information theory criterion. It is shown that even in the presence of strong multiple scattering, the time-reversal MUSIC pseudo-spectrum computed using MDL criteria yields accurate estimates of the target locations.

## II. BASIC OF MUSIC ALGORITHM

### A. Multistatic Response Matrix

The multistatic response matrix embeds the information required for the MUSIC algorithm to form an image. Consider an imaging setup that consists of a transmit arrays of  $N_t$  antennas and a receive arrays of  $N_r$  sensors. The transmitters emit a known signal  $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_{N_t}(t)]^T$  to illuminate the scenario of interest, i.e., the  $j$ -th antenna transmits,  $s_j(t)$ ,  $j = 1, \dots, N_t$ , and the resulting backscattered returns are measured by all the receive antennas. Considering the signals in the frequency domain, the received signal is represented as,  $\mathbf{Y}(\omega) = \mathbf{K}(\omega)\mathbf{S}(\omega)$ , where  $\mathbf{Y}(\omega) = [y_1(\omega), \dots, y_{N_r}(\omega)]^T$  and  $\mathbf{S}(\omega)$  are the Fourier transforms of the received and transmitted signals, respectively. The multistatic response matrix  $\mathbf{K} = [K_{j,k}(\omega)]$  [2] of dimension  $N_r \times N_t$  contains the elements  $K_{j,k}(\omega)$  which shows the received signal at the  $j$ -th receive antenna due to an excitation signal of the  $k$ -th transmit antenna at a frequency  $\omega$ . As it can be seen the information about the probed scenario is encoded in the multistatic matrix  $\mathbf{K}(\omega)$ .

### B. Music Algorithm in Microwave Imaging

In microwave imaging, the received wave by  $i$ -th receiver at  $R_i^r$  position from  $j$ -th transmitter can be formulated as,

$$\psi_j(R_i^r, \omega) = \psi_j^{inc}(R_i^r, \omega) + k_0^2(\omega) \int G(R_i^r, r, \omega) O(r, \omega) \psi_j(r, \omega) d_r \quad (1)$$

In which  $G(r, r', \omega)$  is the green function of the background,  $\psi_j^{inc}(r, \omega)$  the incidence wave of  $j$ -th source and  $O(r, \omega)$  is the object distribution function or scattering potential.

In this case the scattered field can be written as (2),

$$\begin{aligned} \psi_j^{scatt}(R_i^r, \omega) &= \psi_j(R_i^r, \omega) - \psi_j^{inc}(R_i^r, \omega) \\ &= k_0^2(\omega) \int G(R_i^r, r, \omega) O(r, \omega) \psi_j(r, \omega) d_r \end{aligned} \quad (2)$$

So we have an  $N_r \times N_t$  dimensional matrix in each frequency  $\omega$  where it's  $i, j$ th element is,

$$k_{i,j}(\omega) = k_0^2(\omega) \int G(R_i^r, r, \omega) O(r, \omega) \psi_j(r, \omega) d_r \quad (3)$$

If we define  $\psi(r, \omega)$  as (4),

$$\begin{aligned} \psi(r, \omega) &= [\psi_1(r, \omega), \psi_2(r, \omega), \dots, \psi_{N_t}(r, \omega)]^T, \\ \mathbf{g}_r(r, \omega) &= [G(R_1^r, r, \omega), G(R_2^r, r, \omega), \dots, G(R_{N_r}^r, r, \omega)]^T \end{aligned} \quad (4)$$

so,

$$\mathbf{K}(\omega) = k_0^2(\omega) \int g(r, \omega) O(r, \omega) \psi^T(r, \omega) d_r \quad (5)$$

For running the MUSIC algorithm we have two approximations:

- 1) Born approximation: In which  $\psi_j$  within the integral can be set equal to the incident field  $\psi_j^{inc}(r, \omega)$ .
- 2) Another simplification results in cases where the sensor elements are small relative to the wavelength. In this case the incident waves  $\psi_j^{inc}$  are proportional to the Green's functions; i.e.,

$$\psi_j^{inc}(r, \omega) = e_j(\omega) G(r, R_j^t, \omega) \quad (6)$$

where  $e_j(\omega)$  is the incident pulse, or source excitation, spectrum, and  $R_j^t$  is the transmitter location.

### C. Time Reversing

It can be seen that  $\mathbf{T}(\omega) = \mathbf{K}^*(\omega) \mathbf{K}(\omega)$  works as a time-reversal operator, defined as time-reversal matrix in [2], and it is shown in [4] that for well-resolved scatterers, the eigenvectors of the time-reversal matrix that correspond to the nonzero eigenvalues are associated in a one-to-one manner with the scatterers. In the computational timereversal imaging, "pseudo-spectrum" [4] is computed from  $\mathbf{T}(\omega)$  and the inference on the scenario is made through the created image.

So by time reversing the received signal, we have,

$$\begin{aligned} \mathbf{T} &= \mathbf{K}^H \mathbf{K} = \left[ \sum_{m=1}^M \tau_m g_t^T(X_m) g_r(X_m) \right]^* \left[ \sum_{m=1}^M \tau_m g_r(X_m) g_t^T(X_m) \right] \\ &= \sum_{m=1}^M \sum_{m'=1}^M \tau_m^* \tau_{m'} g_t^*(X_m) g_t^T(X_{m'}) g_r^*(X_m) g_r(X_{m'}) \end{aligned} \quad (7)$$

In MUSIC algorithm the transmitter and receiver spaces may be subdivided into "signal" and "noise" subspaces spanned, respectively, by the singular vectors having nonzero singular values of matrix  $\mathbf{T}$  and the singular vectors having zero singular values. Then by using of orthogonality of signal and noise subspace, Pseudo Spectrum at each point  $X_p$  can be defined as follow [4],

$$P(X_p) = \frac{1}{\sum_{j=M+1}^N | \langle v_j, g(X_p) \rangle |^2} \quad (8)$$

where  $v_j$  is the  $j$ th eigenvector of matrix  $\mathbf{T}$  and  $g(X_p)$  is the green function at each point  $X_p$

For well-resolved scatterers,  $\mathbf{P}(\mathbf{r})$  will have distinct peaks at each of the scatterer locations.

### III. INFORMATION THEORY CRITERIA

Performance of MUSIC will be dramatically degrades in environment with noise and multiple reflections between targets. In such situation eigenvalue are close to each other and it is difficult to separate signal and noise subspace. So we have not the same peaks in pseudospectrum as in real state and some targets may be lost. We solve this problem by an information theory criterion named MDL [5]. But unlike conventional MDL methods, we do this algorithm in the frequency domain on transfer mutrix  $\mathbf{K}$ . So first we determine the number of targets in noisy medium by MDL and then do MUSIC method on data.

#### A. MDL Method

From a sensor array of  $N$  elements,  $n$  observations  $x_i, i = 1, \dots, n$  are made, which is a linear transformation of  $d < N$  source signals ( $s_i$ ), plus noise ( $v_i$ ),

$$\mathbf{X} = \mathbf{A} \mathbf{S} + \mathbf{V} \quad (9)$$

where  $\mathbf{A} \in C^{N \times d}$ , the steering matrix is composed of  $d$  linearly independent column vectors of array response  $\mathbf{a}(k), k = 1, \dots, d$ . It is assumed that Noise be circular Gaussian. To estimate the number of present signals  $d$ , eigenvalues of the correlation matrix  $\mathbf{R} = \frac{1}{n} E(\mathbf{X} \mathbf{X}^H)$  are used. The eigen decomposition of the correlation matrix is,

$$\mathbf{R} \mathbf{v}_i = \lambda_i \mathbf{v}_i \quad (10)$$

where  $\lambda_1 > \dots > \lambda_d > \lambda_{d+1} = \dots = \lambda_L = \sigma^2$  and  $\sigma^2$  is the noise variance.

The MDL estimator of  $d$  is the minimizer of the following criterion,

$$\Lambda(d, N, n) = n(N-d) \log\left(\frac{\alpha_d}{g_d}\right) + \frac{1}{2} d(2N-d) \log(n) \quad (11)$$

where

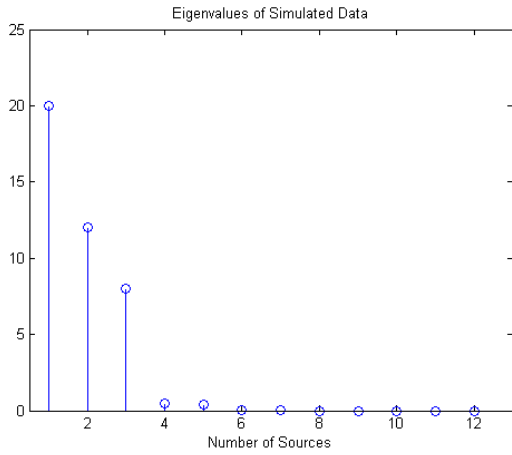
$$\alpha_d = \frac{1}{N-d} \sum_{i=d+1}^N \lambda_i \quad (12)$$

$$g_d = \prod_{i=d+1}^N \lambda_i \left(\frac{1}{N-d}\right) \quad (13)$$

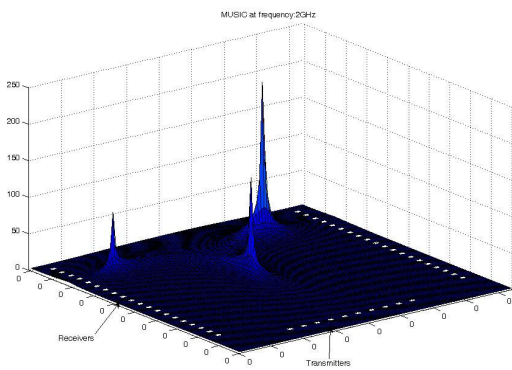
In noisy environments we have a noisy transfer matrix. By finding this matrix in  $n$  distinct frequency and defining the sample correlation matrix as (14),

$$\hat{\mathbf{R}} = \frac{1}{n} \sum_{i=1}^n \mathbf{K}(\omega) \mathbf{K}^H(\omega) \quad (14)$$

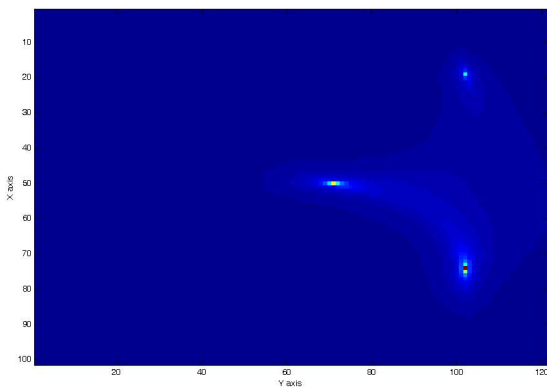
Then run MDL on eigenvalues of sample correlation matrix.



(a)



(b)



(c)

Fig. 1 (a) Eigenvalues (b) Pseudospectrum and (c) Tomography image (all in noise free state)

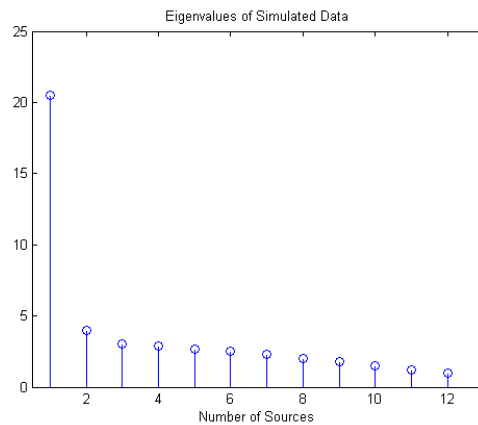
IV. SIMULATIONS

Simulation results are presented for 2D TM electromagnetic wave incident. The MUSIC algorithm is applied to the synthetic data that is generated by method of moment (MOM). There are uniform linear arrays of 10 transmitters with  $10\lambda$  distance and 40 receivers with  $5\lambda$  distance between array elements. This ensemble probes a rectangle environment having  $100 \times 100$  mm sides. In order to obtain a column of K matrix, an Infinite unit current line, serves as the transmitter, radiates wave into the environment and the scattering electric fields at the receiver arrays form the specified column. The current lines occupy one cell in the MOM at desired positions.

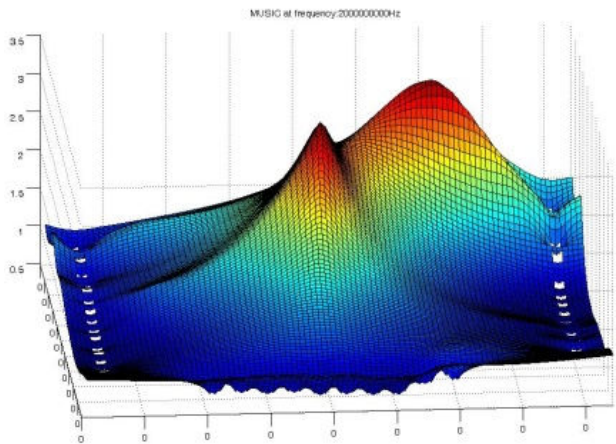
In the first stage we put 3 targets at positions with center (51, 51), (26, 82), (82, 82) in x and y dimension, all in mm, where their distance are  $.3\lambda, .6\lambda, .5\lambda$  respectively. We evaluate transfer matrix at frequency equal to 2GHz.

Fig. 1 shows eigenvalue and pseudospectrum of the received signal when there is no noise or multiple reflections. It can be seen that MUSIC can exactly detect all positions. Fig. 2 shows that performance of MUSIC degrades when we have noise in the system. In this state SNR=-5dB, noise is additive gaussian and algorithm can detect only 2 targets. Using MDL criteria on eigenvalues of transfer matrix we have minimum criteria on  $d=3$  and using this in MUSIC will reach to Fig. 3. For more exploring of the algorithm, let targets close to each other by spacing equal to  $0.1\lambda, 0.3\lambda, 0.4\lambda$  and add a noise to the signal with SNR=-5. Performance of the algorithm without and with MDL is shown in Figs. 4 and 5 respectively. Fig. 5 shows the superiority of our method in estimating the target locations.

Table I shows performance of MUSIC and MUSIC+MDL in finding the number of targets in different SNR ratios. It is evidence from the table that MUSIC+MDL have a good results rather than MUSIC algorithm.



(a)



(b)

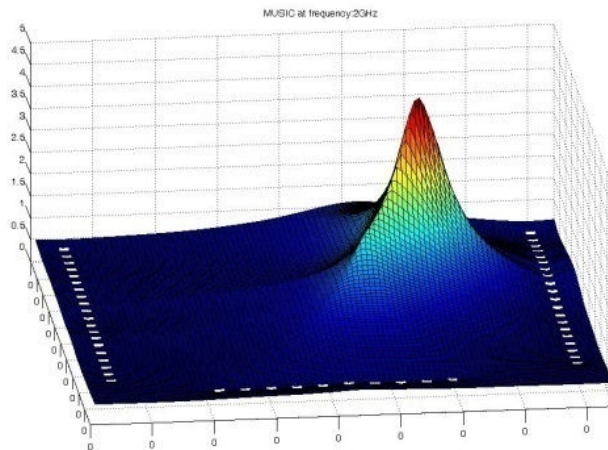
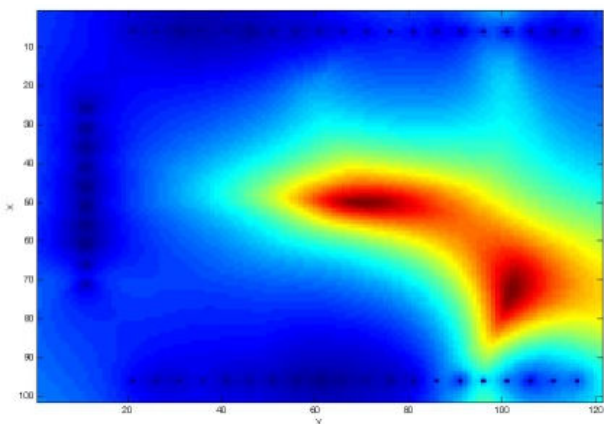


Fig. 4 Pseudospectrum with multiple scattering and noise



(c)

Fig. 2 (a) Eigenvalue (b) Pseudospectrum and (c) Tomography image all in noisy environment

TABLE I  
MUSIC AND MUSIC+MDL PERFORMANCE

SNR		-10	-5	-2	0	5	10
NUMBER of TARGETS	MUSIC	1	1	2	2	3	3
	MUSIC+MDL	2	3	3	3	3	3

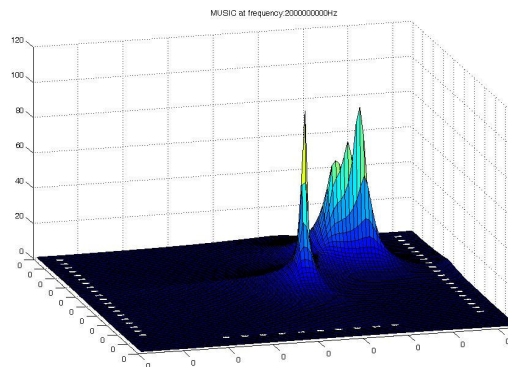


Fig. 5 Pseudospectrum after MDL with multiple scattering and noise

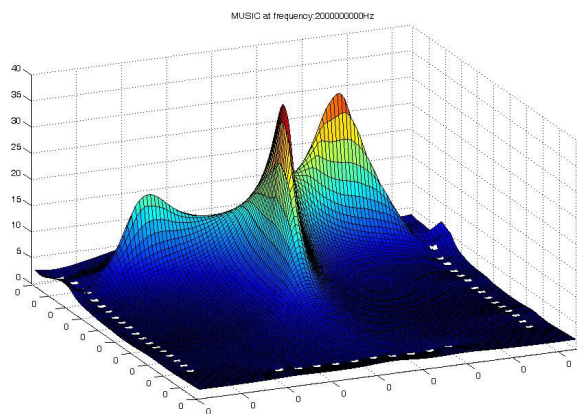


Fig. 3 Pseudospectrum after MDL algorithm

V. CONCLUSION

We derived the generalized theory for transmission mode time-reversal imaging for arbitrary transceiver number and location in noisy and multiple reflection situations.

An accurate performance analysis for the probability of missed detection of the MDL source enumeration method was presented and introduced a novel MDL in the frequency domain. Simulation results show the superiority of the proposed analysis compared with the previous results.

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