

# Implementation of SU-MIMO and MU-MIMO GTD-System under Imperfect CSI Knowledge

Parit Kanjanavirojkul, Kiatwarakorn Keeratishananond, and Prapun Suksompong

**Abstract**—We study the performance of compressed beamforming weights feedback technique in generalized triangular decomposition (GTD) based MIMO system. GTD is a beamforming technique that enjoys QoS flexibility. The technique, however, will perform at its optimum only when the full knowledge of channel state information (CSI) is available at the transmitter. This would be impossible in the real system, where there are channel estimation error and limited feedback. We suggest a way to implement the quantized beamforming weights feedback, which can significantly reduce the feedback data, on GTD-based MIMO system and investigate the performance of the system. Interestingly, we found that compressed beamforming weights feedback does not degrade the BER performance of the system at low input power, while the channel estimation error and quantization do. For comparison, GTD is more sensitive to compression and quantization, while SVD is more sensitive to the channel estimation error. We also explore the performance of GTD-based MU-MIMO system, and find that the BER performance starts to degrade largely at around -20 dB channel estimation error.

**Keywords**—MIMO, MU-MIMO, GTD, Imperfect CSI.

## I. INTRODUCTION

**M**IMO is a famous technique that is frequently used to improve the performance of wireless system. It has already been implemented in 802.11n wireless standard, and become one of the key technology that is used to improve the data rate. In the next generation wireless LAN, MIMO will still play a key role, for improving the system capacity and reliability.

Generalized triangular decomposition (GTD) [1], [2] is one of beamforming techniques used to implement MIMO. Others include singular value decomposition (SVD) [3], [4], [5], [6], [7], QRS [8] and GMD [9]. The advantage of the GTD over other beamforming technique is that the QoS of each data stream can be specified independently. GTD is implemented based on QoS constrain on each data stream. In addition, the input power is minimized. It has also been shown that the GTD technique can be implemented in multiuser setting or MU-MIMO system such as in the next generation wireless LAN [10]. The beamforming technique, however, need channel state information (CSI) for computing preprocessing matrix and equalizer. Several works investigated the effect of imperfect CSI to the performance of systems utilized other beamforming technique, but to our knowledge such impairment to GTD-based MIMO still largely unexplored. The main causes of imperfect CSI are channel estimation error, and limited feedback. [11], [12], [13], [14]

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To reduce the information feedback the 802.11n standard [15] provides three feedback schemes, which are CSI feedback, beamforming matrix feedback and compressed beamforming weight feedback. The performance of each scheme on the SVD beamforming technique has been investigated recently by Sun [16] and it turns out that the compressed beamforming feedback scheme is the most suitable one providing significant diversity gain with moderate complexity and small feedback overhead.

In this article, we modify compressed beamforming weight (CBW) feedback scheme to be able to use in GTD-based MIMO system. In particular, for CBW feedback, the usual implementation compresses unitary precoder matrix in SVD but GTD will not give unitary matrix. Hence the precoder for GTD must be unitarized. The performance of the system in the presence of channel estimation error and quantization is evaluated. We also compare the performance degradation of the system utilized GTD scheme to the conventional SVD scheme in term of BER.

The paper will be arranged as follows, in section II, a brief review of GTD approach is provided, along with the modified CBW technique used. Section III, the performance of the system, comparison to the SVD scheme and the result of channel estimation error in GTD-based MU-MIMO, are shown. Then the conclusion is made in section IV.

## II. SYSTEM MODEL AND BACKGROUND

### A. MIMO system

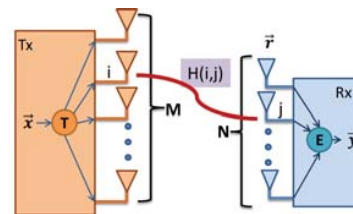


Fig. 1. MIMO System with  $M$  transmitted antennas and  $N$  Received antennas

We consider a MIMO system Fig. 1 which has  $M$  transmitted antennas at a transmitter (Tx) and  $N$  received antennas at a receiver (Rx). The received  $\vec{r} \in \mathbb{C}^{N \times 1}$  signal is given by

$$\vec{r} = \mathbf{H}\mathbf{T}\vec{x} + \vec{n} \quad (1)$$

where  $\vec{x} \in \mathbb{C}^{L \times 1}$  is the intended transmitted data with  $L$  data streams. Precoder matrix  $\mathbf{T} \in \mathbb{C}^{M \times L}$  is applied to  $\vec{x}$  before being transmitted through the antenna array. The channel

is denoted by  $\mathbf{H} \in \mathbb{C}^{N \times M}$  in which the complex channel response of each pair of antennas is represented. The noise  $\vec{n}$  is modeled as zero-mean circular symmetric complex gaussian. Overall input power used in the system will be equal to  $\text{Tr}(\mathbf{T}\mathbf{T}^*)$ , where  $*$  denotes Hermitian transpose operator and  $\text{Tr}$  is the trace of the matrix.

### B. Generalized triangular decomposition

Using GTD, a matrix can be decomposed into a product of 3 matrices  $\mathbf{Q}$ ,  $\mathbf{R}$  and  $\mathbf{P}^*$ . There exists some conditions that a matrix can not be decomposed by GTD. However, we shall follow the approach of Jiang, Li and Hager [1], where in a preprocessing matrix  $\mathbf{F}$  is added to the transmitted signal to generate a virtual channel  $\mathbf{HF}$ , which can then be decomposed. The main advantage of GTD is that we can prescribe the output SNR of each data stream, which means we can specify the QoS of each data stream independently. The method of finding the matrix  $\mathbf{F}$  for a specific channel and the prescribed output SNR, along with the method to find the decomposition is proposed in the mentioned work[1]. The decomposition results in

$$\mathbf{HF} = \mathbf{QRP}^* \quad (2)$$

where  $\mathbf{R} \in \mathbb{C}^{K \times K}$  is the upper triangular matrix.  $\mathbf{Q} \in \mathbb{C}^{N \times K}$  and  $\mathbf{P} \in \mathbb{C}^{M \times K}$  are semi-unitary matrix. At the receiver we set the precoder matrix  $\mathbf{T} = \mathbf{FP}$ , and equalizer  $\mathbf{E}$  to be  $\mathbf{Q}^*$ , the output of the equalizer would become.

$$\begin{aligned} \vec{y} &= \mathbf{E}\vec{r} \\ &= \mathbf{Q}^* \{ (\mathbf{HF})\mathbf{P}\vec{x} + \vec{n} \} \\ &= \mathbf{Q}^* (\mathbf{QRP}^*)\mathbf{P}\vec{x} + \mathbf{Q}^*\vec{n} \\ &= \mathbf{R}\vec{x} + \vec{n} \end{aligned} \quad (3)$$

the noise  $\vec{n}$  remains statistically the same as  $\vec{n}$ . Since  $\mathbf{R}$  is the upper triangular matrix. The transmitted data  $\vec{x}$  can be recovered by successive cancellation process[1]. Suppose we transmit  $L$  data streams at a time.

$$\begin{aligned} \tilde{s}_L &= \frac{y_L}{r_{LL}} \\ \tilde{s}_{L-1} &= \frac{y_{L-1} - \tilde{s}_L r_{(L-1)L}}{r_{(L-1)(L-1)}}, \\ \tilde{s}_{L-2} &= \frac{y_{L-2} - \tilde{s}_{L-1} r_{(L-2)(L-1)} - \tilde{s}_L r_{(L-2)L}}{r_{(L-2)(L-2)}}, \\ &\vdots \\ \tilde{s}_1 &= \frac{y_1 - \tilde{s}_2 r_{12} - \dots - \tilde{s}_L r_{1L}}{r_{11}}, \end{aligned} \quad (4)$$

where  $y_i$  is the  $i^{\text{th}}$  equalized symbol in the vector  $\vec{y}$  and  $r_{ij}$  is an element in matrix  $\mathbf{R}$ . The GTD beamforming scheme can also be extended to use in multiuser setting by the method of block diagonalization, which will cancel out the inter-user interference [10].

### C. Imperfect channel state information

The GTD technique requires channel state information (CSI) at the transmitter. In the real system, however, the CSI is usually corrupted by the channel estimation error and imperfect feedback. For the rest of this section we will discuss the cause imperfect CSI by channel estimation error, and explore the modified CBW scheme in GTD-based MIMO.

1) *Channel Estimation Error*: Channel estimation is done at the receiver after receiving training pilot symbols from the transmitter. There are various channel estimation techniques available for MIMO [17]. There will unavoidably be some estimation error. We model the channel estimation error as zero-mean circular symmetric complex gaussian noise  $\vec{n}_e$  [18]:

$$\mathbf{H}_{est} = \mathbf{H} + \vec{n}_e. \quad (5)$$

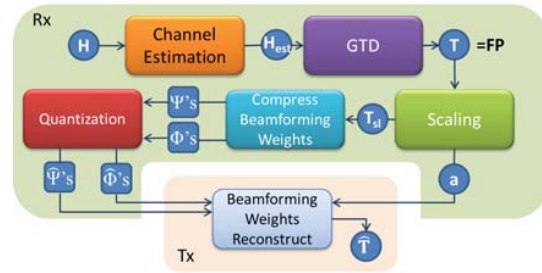


Fig. 2. Compressed beamforming weight (CBW) feedback scheme with channel estimation error

2) *Compressed Beamforming Weights Feedback and Quantization*: Compressed beamforming weights feedback is one of the feedback scheme introduced in 802.11n standard [15]. The scheme can largely reduced the feedback information by performing planar rotation operations using Givens rotations [19]. After the receiver estimates the channel, it will calculate the precoder matrix  $\mathbf{T}$  for the transmitter. Then the matrix  $\mathbf{T}$  is losslessly compressed into a set of angles ( $\Phi$ 's and  $\Psi$ 's). The angles are further quantized. The angles  $\Phi$  are quantized between 0 to  $2\pi$  using  $b+2$  bits and the angles  $\Psi$  are quantized between 0 to  $\pi/2$  using  $b$  bits. Using the compression scheme, the reconstructed precoder at the transmitter would not be exactly the same as the transmitted precoder. They will be different in phase. In particular,

$$\hat{\mathbf{T}} = \mathbf{T}\mathbf{D}^* \quad (6)$$

where  $\mathbf{D}$  which is known at the receiver, is the complex diagonal matrix, in which its element has unit magnitude, introducing phase shift to the signal. We can eliminate the phase shift from the received signal in the successive cancellation process by multiplying the known value of  $d_{ij}$  to each  $r_{ij}$  in equation (4), such that  $d_{ij}$  compensated the phase shift. Convinced by simulation result in [16], where the CBW has good performance in SVD beamforming, we implement the CBW feedback scheme in GTD MIMO.

The compressed beamforming weights feedback need that the matrix to be compressed is unitary. This will not be an issue in SVD based beamforming, since its preprocessing

matrix is unitary. However, for GTD, precoder  $\mathbf{T}$  is not unitary. Fortunately,  $\mathbf{T}\mathbf{T}^* = a\mathbf{I}$ , where  $\mathbf{I}$  is identity matrix and  $a$  is a constant. Therefore, we scale  $\mathbf{T}$  by  $\sqrt{a}$  making it unitary, and this factor has to be feedback as a side information to the transmitter. The transmitter is then multiplied back to the feedback matrix. The compressed beamforming weights feedback in GTD MIMO with channel estimation error system is shown in Fig. 2.

### III. SIMULATION RESULTS

In this section, the effects of channel estimation error and compressed beamforming weights feedback scheme are shown, in terms of BER. To simplify the analysis, we investigate the GTD-MIMO system in a special case where BPSK modulation is used and the SNR of all the three data streams are specified to be equal. The simulation results are divided into three subsections. Section III.A investigates effects of channel estimation error, CBW feedback, and quantization on the GTD SU-MIMO scheme. Section III.B provides the sensitivity comparison of GTD and SVD beamformings to the channel estimation error and CBW. The effect of channel estimation error on the GTD MU-MIMO scheme is shown in section III.C.

#### A. Effects of channel estimation error and compressed beamforming weights feedback

In this subsection,  $3 \times 3$  GTD SU-MIMO with 3 data streams scheme is simulated over Rayleigh flat fading channels. To study the effect of channel estimation error, CBW feedback, and quantization, we simulated seven cases shown in Table I. In Fig. 3 the BER performance is plotted against input power for the seven cases. The BER and input Power are averaged over a large number of channel realizations for each of the prescribed output power. The graph shows average performance of the three data streams. Note that the quantization resolution used in this simulation is  $b = 3$  [16].

From Fig. 3, the BER graph levels off at high input power; that is the imperfect CSI prevents the BER to drop more than the certain level. Comparing case 1, case 2, case 3, and case 4, we notice that with more channel estimation error the BER performance of the system degrades as expected. This is because the inaccurate channel knowledge introduces errors to the precoder and equalizer calculation. This error then distorts the upper triangular matrix  $\mathbf{R}$  resulted from the equalization, allowing interference among the data streams. Moreover, the error also affects the successive cancellation process, which requires the exact knowledge of  $\mathbf{R}$  to cancel out the interference term. Another observation is that applying CBW feedback alone without quantization does not degrade the BER performance, as we can see from case 3 and case 5, in which the estimation error has a power of -15.2 dB. The observed BER plots are not significantly different, at low output SNR. Interestingly, at high output SNR the BER performance of the system with CBW is even better than the system without the scheme. This implies that the compressed beamforming which is a good candidate in SVD beamforming [16] also works well with GTD. Although in GTD we need

one more information (factor  $a$  discussed in section III.C.2) to feedback, the compression technique can still reduce the overall number of information feedback with relatively low performance degradation. For the  $3 \times 3$  system under consideration, the information is reduced from 18 numbers to 7 numbers.

Case	Channel Estimation Error (dB)	CBW	Quantization
1	none	×	×
2	-20	×	×
3	-15.2	×	×
4	-13	×	×
5	-15.2	✓	×
6	-15.2	✓	✓
7	none	✓	✓

TABLE I  
SEVEN SIMULATED CASES OF GTD-MIMO WITH IMPERFECT CSI :  
✓ = IMPLEMENTED × = NOT IMPLEMENTED

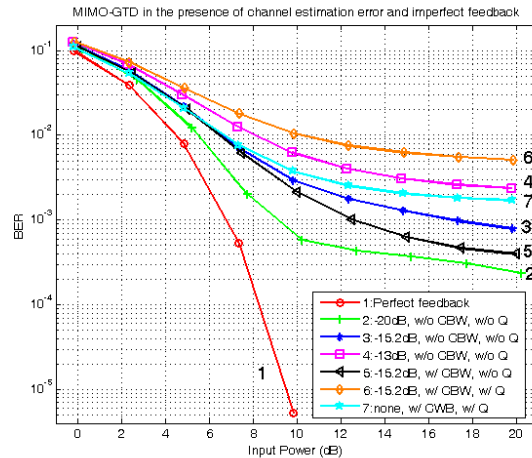


Fig. 3. BER performance of GTD MIMO with channel estimation error, CBW feedback, and quantization

Our last observation is made in regard to the effect of quantization. In our simulation we use quantization with  $b = 3$ . From case 5 and case 6, there are channel estimation error and compression, but in case 5 the compressed data is not quantized, while it is quantized in case 6. Fig. 3 shows degradation in BER performance, particularly at high Input Power. One of possible reason is that the feedback precoder using compressed beamforming need to be scaled down by factor  $\sqrt{a}$ , compressed, quantized then feedback to the transmitter, where it is uncompressed and amplified to its original value. This amplification would also amplify the error caused by quantization. At the high input power this amplification is more and results in large signal degradation.

### B. Comparison of GTD and SVD system under imperfect CSI

In this subsection, we compare the BER performance degradation between GTD and SVD in  $3 \times 3$  MIMO system. We consider the case when all the three data streams of GTD is set to have equal prescribed output SNR, while we have no control over the quality of each data stream of SVD. However, we know that the first data stream of SVD will have the best performance, and the third data stream has the worse performance. The simulation is done over Rayleigh flat fading channels. We plot the BER **degradation** of GTD averaged over three streams, all the three streams of SVD and BER of SVD averaged over three streams versus input power. Note that the quantization used in this simulation is  $b = 3$  bits. Fig. 4 shows the performance degradation of GTD and SVD MIMO in CBW feedback with quantization and channel estimation error. From the graph, we can see that the performance degradation of GTD is more than the degradation of the second stream of SVD, while data stream 1 of SVD has more degradation and data stream 3 of SVD have less degradation. This implies that BER performance of GTD degrades more than SVD in the present of both channel estimation error and CBW feedback. Fig. 5 shows the performance degradation of SVD and GTD, when only -15.2dB channel estimation error is taken into account. The GTD BER degradation for all three streams are the same and they are less than the first two streams of SVD. From this, we may conclude that SVD is more sensitive to the variation of channel estimation error. On the other hand, when considering the average performance of all streams, SVD-based system seems to suffer less degradation. Fig. 6 compares performance degradation, when we consider only the compressed feedback and quantization. At high SNR, the GTD degrades more compare to the second and the third data stream of the SVD. This means GTD is more sensitive to beamforming matrix compression and quantization. However, the average performance of all streams of SVD still suffer less degradation from quantization.

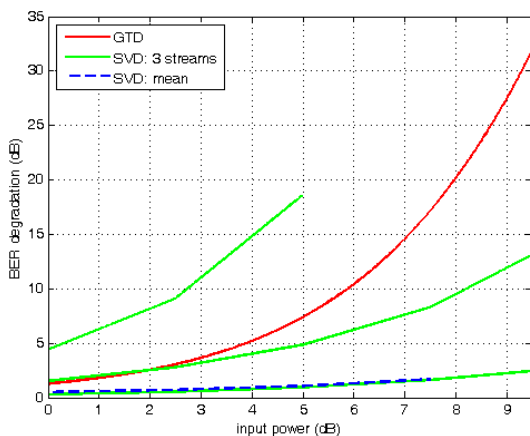


Fig. 4. BER Performance Degradation of GTD and SVD in quantized CBW feedback with channel estimation error (-15.2 dB)

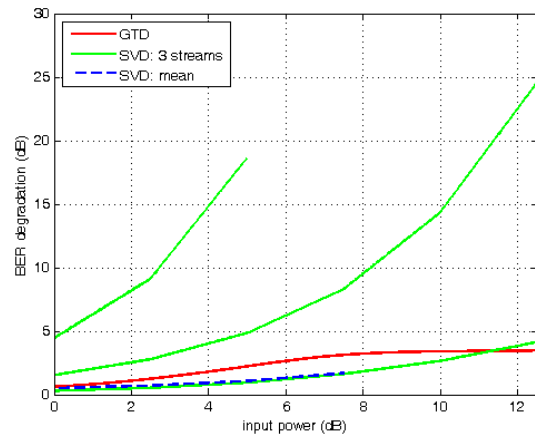


Fig. 5. BER Performance Degradation of GTD and SVD in unquantized CBW feedback with channel estimation error (-15.2 dB) : SVD is more sensitive to channel estimation error

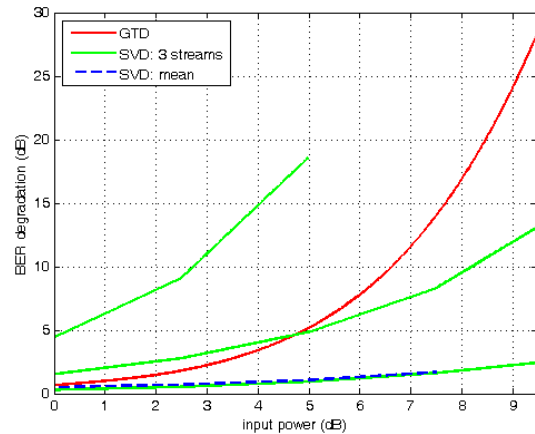


Fig. 6. BER Performance Degradation of GTD and SVD in quantized CBW feedback with no channel estimation error : GTD is more sensitive to CBW feedback

### C. Compressed Beamforming in GTD MU-MIMO

In this subsection we show the effect of channel estimation error in MU-MIMO using GTD beamforming. We implement a GTD  $4 \times (2,2)$  two-user MIMO system, where the MU-MIMO system is realized by block diagonalization technique [10]. The causes of error degradation are the same as in SU-MIMO case discussed in section III.A. and III.B., except that in the multi-user setting, block diagonalization technique is used. CSI is also required to null out the inter-user interference, so the channel estimation error will also introduce inter-user interference to the system. The BER performance of a user with channel estimation error are shown in Fig. 7. With channel estimation error power of -40 and -30 dB the BER performance degrades a little bit, but it degrades largely when the noise power is -20 dB.



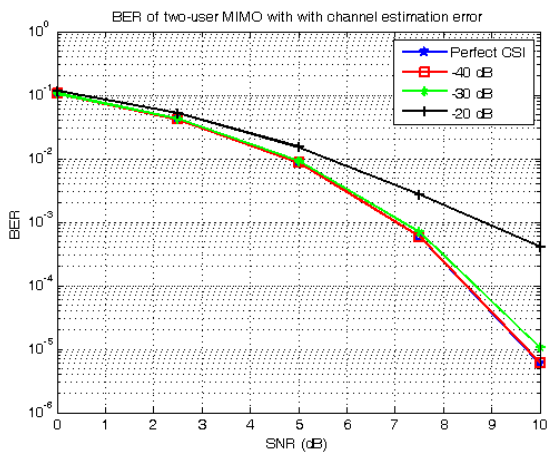


Fig. 7. BER Performance of  $4 \times (2,2)$  MU-MIMO GTD system with channel estimation error of -40, -30 and -20 dB : the performance largely degrade at -20

#### IV. CONCLUSION

The GTD beamforming MIMO using compressed beamforming weights feedback scheme is introduced. First, we investigate the effect of channel estimation error, CBW feedback, and quantization of the system. It turns out that channel estimation error has obvious effect on the BER performance, especially at high input power. The CBW feedback alone doesn't degrade the BER much, but when the quantization comes into account the BER degrades considerably, particularly at high input power, this is because at the transmitter the amplification needed for the precoder, amplify the error causes by quantization. When comparing GTD to SVD with 3 data streams GTD suffer more BER performance degradation compared to SVD beamforming in the presence of CBW feedback and channel estimation error scheme. We also found that SVD is more sensitive to channel estimation error, while GTD is more sensitive to compression and quantization. However, considering average performance of all data streams, BER performance of SVD always degrade less than GTD. We also show the effect of channel estimation error in MU-MIMO setting, where the system degrade largely at -20dB estimation error power.

#### ACKNOWLEDGMENT

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#### REFERENCES

- [1] Y. Jiang, J. Li, and W. Hager, *Transceiver Design Using Generalized Triangular Decomposition for MIMO Communications with QoS Constraints*, Conference Record of the Thirty-Eighth Asilomar Conference on Signal, Systems and Computers, pp 1154-1157, Nov.2004.
- [2] Y. Jiang, W. Hager, and J. Li, "Generalized triangular decomposition, Math. Comput., pp. 10371056, Oct. 2007.

- [3] F. Wang, X. Liu and M.E. Bialkowski, "Investigation into SVD based Beamforming over Rician MIMO Channels", 5<sup>th</sup> International Conference on Wireless Communications, Networking and mobile computing, pp.1-4, Sept. 2009.
- [4] M. Raja, P. Muthuchidambaramanathan, "Performance Analysis of Colosed-Loop MIMO system", International Journal of Computer Applications, Vol.4, No.12, Aug.2010.
- [5] G. Lebrun, T. Ying and M. Faulkner "MIMO transmission over a time-varying TDD channel using SVD", ELECTRONICS LETTERS, Vol.37, No.22, pp.1363-1364, Oct. 2001.
- [6] G. Lebrun, J. Gao, and M. Faulkner "MIMO Transmission Over a Time-Varying Channel Using SVD", IEEE Transactions on Wireless Communications, Vol.4, No.2, pp.757-764, Mar. 2005.
- [7] A. Cano-Gutierrez, M. Stojanovic, J. Vidal "EFFECT OF CHANNEL ESTIMATION ERROR ON THE PERFORMANCE OF SVD-BASED MIMO COMMUNICATION SYSTEMS", 15th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, Vol.1, pp.508, Jan. 2005.
- [8] C. Yuen, S. Sun and J. Zhang, "Comparative Study of SVD and QRS in Closed-loop Beamforming System", Military Communications Conference, Oct. 2007.
- [9] J-K. Zhang, A. Kavcic, and K. M. Wong, "Equal-diagonal qr decomposition and its application to precoder design for successive-cancellation detection", IEEE Trans. on Signal Processing, vol.51, pp.154-172, Jan. 2005.
- [10] P. Kanjanavirojkul, K. Keeratishananond, and P. Suksompong "Implementation and Performance of Generalized Triangular Decomposition in Multi-User MIMO, The International Conference on Information and Communication Technology for Embedded Systems, Jan. 2011.
- [11] D. J. Love, R. W. Heath Jr., W. Santipach, M.L.Honig, "What Is the Value of Limited Feedback for MIMO Channels?", IEEE Communications Magazine, Vol.42, Issue 10, pp.54-59, Oct. 2004.
- [12] Y. Chang, K. Araki "Robust Design for Multiuser Block Diagonalization MIMO Downlink System with CSI Feedback Delay", IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications, 2009.
- [13] K. Kobayashi, T. Ohtsuki, T. Kaneko "MIMO Systems in the Presence of Feedback Delay", IEEE International Conference on Communications, June 2006.
- [14] B. Makki, T. Eriksson Data "Transmission in the Presence of Noisy Channel State Feedback and Outage Probability Constraint", International Symposium on Information Theory and its Applications (ISITA), Oct 2010.
- [15] IEEE802.11n-2009, "IEEE Standard for Local and Metropolitan Area Networks - Telecommunications and Information Exchange between Systems - Local and Metropolitan Area Networks"
- [16] X. Sun, L. J. Cimini, L. J. Greenstein, D. S. Chan, and B. Douglas, "Performance Evaluation of Quantized Feedback Beamforming in IEEE 802.11n Wireless Networks, 41<sup>st</sup> Annual Conference on Information Sciences and Systems, Mar. 2007.
- [17] Y. Peng, S. Armour, and J. McGeehan "Performance Evaluation of Channel Estimation Techniques for MIMO-OFDMA systems with Adaptive Sub-carrier Allocation", IEEE Wireless Communications and Networking Conference, Apr. 2006.
- [18] T.Yoo, and A. Goldsmith "Capacity of Fading MIMO Channels with Channel Estimation Error", IEEE International Conference on Communications, Jun. 2004.
- [19] E. Perahia, and R Stacey "Next Generation Wireless LANs", 1<sup>st</sup> ed. pp. 254-262, New York, US: Cambridge University Press, 2008.