Modeling of Cross Flow Classifier with Water Injection

E. Pikushchak, J. Dueck, and L. Minkov

***Abstract*—**In hydrocyclones, the particle separation efficiency is limited by the suspended fine particles, which are discharged with the coarse product in the underflow. It is well known that injecting water in the conical part of the cyclone reduces the fine particle fraction in the underflow. This paper presents a mathematical model that simulates the water injection in the conical component. The model accounts for the fluid flow and the particle motion. Particle interaction, due to hindered settling caused by increased density and viscosity of the suspension, and fine particle entrainment by settling coarse particles are included in the model. Water injection in the conical part of the hydrocyclone is performed to reduce fine particle discharge in the underflow. The model demonstrates the impact of the injection rate, injection velocity, and injection location on the shape of the partition curve. The simulations are compared with experimental data of a 50-mm cyclone.

***Keywords*—**Classification, fine particle processing, hydrocyclone, water injection.

I. INTRODUCTION

ONE aspect of hydrocyclone development of current interest is the improvement of the separation characteristics, especially the reduction of fine particles in the coarse product [5]. One common method to address this is the injection of water in the conical part of the cyclone [5]-[7], [9], [11]. It is assumed that this approach results in a radial fluid flow, which transports fine particles from the sediment at the cyclone wall to the cyclone center. The fine particles are then collected in the inner swirl vortex, and discharged in the overflow of the cyclone. The water injection in the conical component significantly influences the flow conditions, and it is important to implement this method so that the separation is not destroyed. The wash effect depends on the location and direction of the injection, as well as on the number of injection points and the injected water flow rate.

II. FORMULATION OF THE SEPARATION MODEL

Fig. 1a shows a scheme of the model of turbulent cross flow classification introduced by Schubert and Neeße [9], [10].

The following simplifying assumptions are made:

1) The main flow in the apparatus is positioned so that it crosses the direction of the separation field, i.e., the direction

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of the particle sedimentation with the settling velocity $V_{s,j}$ of the j-th size fraction.

2) The Reynolds numbers $\mathrm{Re} = U_i h / \nu$ indicate turbulent flow conditions. The turbulent particle transport is characterized by the turbulent diffusion coefficient D_t.

3) At the end of the classifier, the vertical size distributions are cut at height h_u. The underflow beyond the cut off should contain the coarse particles, and the overflow should have more fine particles.

4) The model is completed by the water injection not far from the apparatus end and produces a current opposite to the settling direction with the velocity $V_{in,0}$. This counter current transports the fine particles to the overflow.
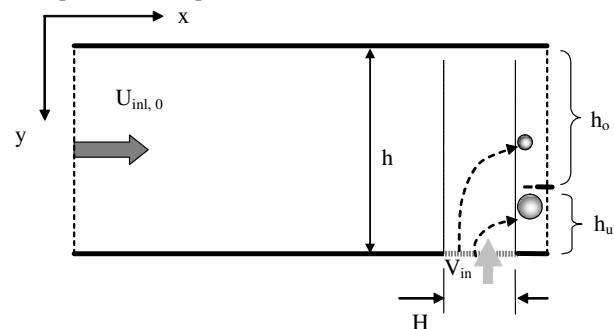

Fig.1. Scheme of cross flow classifier with water injection.

The given model is applicable to the hydrocyclone. Here, in the axis-symmetrical flow, settle the particles in a centrifugal field in the radial y-direction. The cut off is executed by the locus of zero axial velocity between the outer and the inner vortex. In contrast to the cross flow model of Fig. 1, the hydrocyclone overflow and underflow have opposite directions. The water injection is introduced across to the main flow at a distance H from the underflow.

This model assumes that particles are non-inertial:

$$\frac{d_j^2}{\nu} \left/ \frac{d_j}{\left| V_{in} - V_{s,j} \right|} \right. \ll 1 \tag{1}$$

The transport equation, which describes the development of the local concentration c_j of the j-th size fraction (particle diameter d_j) in the apparatus, is:

$$\frac{\partial U_{inl}(x) c_j}{\partial x} + \frac{\partial}{\partial y} \left[\left(V_{s,j} + V_{in} \right) c_j - D \frac{\partial c_j}{\partial y} \right] = 0 . \tag{2}$$

The boundary conditions are:

$$(V_{s,j} + V_{in})c_j - D\frac{\partial c_j}{\partial y} = 0 \text{ for } y = h \text{ and } y = 0. \qquad (3)$$

The condition at entry is: $c_j\big|_{x=0} = c_{j,0}$. $\qquad (4)$

Turbulent diffusion coefficient D_t is constant.

III. MODEL OF JET INJECTION

Under simplifying conditions, a soft injection can be modeled as jet flowing along the bottom of the apparatus. The component of the injection velocity along the jet describes a linear function of the coordinates across the main flow in the apparatus:

$$V_{in}(y) = \begin{cases} 0, & 0 < x \le L - H, \\ \left(-\dfrac{y}{h}\right)V_{in,0}, & L - H < x \le L \end{cases} \qquad (5)$$

L - apparatus length.

The volume conservation equation and Equation (5) are used to determine the component of the velocity along the axis of the apparatus:

$$\frac{U_{inl}(x)}{U_{inl,0}} = \begin{cases} 1, & 0 < x \le L - H \\ 1 + \dfrac{V_{in,0}}{U_{inl,0}}\left(\dfrac{x - (L-H)}{h}\right), & L - H < x \le L \end{cases} / \qquad (6)$$

V_r can be approximated by assuming that the injected water rate Q_{in} flows through the area of a cylinder having a diameter d_{inj} along the underflow wall. The injected water flow can be described by: $Q_{in} = \pi d_u \pi d_{in} V_r$.

It follows that: $V_r = \dfrac{Q_{in}}{\pi^2 d_{in} d_u}$.

IV. DEFINITION OF THE PARTITION FUNCTION

As shown in Fig. 1, the volume flux ratio of the overflow Q_o and underflow Q_u (volume split S) can be computed as follows:

$$S = \frac{\displaystyle\int_0^{h_o} U_{inl}(x,y)dy}{\displaystyle\int_{h_o}^{h} U_{inl}(x,y)dy} \qquad (7)$$

When there is no injection: $U_i(x) = U_{i,0}$ and $S_0 = \dfrac{h_0}{h_u} > 1$.

The particle fluxes of the j-size fraction $R_{un,j}$ in the underflow and in the overflow $R_{ov,j}$ are:

$$R_{un,j} = \int_{h_o}^{h} U_{inl}(L)c_j(L,y)dy \qquad (8)$$

$$R_{ov,j} = \int_0^{h_o} U_{inl}(L)c_j(L,y)dy \qquad (9)$$

The partition function $T(d_j)$ determines the part of every size fraction that is discharged through the under flow:

$$T(d_j) = \frac{R_{un,j}}{R_{un,j} + R_{ov,j}}. \qquad (10)$$

The partition function is usually characterized by the following parameters:
1) the d_{50} – the cut size with a 50% fractional recovery in the underflow;
2) the proportion T_0 of the smallest particles that are misplaced in the coarse underflow. This value is important to characterize of the washing effect of the water injection.

V. MODEL OF PARTICLE SETTLING IN A POLYDISPERSE SUSPENSION

To apply the proposed model for concentrated suspensions, the peculiarities of particle sedimentation under these conditions must be taken account.

There are numerous [5] experimental and theoretical results on the settling of dense suspensions. These studies focus on the settling behavior of polydisperse suspensions. It has been observed that the settling velocity of fine particles increases in the presence of coarser fractions. In this work, the investigations of Dueck et al. [2], Dueck and Neesse [1], and Minkov and Dueck [3], [4], [8] on the disturbed settling of dense suspensions are applied.

Polydisperse settling suspensions include the following inter-particle effects:
1. Hindered settling due to an increased ''effective'' density and viscosity of the fluid.
2. Counter flow of the displaced fluid caused by settling particles.
3. Entrainment of fine particles in the boundary layer range of coarse settling particles.

The complete equation for the settling velocity of j-fraction particles is summarized [1, 5] as follows:

$$V_{s,j} = V_{h,j}\left[\begin{aligned} &1 + A(c)c_V^{1/3}\cdot\frac{f_e(d_j)}{d_j^2} - \\ &-c_V\sum_{i=1}^n\left(1 + A(c)c_V^{1/3}\cdot\frac{f_e(d_i)}{d_i^2}\right)\frac{d_i^2}{d_j^2}\Delta m_i \end{aligned} \right], \qquad (11)$$

where $V_{h,j} = V_{St,j}(1-c_V)(1-c_V/0.6)^{1.5}$ - is the function describing the hindering settling due to higher values of viscosity and density, $f_e(d_j) = \left(\displaystyle\sum_{d_i > \beta d_j}^{n} d_i^6\Delta m_i\right)^{1/3}$ is entraining function with $\Delta m_i = \dfrac{c_i}{c_V}$, $A(c_V) = 2.5c_V^{1/3}\exp\left(-[5c_V]^3\right)$ is an empiric correction function from experiments. Herewith c_V is total volume solids concentration, β is parameter characterizing minimal particle size that can entrain finer

particles. $V_{St,j} = b\dfrac{gd_j^2}{18\mu_L}\left(\rho_p - \rho_L\right)$ is the Stokes settling rate, where b is centrifugal number (ratio of the centrifugal acceleration to the gravitational one), μ_L is the liquid viscosity, ρ_p is solid density, ρ_L is the liquid density.

VI. SIMULATION

The partition curves were computed using the numerical solutions of Equations (2), (5), and (10). The values from the above experiments were used as input. Five injection points and an injection diameter $d_{in} = 2.5$-mm were chosen for the adjusting of the injection velocity.

The number of size fractions in the feed and in the discharged overflow and underflow was set to 51.

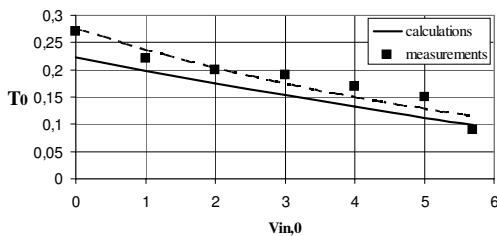The dependences of T_0 and d_{50} on the injection rate are shown in Fig. 2 and 3.

Fig.2. Comparison of calculated and measured values of T_0 as a function of the injected water rate.
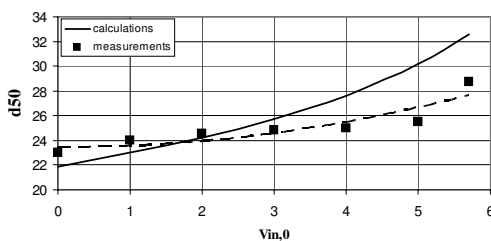

Fig.3. Comparison of calculated and measured values of the cut sizes d_{50} as a function of the injected water rate.

Both dependencies show a good agreement between calculations and experiments. Nevertheless, the measured increase of the cut size with growing injection velocity is lower than model predictions. The model does not account for the dilution caused by the injection, which improves the settling of the coarse particles.

A possible explanation for this effect is based on the partition function Equation (10). A constant diffusion coefficient is assumed to exist at low injection rates.

VII. CONCLUSIONS

The separation in a cyclone with water injection can be described using a modified model of turbulent cross flow classification. This model is the first attempt to simulate the fish-hook of the partition curve based on physics of interaction of settling particles. Hereby, the fine particle entrainment by coarse particles is the most important effect. The cyclone experiments with water injection indicate that there is an optimum effect at a certain injection rate.

The separation model indicates that the optimum is a result of two superposing effects. For low injection rate and constant turbulent diffusion coefficient the separation model delivers an increasing of washing effect caused by the radial countercurrent. This results in the displacement of fine particles to the cyclone center. At higher injection rates, more turbulence is produced, which disturbs the separation. This influence can be considered using an increased turbulent diffusion coefficient.

The optimal water injection at the apex is sensitive to variations in the feed. Therefore, the hydrocyclone operation with water injection must be stabilized at the optimum via a controlled water injection. This control is based on the measurement of the discharge shape of the underflow. This procedure enables the application of water injection even for small 50-mm cyclones. Possible applications of controlled water injection include desliming before flotation, kaolin preparation, or soil washing.

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