International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:6, No:8, 2012

Investigating the Performance of Minimax Search and Aggregate Mahalanobis Distance Function in Evolving an Ayo/Awale Player

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Abstract**—**In this paper we describe a hybrid technique of Minimax search and aggregate Mahalanobis distance function synthesis to evolve Awale game player. The hybrid technique helps to suggest a move in a short amount of time without looking into endgame database. However, the effectiveness of the technique is heavily dependent on the training dataset of the Awale strategies utilized. The evolved player was tested against Awale shareware program and the result is appealing.

Keywords**—**Minimax Search, Mahalanobis Distance, Strategic Game, Awale

I. INTRODUCTION

THE design of algorithms to play games by computers is generally an important problem in computer science. In particular, it is exhilarating to design heuristics for playing Awale, which is a member of Mancala family of games [1]. Awale is a two person zero sum, seed sowing board game with 6 holes on North and South sides respectively. Each hole contains 4 seeds at the commencement of a game, so there are 48 seeds altogether. As the game progresses, the current player selects all seeds from a non-empty hole and sows them counter-clockwise into the corresponding holes, but excluding the starting hole. If the last seed is sown into a hole of the opponent resulting into 2 or 3 seeds, the player captures all the consecutive 2 or 3 seeds. This capturing principle is called 2-3 rules, which generally varies for different variants of Mancala games. However, an important consideration in Awale is that a player cannot capture all seeds on opponent side. The Awale game comes to an end according to the following fundamental rules. First, a player captures more than 24 seeds to win the game. Second, both players have equally captured 24 seeds leading to a draw. Third, whenever there are fewer seeds, say 2 to 3 endlessly circulating on the board so that neither player can ever capture more seeds. Fourth, the player with the highest number of seeds wins the game and equal number of seeds captured by both players connotes a draw. The solution of Awale has been obtained to be draw, albeit the use of more intensive computational resources [2]. The state space of Awale game contains 889,063,398,406 positions, which makes it impossible for processing power and memory of a single Von Neumann computer to be used to search the entire state space.

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The problem was extremely challenging even on a parallel computer [2]. The objective of this study is to describe a hybrid technique of Minimax search and aggregate Mahalanobis distance function synthesis to evolve Awale game player. The remainder of this paper is for the sake of easy readability, succinctly organized as follows. In Section II, we discuss the related studies. In Section III, we describe the proposed hybrid technique. In Section IV, we present the experimentation result. In Section V, we give a brief conclusion.

II. RELATED STUDIES

Minimax Search Algorithm (MSA) [3] is generally employed for game playing and it has been successfully applied to Chess game playing [4-6]. MSA constructs a game tree and then employs backward induction to predict the game value and has been applied to Awale [7]. Endgame databases have been offered for evolving Awale player [8]. Retrograde analysis was proposed to find optimal play for all possible board positions of Awale [2, 9]. However, both endgame databases and retrograde analysis techniques can be expensive to implement because Awale positions occur in several billions, so more storage space is demanded. Consequently, such techniques cannot be easily implemented on a small memory device like wireless handset.

In order to design more storage efficient algorithms to play Awale, researchers are still interested in new techniques. Lithindion is an artificial Awale player that uses a combination of alpha-beta search algorithm and endgame database [7]. Marvin is an Awale player that uses a hybrid technique of depth-first and breadth-first searches [10]. Meta-heuristic based hybrid technique was proposed to mine endgame databases for relevant features that are useful in the construction of a static evaluation function [11, 12]. The evaluation function was a linear combination of certain Awale features with associated weights representing the current game position. The weights of the evaluation function were evolved using meta-heuristic technique and the output of the evaluation function was used in Minimax search [11, 12].

The data mining technique was proposed for evaluation function design in Awale [13]. However, the main difficulty posed by Minimax search is how to develop and apply an evaluation function to a game tree. Evaluation function design is an important consideration in games and computer game programs are differentiated by the quality of their evaluation functions.

In our previous studies, we showed that hybrid heuristic techniques can be promising for evolving Awale player. The refinement assisted Minimax [14] and Minimax-CBR heuristic [15] were offered for evolving Awale. In particular, the hybrid refinement technique provides a simple procedure to synthesize Minimax search with machine learning techniques. The primary objective of this current study is to show how Minimax search can be combined with aggregate Mahalanobis distance function to evolve Awale game player.

III. The Hybrid Technique

The fundamental principle underlying the proposed hybrid technique is to avoid looking into a dataset of Awale strategies for suggesting a move. This can be accomplished by training a machine learning system to predict the outcome of such a move. In this way, Awale player is evolved that fits into the main memory of a computer.

The Minimax search technique employed in this study is synonymously related to the following optimization scheme. The Max player tries as much as possible to increase the minimum value of the game, while Min player tends to decrease its maximum value as both players play towards optimality. This process can be formally described by the following extended Stockman formula [15-18]:

$$f(n)=\begin{cases} \max\{f(c) \mid c \text{ is a child node of } n\}, -f(n), \text{if } n \text{ is a Max node} \\ \min\{f(c) \mid c \text{ is a child node of } n\}, +f(n), \text{if } n \text{ is a Min node} \end{cases} \quad (1)$$

Our previous hybrid Minimax case based reasoning (Minimax-CBR) heuristic technique implemented three principal components, build Game Tree, compute Game value and predict Strategy [15]. The build Game Tree is the component that builds the game tree and compute Game value uses Equation (1) to compute the game value. The predict Strategy is the component that uses a refinement procedure [14] to deal with equipotent strategies that can result from computing game value. A refinement is a mapping that accepts a set of strategies, which are to be evaluated and the mapping returns a strategy with the best advantage [14]. This current work implements aggregate Mahalanobis distance function as refinement procedure. The choice of Mahalanobis distance function as refinement procedure was motivated by the Probabilistic Distance Clustering (pd-clustering) technique [19]. Recap that the general problem of data clustering is to partition a dataset into m clusters of similar data points.

The majority of the existing clustering techniques can be classified into distance based clustering such as k-means [20] and probabilistic based clustering such as expectation maximization [21]. The third category of clustering technique is the pd-clustering, which relates probability and distance using a simple inverse principle.

The pd-clustering principle is illustrated as follows. Let $x=(x_1,x_2,...,x_N)\in\mathbb{R}^N$ be a given vector of data points and suppose a dataset D consists of N data points $\{x_1,x_2,...,x_N\}$. For each $x\in D$, cluster centroid c_k and constant K, the probability $p_i(x)$ that x belongs to D is given as:

$$\frac{p_k(x)d_k(x)}{q_k}=K \quad (2)$$

Equation (2) has been shown to be the solution of the following extremal problem [19]:

$$\min\left\{\sum_{i=1}^{N}\left(\frac{d_1(x)*p_1^2*x()}{q_1}+\frac{d_2(x)*p_2^2*x()}{}\right) \,\middle|\, p_1(x)+p_2(x)=1, p_1(x), p_2(x)\geq 0\right\} \quad (3)$$

Where $d_1(x)$ and $d_2(x)$ are distances of the data point x to the clusters of sizes q_1 and q_2 and $p_1(x)$ and $p_2(x)$ are the cluster probabilities. In order to solve Equation (3), the Lagrangian of the problem is defined as:

$$L(p_1(x),p_2(x),\lambda)=\sum_{i=1}^{N}\left(\frac{d_1(x)*p_1^2*x()}{q_1}+\frac{d_2(x)*p_2^2*x()}{}\right)-\lambda(p_1(x)+p_2(x)-1) \quad (4)$$

The distance metric d(x, y) measures the closeness of the vectors x and y and is usually given as:

$$d(x,y)=\|x-y\|, \forall x, y\in\Re^n \quad (5)$$

Where ||.|| is a norm that can be Chebychev,Proscrute, Euclidean or mahalanobis Mahalanobis. The Mahalanobis metric is generally preferred to the Euclidean because it is consistent across conditions and it pays equal attention to all components. The Mahalanobis distance function is written as [19, 22, 23]:

$$d(x,c_k)=\left\{(x-c_k)^T\textstyle\sum_k^{-1}(x-c_k)\right\}^{1/2} \quad (6)$$

Where A^T means transpose vector of A and $\sum_k^{-1}$ is the inverse matrix of the covariance matrix $\sum_k$ given by

$$\sum\nolimits_k=\frac{\sum_i^N u_k(x_i)(x_i-c_k)(x_i-c_k)^T}{\sum_{i=1}^N u_k(x_i)} \quad (7)$$

The proposed hybrid system was designed such that Awale strategies were classified into two clusters ς_1 and ς_2 of good and bad strategies respectively. A good strategy will always lead a player towards winning whilst a bad strategy leads the player towards loosing the game. In order to provide an understanding of good and bad Awale strategies, consider the following Tchoucallion strategy {7, 5, 3, 1, 2, 2, 0, 1, 0, 0, 0, 0}, which is fictitious Awale strategy [24, 25]. The current player, Max will always make a move to capture (n-1) seeds such that the opponent, Min player has no choice than to keep making on available move. The parameter n is the total number of seeds on board. In order to prevent the Min player from ever capturing seeds as the game progresses, the Max player will try to make a good move each time. According to the Tchoucallion strategy example, a good strategy is {7, 5, 3, 1, 2, 0, 1, 0, 0, 0, 0, 0}, which is obtained by making move 6 and every alternative strategy is regarded as a bad strategy. Based upon this Tchoucallion example, the set of bad strategies overwhelm the set of good strategies.

Given an Awale strategy x the refinement procedure calculates Mahalanobis distances $d(x,c_1)$ and $d(x,c_2)$ between x and the clusters of good and bad Awale strategies. The clusters are represented by their centroids c_1 and c_2 respectively. The aggregate Mahalanobis distance D_x^m of the strategy x to the clusters ς_1 and ς_2 is then calculated as follows:

$$D_x^m = \frac{\{(x-c_2)^T \sum_2^{-1}(x-c_2)\}^{1/2}}{\{(x-c_2)^T \sum_2^{-1}(x-c_2)\}^{1/2} + \{(x-c_1)^T \sum_1^{-1}(x-c_1)\}^{1/2}} \qquad (8)$$

The value of D_x^m lies between 0 and 1, with value close to 0 suggesting that x is the worst strategy and value close to 1 suggesting that x is the best strategy respectively. The decision rule of the hybrid system suggests a strategy with the highest aggregate Mahalanobis distance.

IV. Experimental Results

The hybrid technique was implemented in C++ Builder 6 integrated development environment running on Microsoft Windows XP Professional Version 2002 operating system install on Intel® Core ™i7 CPU 870@293 GHz, 3.18 GB of RAM. In order test the effectiveness of the hybrid system, we organized a tournament between the hybrid system, Awale Amateur and Awale grandmaster, which implemented Minimax search algorithm at search depths of 12 to 20. The tournament was organized such that the hybrid system was allowed to play against the two versions of Awale shareware. A match between the hybrid system and its two opponents consists of 10 games and each player started 5 times, hence, a tournament counted 20 games. However, no time restriction was imposed, but we accepted a default search depth of 12 for Awale grandmaster to increase response time. The hybrid system used a search depth of 6, which was the maximum implemented and the same computer was used for the tournament. In order to compare the performance of the hybrid system with Awale shareware [26], the average seeds captured, average moves to complete a match and their standard deviations were the performance metrics used. The Gaussian probability density function that takes sample mean and standard deviation as arguments was used to discriminate the performance of the hybrid system from Awale shareware. Figure 1 shows that the proposed hybrid system consistently performes better than Awale Amateur as it captures more than 24 seeds throughout the game. The minimum and maximum seeds captured by the hybrid system are 25 and 28 while Awale Amateur recorded 2 and 21.

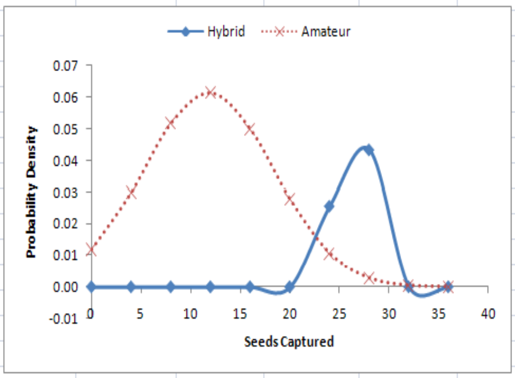

Fig. 1 Performance of Hybrid and Awale Amateur using Seeds Captured Metric

The performance of the hybrid system was also compared with the Awale grandmaster. Figure 2 shows this result, wherein it can be seen that Awale grandmaster consistently defeated the hybrid system. However, this result was not as bad as that of the Awale Amateur because the hybrid system did not captured less than 15 seeds on average. In fact the minimum and maximum seeds capatured by Awale grandmaster are 26 and 28 whilst the hybrid system recorded 12 and 16 respectively.

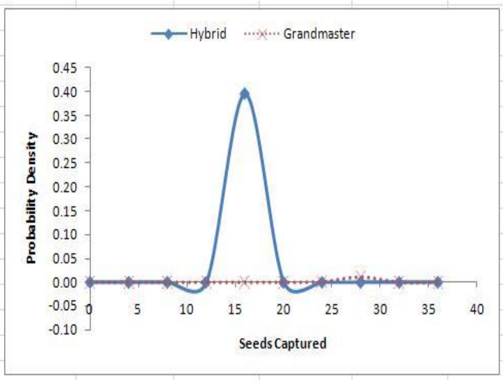

Fig. 2 Performance of Hybrid and Awale Grandmaster using Seeds Captured Metric

Finally, the performance of the hybrid system was determined in terms of the move metric to examine the strength of the hybrid system towards suggesting good moves. Figure 3 shows this result, wherein it can be seen that it takes longer time to complete a match between the hybrid system and Awale amateur when compared to the match between the hybrid system and Awale grandmaster. The ability of the hybrid system to suggest good moves is better than that of Amateur. The minimum and maximum moves recorded for the match between the hybrid system versus Awale amateur are 25 and 96 whilst that of the hybrid system and Awale grandmaster are 33 and 48 respectively. In general, it can be concluded from this analysis of the experimental results of testing the performance of the hybrid system that its performance is satisfactory. Moreover, the hybrid system cannot be classified as Amateur player because it outperforms the Awale Amateur. However, this study gives room for further improvement on the performance of the proposed hybrid system.

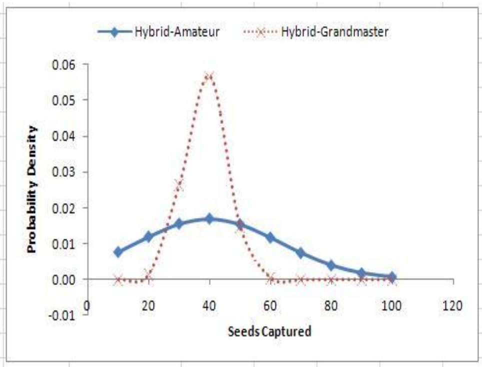

Fig. 3 Performance of Hybrid and Awale using Move Metric

V. Conclusion

In this paper we have described a hybrid technique based upon Minimax search and aggregate Mahalanobis distance function to evolve Awale player that can play at a reasonable level. The hybrid technique was implemented and tested against Awale shareware and the result was satisfactory. In the future, we hope to further refine the hybrid technique towards improving its performance.

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