

# Optimal Control Strategies for Speed Control of Permanent-Magnet Synchronous Motor Drives

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**Abstract**—The permanent magnet synchronous motor (PMSM) is very useful in many applications. Vector control of PMSM is popular kind of its control. In this paper, at first an optimal vector control for PMSM is designed and then results are compared with conventional vector control. Then, it is assumed that the measurements are noisy and linear quadratic Gaussian (LQG) methodology is used to filter the noises. The results of noisy optimal vector control and filtered optimal vector control are compared to each other. Nonlinearity of PMSM and existence of inverter in its control circuit caused that the system is nonlinear and time-variant. With deriving average model, the system is changed to nonlinear time-invariant and then the nonlinear system is converted to linear system by linearization of model around average values. This model is used to optimize vector control then two optimal vector controls are compared to each other. Simulation results show that the performance and robustness to noise of the control system has been highly improved.

**Keywords**—Kalman filter, Linear quadratic Gaussian (LQG), Linear quadratic regulator (LQR), Permanent-Magnet synchronous motor (PMSM).

## NOMECLATURE

$i_d, i_q$	d-q Stator currents
$v_d, v_q$	d-q Stator voltages
$\omega_r$	Electrical speed (rad/s)
$\theta_r$	Electrical position (rad)
$T_L$	Load torque (N.m)
$R$	Phase resistance ( $\Omega$ )
$L$	Phase inductance (Henry)
$J$	Rotor inertia ( $\text{kg.m}^2$ )
$D$	Damping coefficient (N.m.s/rad)
$\varphi$	Permanent magnet flux linkage (N.m/A)
$N$	Number of pole pairs

## I. INTRODUCTION

VECTOR control technique, incorporating fast signal processing and fast power electronics, have made possible the application of ac motors drives in high performance tasks where traditionally only dc servo drives were applied. A permanent magnet synchronous motor (PMSM) employing vector control is especially favorable for high performance servo drive applications because it fulfills the design criteria of high performance servo drive, such as compact structure, high air-gap flux density, high power to

inertia ratio, high torque to inertia ratio and high torque capability [1]. As the PM ac motor is replacing the conventional dc motors for small output power rating variable speed control system, the performance of PM ac motors which uses vector control and has quick transient response at same time, must be improved [2]. One way to improved the response of the system is linear quadratic regulator (LQR). The linear quadratic regulator (LQR) is an optimal control methodology that can be employed in wide range of applications. The quadratic cost function provides the designer with lots of flexibility to perform trade off among various performance criteria. The relationship between cost function weights and performance criteria hold even for high order and multiple input systems, where classical control becomes cumbersome. A major limitation of LQR is that the entire state must be measured exactly when generating the control. This limitation becomes increasingly troublesome for high order systems, where measuring all states can be very expensive. In addition no measurement is ever exact. Therefore, an optimal design methodology that results in controllers that utilize noisy, partial state information is desirable. The linear quadratic Gaussian (LQG) methodology provides a means of designing such controller [3].

The main contribution of this paper is optimal vector control of PMSMs that is essential for high precision applications such as servo drive. The rest of the paper is structured as follows. The linearized models of PMSM are presented in section II. Section III introduces the optimal strategies for speed control of PMSM through linear quadratic regulator (LQR) and linear quadratic Gaussian (LQG) methodologies. The computer simulation results are presented in section IV. Finally, section V concludes the paper.

## II. STATE SPACE MODELS AND LINEARIZED MODELS OF PMSM

The dynamic model for the PMSM in the d-q transformed rotor reference frame is given in state space as follow [4]:

$$\frac{d}{dt} \begin{bmatrix} i_q \\ i_d \\ \omega \end{bmatrix} = \begin{bmatrix} -R/L & 0 & -\varphi/L \\ 0 & -R/L & 0 \\ 1.5N^2\varphi/J & 0 & -D/J \end{bmatrix} \begin{bmatrix} i_q \\ i_d \\ \omega \end{bmatrix} + \begin{bmatrix} -\omega i_d \\ \omega i_q \\ 0 \end{bmatrix} + \begin{bmatrix} 1/L & 0 \\ 0 & 1/L \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_q \\ v_d \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -N/J \end{bmatrix} T_L \quad (1)$$

The basic principle in controlling the PMSM is based on field orientation. This is obtained by letting the permanent magnet flux linkage be aligned the d-axis and stator current vector is kept along q-axis direction. This means the value of  $i_d$  is kept zero in order to achieve the field orientation condition. Since the permanent magnet flux is constant, therefore the electromagnetic torque is linearly proportional to the q-axis current which is determined by closed loop control. As a result, maximum torque per ampere can be obtained from the machine in addition to achievement of high dynamic performance. Applying the field orientation concept by letting  $i_d = 0$  in (1) the linearized model of PMSM can be described in state space form as [5]:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \quad (2)$$

Where:

$$x(t) = [i_q \quad \omega_r \quad \theta_r]^T, \quad u(t) = v_q, \quad d(t) = T_L \quad (3)$$

$$A = \begin{bmatrix} -R/L & -\phi/L & 0 \\ 1.5N^2\phi/J & -D/J & 0 \\ 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ -N/J \\ 0 \end{bmatrix} \quad (4)$$

Above, the switching effect of inverter is neglected. As a matter of fact, because of nonlinearity of PMSM and existence of inverter, system is nonlinear and time-variant (due to periodicity of function  $v_d$  and  $v_q$  and they have switching form) [6]:

$$\dot{x}(t) = f(x(t), t) \quad (5)$$

$$f = \begin{bmatrix} \frac{v_d}{L} - \frac{R}{L}i_d + \omega_r i_q \\ \frac{v_q}{L} - \frac{R}{L}i_q - \frac{\phi}{L}\omega_r - \omega_r i_d \\ \frac{1.5N^2\phi}{J}i_q - \frac{N}{J}T_L - \frac{D}{J}\omega_r \\ \omega_r \end{bmatrix} \quad (6)$$

After deriving average model,  $v_d$  and  $v_q$  are changed to constant value. Consequently the system is changed to nonlinear time-invariant. Optimal output feedback problem can be solved for system by linearization around average values. In other word equilibrium operating point is defined by average model. Where  $\Delta x$  is small changes around average point with resulted by jacobian.

$$\Delta \dot{x}(t) = \begin{bmatrix} -R/L & \bar{\omega}_r & \bar{i}_q & 0 \\ -\bar{\omega}_r & -R/L & -\bar{i}_d - \phi/L & 0 \\ 0 & 1.5N^2\phi/J & -D/J & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Delta x(t) + \begin{bmatrix} 1/L & 0 \\ 0 & 1/L \\ 0 & 0 \end{bmatrix} \Delta u(t) + \begin{bmatrix} 0 \\ 0 \\ -N/J \\ 0 \end{bmatrix} \Delta d(t) \quad (7)$$

Where:

$$\Delta x(t) = [i_d \quad i_q \quad \omega_r \quad \theta_r]^T, \quad \Delta u(t) = [v_d \quad v_q]^T \quad (8)$$

In the next section, optimal strategy will be summarized.

### III. OPTIMAL CONTROL STRATEGY

#### A. Linear Quadratic Regulator (LQR)

The linear quadratic regulator (LQR) is an optimal control problem where the state equation of the plant is linear, the cost function is quadratic and test conditions consist of initial condition on the state and no disturbance input. The plant equation is:

$$\dot{x} = Ax + Bu \quad (9)$$

$$y = Cx \quad (10)$$

A reasonable cost function to use when the control system is designed to operate for long time period is:

$$J(x(t), u(t)) = 1/2 \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt \quad (11)$$

One method of finding the optimal feedback gain matrix utilizes a nonlinear matrix differential equation, known as Riccati equation:

$$\dot{p}(t) = -p(t)A - A^T p(t) - Q + p(t)BR^{-1}B^T p(t) \quad (12)$$

$P(t)$  must satisfy the above equation. The solution of the optimal control problem can be reduced to finding the matrix  $p(t)$ , since the optimal control is given [3]:

$$u(t) = -R^{-1}B^T p(t)x(t) = -k(t)x(t) \quad (13)$$

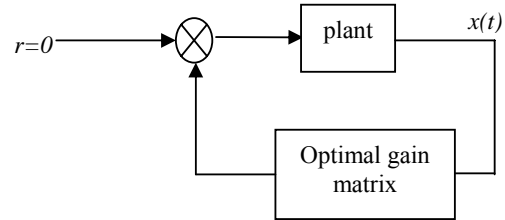


Fig. 1 Block diagram of optimal control

#### B. Linear Quadratic Gaussian (LQG)

Linear quadratic Gaussian (LQG) control refers to an optimal control problem where the plant model is linear, the cost function is quadratic and the test condition consist of random initial conditions, a white noise disturbance input, and white measurement noise. The plant is described by the following [3]:

$$\dot{x}(t) = Ax(t) + B_u u(t) + B_w w(t) \quad (14)$$

where  $u(t)$  is the control input and  $w(t)$  is a random disturbance input known as plant noise. The measurement available for feedback is:

$$m(t) = c_m x(t) + v(t) \quad (15)$$

where  $v(t)$  is a random signal known as measurement noise.

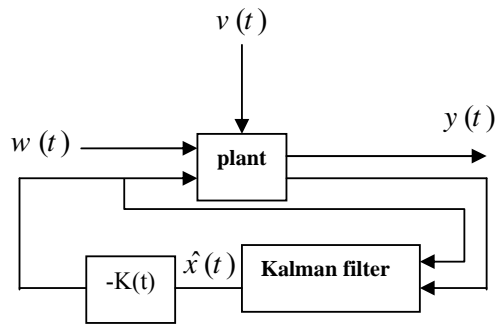


Fig. 2 A Linear quadratic Gaussian optimal control system

The state model for the optimal controller is:

$$\dot{\hat{x}} = [A - G(t)C_m - B_u K(t)]\hat{x}(t) + G(t)m(t) \quad (16)$$

$$u(t) = -K(t)\hat{x}(t) \quad (17)$$

Where  $G(t)$  is Kalman gain. The state feedback gain  $K(t)$  is found by solving the following [3]:

$$\dot{p}(t) = -p(t)A - A^T p(t) - Q + p(t)B_u R^{-1} B_u^T p(t) \quad (18)$$

$$K(t) = R^{-1} B_u^T p(t) \quad (19)$$

In summary, the solution of linear quadratic Gaussian optimal control problem can be broken into two parts: (1) find the linear quadratic regulator feedback gains that minimize the cost assuming perfect state information. (2) generate a Kalman filter to estimate the state. This is a remarkable result known as the stochastic separation principle [3].

#### IV. SIMULATION RESULTS

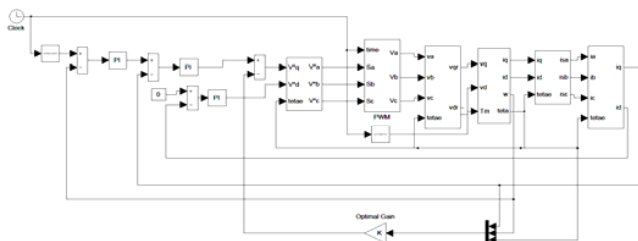


Fig. 3 Block diagram of optimal vector control

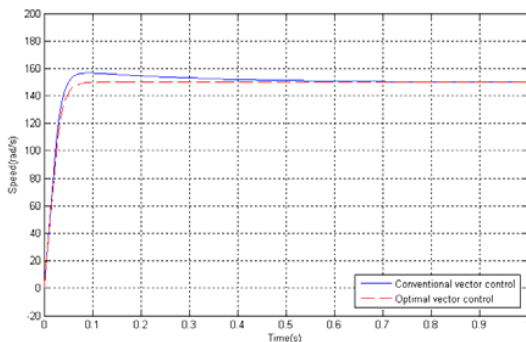


Fig. 4 Optimal and conventional vector control

Speed references is set to 150rad/s and settling time of actual speed is 0.1s in optimal vector control but in

conventional vector control, settling time of actual speed is 0.5s (with the same PI parameters). Speed overshoot in optimal vector control is 0.033% but in conventional method, its overshoot is 4.58% (Fig. 4).

Electromagnetic torque overshoot has 3.73% improvement in comparison of optimal vector control and conventional vector control.

At  $t=1s$ , speed reference is changed from 150rad/s to 180rad/s. Settling time of actual speed in optimal method is 0.07s and speed overshoot is 0% approximately but in conventional method settling time of actual speed is 0.4s and it has 0.8% overshoot (Fig. 5). Electromagnetic torque overshoot while speed is changing, has 1.94% improvement in comparison of conventional method.

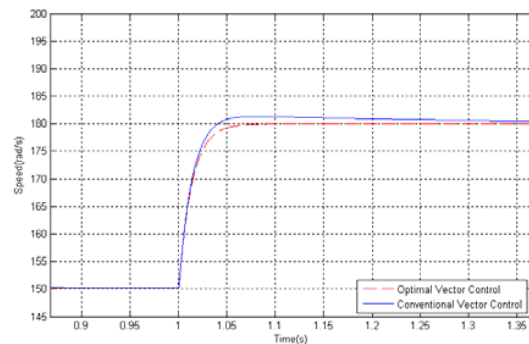


Fig. 5 Speed of optimal and conventional vector control

At  $t=2s$ , speed reference is changed from 180rad/s to 150rad/s. speed settling time and speed undershoot in optimal method are, 0.1s and 0% respectively. But in conventional method these parameters are 0.8s and 0.83% respectively (Fig. 6). Also electromagnetic torque overshoot has 2.72% improvement in comparison of conventional method.

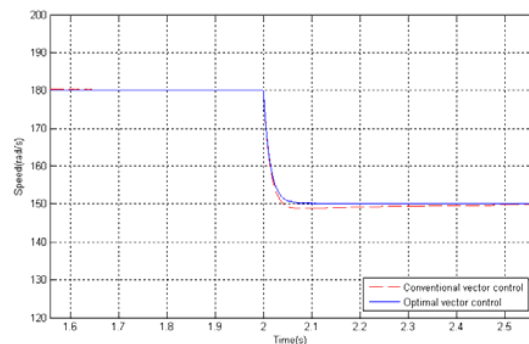


Fig. 6 Speed variation in optimal and conventional vector control

Then it is assumed that system has process and measurements noises and Kalman filter is used to reduce noise effect (Fig. 7).

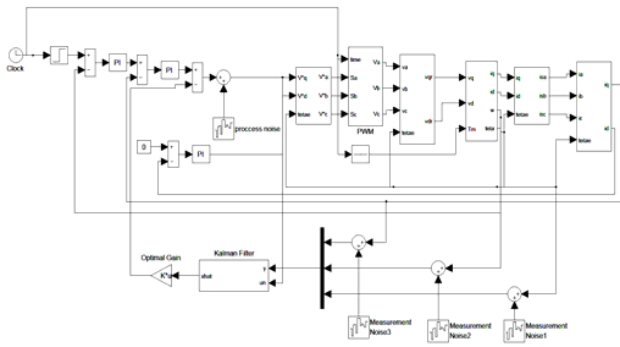


Fig. 7 Block diagram of linear quadratic Gaussian

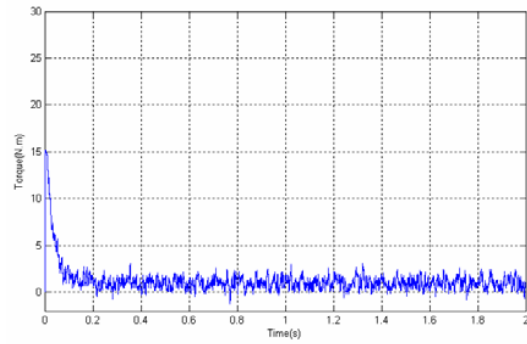


Fig. 10 Electromagnetic torque for LQG under noisy condition

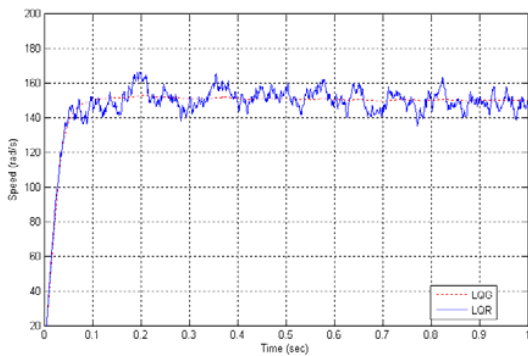


Fig. 8 Speed for LQR and LQG under noisy condition

Considering nonlinearity of PMSM and time-variant of inverter and linearization of model around average value is trend to changes block diagram of optimal vector control as below:

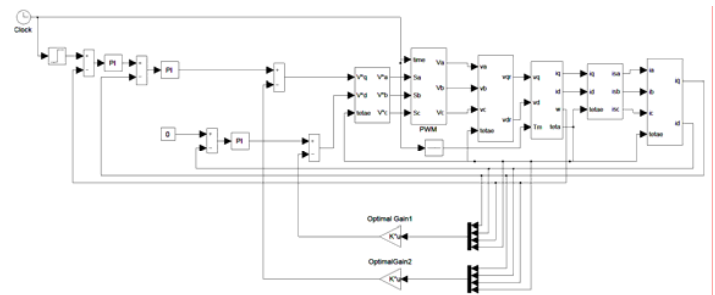


Fig. 11 Block diagram of optimal vector control

Fig. 8, Fig. 9 and Fig. 10 show speed and electromagnetic torque for LQR and LQG under noisy condition.

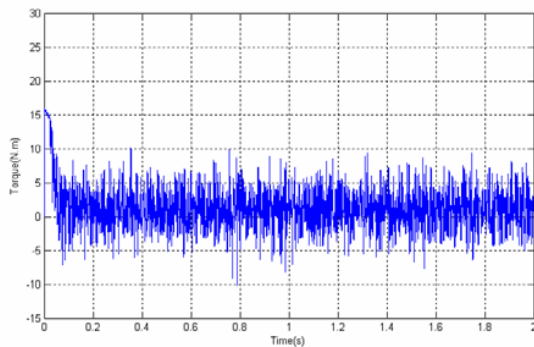


Fig. 9 Electromagnetic torque for LQR under noisy condition

Speed reference is set to 150rad/s and settling time for actual speed is 0.05s.

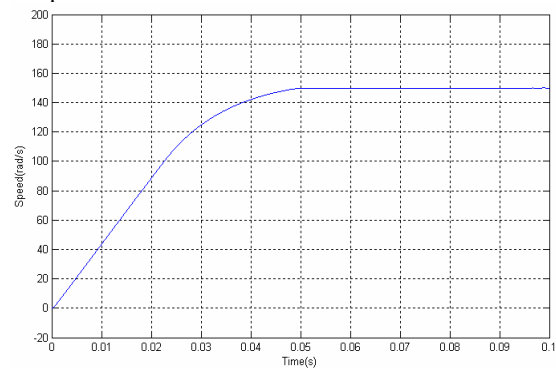


Fig. 12 Actual speed

Fig. 13 shows actual speed comparison of conventional vector control and two optimal vector control methods.

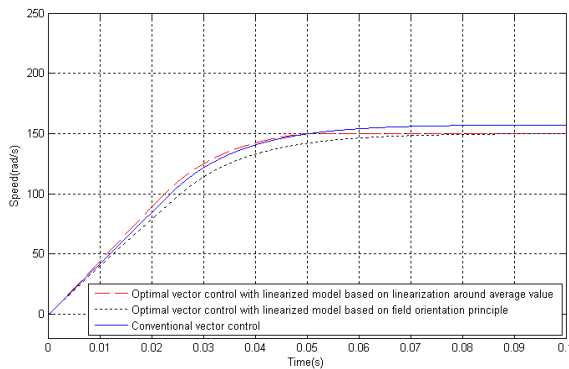


Fig. 13 Actual speed comparison

## V. CONCLUSION

In this paper, we investigated the LQR and LQG methodologies in vector control of PMSMs. The simulation results showed that the proposed controllers has better performance for the sake of design criteria like overshoot and settling time of the step response. Moreover, the LQG controller shows more robustness against process and measurement noises. Considering the nonlinearity of PMSM and existence of inverter in its control circuit that is caused the optimal gains with conventional optimal control theory can not be found. The solution to this problem is using average values and linearization around average values and simulation results showed that performance of system is improved by this method.

## APPENDIX

Motor parameter used in the simulation:

PMSM	INDRAMAT-MAC090B
Stator resistance	0.97 ohm
Stator inductance	5.1 mH
Permanent magnet flux	0.121 N.m/A
Moment of inertia	0.0036 kg.m <sup>2</sup>
Friction coefficient	0.0221 N.m.s/rad

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