

# Mitigating the clipping noise by using the oversampling scheme in OFDM systems

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**Abstract**—In an Orthogonal Frequency Division Multiplexing (OFDM) systems, the Peak to Average power Ratio (PAR) is high. The clipping signal scheme is a useful and simple method to reduce the PAR. However, it introduces additional noise that degrades the systems performance. We propose an oversampling scheme to deal with the received signal in order to reduce the clipping noise by using finite impulse response (FIR) filter. Coefficients of filter are obtained by correlation function of the received signal and the oversampling information at receiver. The performance of the proposed technique is evaluated for frequency selective channel. Results show that the proposed scheme can mitigate the clipping noise significantly for OFDM systems and in order to maintain the system's capacity, the clipping ratio should be larger than 2.5.

**Keywords**—Orthogonal Frequency Division Multiplexing (OFDM), peak-to-average power ratio (PAR), oversampling.

## I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) technique, known to be well-suited for high rate transmission on time-dispersive channels, has been accepted for many applications. However, one of the limitations of using OFDM is the high peak-to-average power ratio (PAR) of the transmitted signal. A large PAR leads to disadvantages such as increasing complexity of the analog-to-digital converter (A/D) and reducing efficiency of the radio frequency (RF) amplifier. If power amplifiers are not operated with large linear-power back-offs, it is impossible to keep the out-of-band power below imposed limits. This leads to very inefficient amplification, expensive transmitters and causes intermodulation among the subcarriers and undesired out-of-band radiation. The PAR reduction techniques are therefore of great importance for OFDM systems.

Recently, a large number of methods to reduce the PAR have been proposed. The side information (SI) must be transmitted to the receiver for recovering signal while using partial transmit sequences [1,2,3], selected mapping [2,4,5] or selective scrambling [6] method, so the channel efficiency drops a little. The Golay sequences technique [7, 8, 9] can reduce the PAR with little computational cost at the transmitter,

but designing the low PAR codes while maintaining a reasonable coding rate becomes quite difficult as the number of subcarriers increases. The active constellation extension [10, 11] algorithm becomes computationally challenging for the systems with a large number of subcarriers. The clipping signal scheme [12, 13, 14] is simplest and little cost method for PAR reduction. However, this introduces additive clipping noise that degrades the performance of systems. The oversampling scheme [15] for the clipped OFDM signal was performed, and used the band-pass filter to reduce the out-band noise at transmitter. The problem arises, however, that band-pass filtering the clipped OFDM signal samples results in considerable regrowth of peak power. So the degradation in bit-error performance was not significantly alleviated. We propose an oversampling method to deal with the received signal, (not the transmitted clipped signal that is discussed in reference [15]), followed by finite impulse response (FIR) filter and minimize mean square error principle to mitigate the clipping noise at receiver. Simulation results show that the proposed scheme has significant improvement on bit-error performance.

This paper is organized as follows. It starts with a systems model of OFDM transmission and describes the fundamentals of the clipping scheme in Section 2. Section 3 expresses the performances of the proposed scheme. Some simulation results and conclusions are given in section 4 and section 5.

## II. SYSTEMS DESCRIPTION

In an OFDM systems,  $N$  symbols,  $\{X_n, n = 0, 1, \dots, N-1\}$ , are modulated to  $N$  orthogonal subcarriers. The baseband subcarrier frequency is selected as  $f_n = n\Delta f = n/T$ , where  $T$  is the OFDM symbol time duration. A continuous-time domain OFDM baseband signal  $y(t)$  can be expressed as follows.

$$y(t) = \sum_{n=0}^{N-1} X_n \exp(j2\pi f_n t), \quad 0 \leq t < T \quad (1)$$

Sampling the above OFDM symbol at the sampling period  $\Delta t = T/M$ ,  $M > N$  (when  $M=N$ , the sampling rate is Nyquist rate), the discrete-time OFDM signal sampled at time instant  $t = k\Delta t$ ,  $k = 0, 1, \dots, M$ , is then expressed

$$y_k = y(k\Delta t) = \sum_{n=0}^{N-1} X_n \exp(j2\pi n k/M) . \quad (2)$$

The above equation can be written as

$$y_k = \sum_{n=0}^{M-1} \hat{X}_n \exp(j2\pi n k/M) . \quad (3)$$

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$$\text{where } \hat{X}_n = \begin{cases} X_n & \text{for } n < N, \\ 0 & \text{for } n \geq N. \end{cases}$$

Because of the large PAR for  $y(t)$ , we consider a clipping scheme of the baseband, so the clipped signal  $y_{tran}(t)$  can be obtained as follows.

$$y_{tran}(t) = \sum_{i=0}^{N-1} y_{tran}^i(t) = \begin{cases} y(t) & |y(t)| < \lambda \\ \lambda \arg(y(t)) & |y(t)| \geq \lambda \end{cases} \quad (4)$$

Where  $\lambda$  is the maximum permissible amplitude because of the limited power at transmitter,  $\arg(y(t))$  is the phase of  $y(t)$ .

We define the clipping ratio  $\gamma$  as the max-permitted power value (MPTV)  $\lambda^2$  to the average power of the clipped OFDM signal, that is

$$\gamma = \frac{\lambda^2}{E[|y(t)|^2]}. \quad (5)$$

The received continuous-time signal at receiver is expressed as

$$y_{re}(t) = \eta(t) \otimes y_{tran}(t) + n(t). \quad (6)$$

Where  $\eta(t)$  is the complex channel impulse response,  $n(t)$  is channel additive white Gaussian noise (AWGN), and  $\otimes$  denotes the convolution, respectively. Because OFDM technology increases the symbol time duration, the every subchannel is flat fading subchannel, although OFDM channel is also frequency selective fading channel. Assuming the variety of radio channel is slow compared with the time  $T$ , so that  $n(t)$  stays constancy within our concerned time slot. Then the recovered signal  $y_{re}(t)$  can be expressed as

$$y_{re}(t) = \sum_{i=0}^{N-1} c_i y_{tran}^i(t) + n_i(t) \quad (6^*)$$

Where  $c_i$  is the link loss of the  $i$ th subchannel,  $n_i(t)$  is the  $i$ th subchannel AWGN. Assuming the perfect channel state information (CSI) is known to the receiver, that is  $c_i, i=0, \dots, N-1$  is known to the receiver. The M-point ( $M > N$ ) sampled signals  $y_{re}(m)$  for the continuous-time signal  $y_{re}(t)$  are as follows.

$$\begin{aligned} y_{re}(m) &= \sum_{i=0}^{N-1} [c_i y_{tran}^i(mT/M) + n_i(mT/M)] \\ &= \sum_{i=0}^{N-1} c_i y_{tran}^i(m) + \sum_{i=0}^{N-1} n_i(m) \\ & \quad m = 0, 1, \dots, M-1 \end{aligned} \quad (7)$$

Do M-point DFT (Discrete Fourier Transform), (7) can be transformed as

$$Z_n = c_n \tilde{Y}_n + \Omega_n \quad n = 0, 1, \dots, M-1 \quad (8)$$

Where

$$\tilde{Y}_n = DFT(M, \sum_{i=0}^{N-1} y_{tran}^i(m)), \Omega_n = DFT(M, \sum_{i=0}^{N-1} n_i(m))$$

$$Z_n = DFT(M, y_{re}(m)).$$

Considering (4), the relationship between the original signal  $y(m)$  and the clipped signal  $y_{tran}(m)$  can be expressed as

$$y_{tran}(m) = y(m) - e(m). \quad (9)$$

Where  $e(m)$  is the error item because of the clipping process. Do M-point DFT to (9), we get

$$\tilde{Y}_n = \hat{X}_n - E_n. \quad (10)$$

Where  $E_n = DFT(M, e(m))$ .

Substituting (10) to (8), then we get equation (11).

$$\begin{bmatrix} Z_0 \\ \vdots \\ Z_{N-1} \\ Z_N \\ \vdots \\ Z_{M-1} \end{bmatrix}_{M \times 1} = \begin{bmatrix} c_0 X_0 \\ \vdots \\ c_{N-1} X_{N-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} c_0 E_0 \\ \vdots \\ c_{N-1} E_{N-1} \\ c_N E_N \\ \vdots \\ c_{M-1} E_{M-1} \end{bmatrix} + \begin{bmatrix} \Omega_0 \\ \vdots \\ \Omega_{N-1} \\ \Omega_N \\ \vdots \\ \Omega_{M-1} \end{bmatrix}. \quad (11)$$

Considering equation (11),  $Z$  can be obtained from received signals. Our aim is to recover the original signal,  $\hat{X}$  (or  $X$ ), from  $Z$ . We use the L-order FIR filter to recover  $\hat{X}$ . That is

$$\frac{1}{c_k} \sum_{l=0}^{L-1} \beta_l \frac{1}{c_{k-l}} Z(k-l) = \tilde{X}_k \quad k = 0, \dots, M-1 \quad (12)$$

Where  $\beta_l$  is the coefficient of the FIR filter.  $\tilde{X}_k$  is the estimate value of  $\hat{X}_k$ ,  $Z(k)$  is  $Z_k$ . The mean square error (MSE) between  $\tilde{X}_k$  and  $\hat{X}_k$  can be expressed as

$$E[|\tilde{X}_k - \hat{X}_k|^2] = E\left[\left|\frac{1}{c_k} \sum_{l=0}^{L-1} \beta_l Z(k-l) - \hat{X}_k\right|^2\right]. \quad (13)$$

The optimum  $\beta_l$  ( $l = 0, \dots, L-1$ ) are calculated to minimize MSE (MMSE) given by equation (13), and expressed as

$$\{\beta_0^*, \beta_1^*, \dots, \beta_{L-1}^*\} = \underset{\{\beta_0, \dots, \beta_{L-1}\}}{\text{arg min}} E[|\tilde{X}_k - \hat{X}_k|^2]. \quad (14)$$

The necessary condition to minimize MSE with regard to  $\beta_l$  is that its partial derivative of  $\beta_l$  has zero value. Then we get equation (15).

$$\sum_l \beta_l^* \Psi_{ZZ}(l-k) = c_k^* \Psi_{Z\hat{X}}(k) \quad (15)$$

Where  $c_k^*$  is the conjugation of  $c_k$ , the auto-correlation function,  $\Psi_{ZZ}(j)$ , is defined

$$\Psi_{ZZ}(j) = \sum_m Z(m)Z^*(m+j). \quad (16)$$

The cross-correlation,  $\Psi_{Z\hat{X}}(j)$ , is defined as equation (17).

$$\Psi_{Z\hat{X}}(j) = \sum_m Z(m)\hat{X}^*(m+j) \quad (17)$$

Because the number of non-zero values of  $\hat{X}_k$  is only N, so

$$\Psi_{Z\hat{X}}(j) = 0 \text{ when } j = -M+1, \dots, -N. \quad (18)$$

Substituting (18) into (15), we have

$$\sum_l \beta_l^* \Psi_{ZZ}(l-k) = 0 \quad k = -M+1, \dots, -N \quad (19)$$

By solving (19), we can obtain L non-singular values. These L values are optimum solutions of  $\beta_l^*$ ,  $l = 0, 1, \dots, L-1$ . Let these L optimum solutions substitute into equation (12),  $\tilde{X}_k$  is determined. The original signal ( $\hat{X}_k$ ) is estimated finally.

In order to guarantee L non-singular values of equation (12), the number of sampling point satisfies the following inequation.

$$M \geq N + L \quad (20)$$

### III. SYSTEM PERFORMANCE

#### A. The capacity of the systems

In evidence, the clipping process reduces the output power. For large N,  $y(t)$  amplitude is Rayleigh distributed. For clipping ratio  $\gamma$ , the input power  $P_m$  and the output power  $P_{out}$  of the clipping processor is expressed as follows.

$$P_{out} = (1 - e^{-\gamma^2})P_m \quad (21)$$

The inverse of Signal power to Interference and Noise power Rate (SINR) of the  $k$ th subcarrier is then given as [14]

$$SINR_k^{-1} = \frac{P_{w,k}}{P_{out,k}} + \frac{\sum_{k=0}^{N-1} P_{noise,k}}{N \sum_{k=0}^{N-1} (P_{out,k} + P_{w,k})} \left\{ \frac{\sum_{k=0}^{N-1} P_{in,k}}{P_{in,k}} + \frac{\sum_{k=0}^{N-1} P_{w,k}}{P_{out,k}} \right\}. \quad (22)$$

Where  $P_{in,k}$ ,  $P_{out,k}$  denote the average (useful) signal power of the  $k$ th subcarrier before and after the clipping processor, respectively.  $P_{w,k}$ ,  $P_{noise,k}$  are the average clipping noise power and AWGN power of the  $k$ th subcarrier, respectively. We further assume that the power allocation of each subcarrier is equal, i.e.,  $P_{in,k} = \sum_{k=0}^{N-1} P_{in,k} / N = P_{in} / N$  for all  $k$ . Then, (22)

reduces to

$$SINR_k^{-1} = \frac{e^{-\gamma^2}}{1 - e^{-\gamma^2}} + \frac{\sum_{k=0}^{N-1} P_{noise,k}}{\sum_{k=0}^{N-1} (P_{out,k} + P_{w,k})} \left\{ 1 + \frac{1}{N} \frac{N e^{-\gamma^2}}{1 - e^{-\gamma^2}} \right\} \quad (23)$$

$$= \frac{e^{-\gamma^2}}{1 - e^{-\gamma^2}} + \frac{1}{SNR_c} \left( \frac{1}{1 - e^{-\gamma^2}} \right)$$

Where  $SNR_c = \sum_{k=0}^{N-1} (P_{out,k} + P_{w,k}) / \sum_{k=0}^{N-1} P_{noise,k}$  is the channel signal to noise ratio.

The average channel capacity (ACC) of OFDM is

$$C = \frac{1}{N} W \sum_{k=0}^{N-1} \log_2(1 + SINR_k) = W \log_2 \left( 1 + \left( \frac{e^{-\gamma^2}}{1 - e^{-\gamma^2}} + \frac{1}{SNR_c} \frac{1}{1 - e^{-\gamma^2}} \right)^{-1} \right) \quad (24)$$

$$= W \log_2 \left( 1 + \left( e^{-\gamma^2} + \frac{1}{SNR_c} \right) \frac{1}{1 - e^{-\gamma^2}} \right)^{-1}$$

Where  $W$  is the bandwidth of subcarrier.

#### B. The bit error rate of the systems

When the signal is clipped, it generally causes performance degradation associated with the nonlinear distortion due to the clipping process. In this section, we analyze the bite error rate (BER) performance for uncoded OFDM systems.

For large N,  $y(t)$  amplitude is Rayleigh distributed. For the clipping ratio  $\gamma$ , the probability that  $|y(t)|^2 \geq \gamma \cdot E[|y(t)|^2]$  is  $p_r(\gamma) = p_r(|y(t)|^2 \geq \gamma \cdot E[|y(t)|^2]) = 1 - (1 - e^{-\gamma^2})^M$ . (25)

Considering (7) and (8) and assuming the error probability of  $y_k$  is as same as the error probability of  $X_k$ , the systems BER,

$$p_r^{sys}, \text{ is}$$

$$p_r^{sys} = (1 - p_r(\gamma))p_e + p_r(\gamma)(1 - (1 - p_e)^{J+1}). \quad (26)$$

Where  $p_e$  is the BER of the ideal clipping ratio ( $\gamma \rightarrow \infty$ ),  $J$  is the number of error points in oversampling process.

For large N,  $J$  can be approximately obtained as

$$J \approx M \cdot p_r(\gamma). \quad (27)$$

Substituting (27) into (26), considering  $(1 - x)^k \approx 1 - kx$  when  $x$  is relatively small, we get

$$p_r^{sys} \approx (1 - p_r(\gamma))p_e + p_r(\gamma)(J+1)p_e = [1 + Jp_r(\gamma)]p_e$$

$$\approx [1 + Mp_r(\gamma)^2]p_e \quad (28)$$

### IV. SIMULATIONS

To justify the proposed algorithm, assuming there are not other factors (such as frequency set-off) contribute to the interference. The number of subcarriers  $N=512$ ,  $L=N$  and oversampling ratio equals to 2. The subchannel loss is complex Gaussian random variable that the variance is one.

Fig.1 describes the average channel capacity (ACC) of OFDM verses the clipping ratio over the frequency selective fading channel at  $SINR=17$ dB. From Fig.1, we can deduce that when the clipping ratio is not less than 2.5, the ACC is very close to the theory limit (that is  $\gamma \rightarrow \infty$ ) of OFDM, when the clipping ratio is relatively small, the ACC declines very fast. So in order to maintain the ACC at a relatively high value, the clipping ratio should not be less than 2.5.

Fig.2 shows the BER of the clipped systems to the clipping ratio  $\gamma$  with BPSK modulation when the system  $SINR=10$ dB for the proposed scheme and Ochiai's scheme (reference [15]).

When  $\gamma$  is small (that is the clipping process becomes severe) the BER of proposed scheme outperforms that of the Ochiai's scheme. When  $\gamma$  is larger than 9.0, the BER performance of the proposed scheme is as same as that of Ochiai's scheme.

Fig.3 shows the performance of the BER to SINR in clipped systems when  $\gamma$  equals to 2.0, 4.0, respectively. When BER is less than  $10^{-3}$ , the proposed scheme is superior to Ochiai's scheme. The BER performance of  $\gamma=4.0$  surpasses that of  $\gamma=2.0$ .

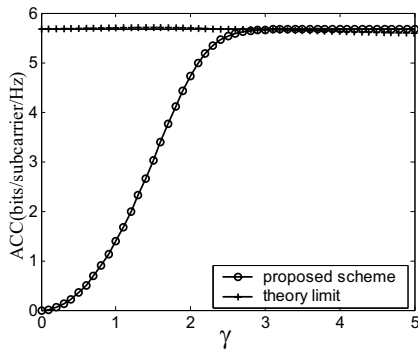


Fig.1 the average channel capacity (ACC) of OFDM to clipping ratio

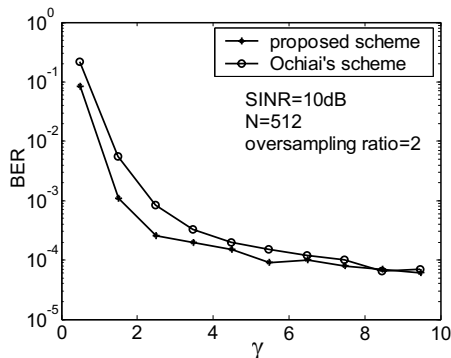


Fig.2 the BER to  $\gamma$  with BPSK under complex Gaussian channel

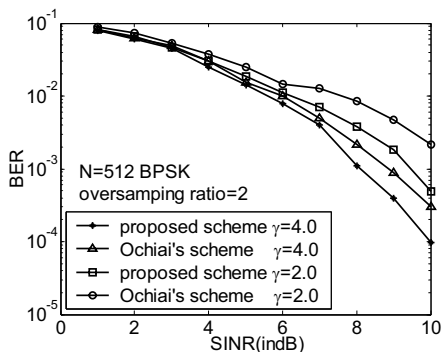


Fig.3 the BER to SINR with BPSK under complex Gaussian channel

## V. CONCLUSIONS

An oversampling scheme is proposed to recover the original signal when the clipping scheme is used and then introduces the

clipping noise. Using FIR filter and MMSE principle, we can mitigate the clipping noise. Simulation shows that BER performance of the proposed scheme is superior to that of Ochiai's scheme at system BER performance and in order to maintain the capacity of system, the clipping ratio should be larger than 2.5.

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