

# Improved Hill Climbing and Simulated Annealing Algorithms for Size Optimization of Trusses

Morteza Kazemi Torbaghan, Seyed Mehran Kazemi, Rahele Zhiani, and Fakhriye Hamed

**Abstract**—Truss optimization problem has been vastly studied during the past 30 years and many different methods have been proposed for this problem. Even though most of these methods assume that the design variables are continuously valued, in reality, the design variables of optimization problems such as cross-sectional areas are discretely valued. In this paper, an improved hill climbing and an improved simulated annealing algorithm have been proposed to solve the truss optimization problem with discrete values for cross-sectional areas. Obtained results have been compared to other methods in the literature and the comparison represents that the proposed methods can be used more efficiently than other proposed methods.

**Keywords**—Size Optimization of Trusses, Hill Climbing, Simulated Annealing.

## I. INTRODUCTION

IN recent years, many algorithms have been developed to solve the structural engineering optimization problem. These optimization techniques can be categorized into classical and heuristic search methods. Linear programming, non-linear programming and optimality criteria are examples of classical optimization methods [1]. Most of the proposed methods for structural optimization problem are based on the assumption that the design variables are continuously valued. In reality, however, the design variables such as cross-sectional areas are discretely valued and they are chosen from a list of discrete variables [2].

In both continuous and discrete cases, different methods have been proposed to solve the problem. Evolutionary algorithms such as genetic algorithms have been mostly used in both cases. Rajeev and Krishnamoorthy in 1992 [3] and then Wu and Chow [4] in 1995 proposed a genetic algorithm approach for discrete optimization of trusses. Cheng [5] in 2010 integrated the concepts of genetic algorithms and the finite element method to propose an efficient algorithm for optimal design of steel truss arch bridges. Other methods have been also used to solve the problem. Li et al. [6] in 2006 used

a particle swarm optimization algorithm to solve the problem. Other stochastic search techniques based on natural phenomena were suggested by Saka [7] in 2007. Lamberty [8] in 2008 proposed an efficient simulated annealing method for design optimization of trusses. Assari et al. [9] in 2012 presented an improved big bang – big crunch algorithm for size optimization of trusses. There are also lots of other methods applied to this optimization problem.

In this paper, an improved hill climbing and also an improved simulated annealing algorithm have been proposed to solve the size optimization of trusses assuming discrete values for cross-sectional areas. The proposed algorithms use hill climbing and simulated annealing iteratively to find the optimum design for a given truss.

The rest of this paper is organized as follows: in section II, the problem of size optimization for trusses has been formulated. Section III explains hill climbing algorithm. Section IV uses the explanation in section III to explain simulated annealing algorithm. In section V, the proposed algorithms to improve hill climbing and simulated annealing has been described. Section VI is related to experimental results. Finally, section VII shows the conclusion and summarizes the paper.

## II. PROBLEM FORMULATION

The optimization problem is the minimization of the weight of the structure subject to stress, displacement and minimum member size constrains. The objective function is:

$$w = \sum_{i=1}^n \gamma_i L_i A_i$$

where  $\gamma_i$  is the material density of the member,  $L_i$  is the length and  $A_i$  is the cross-sectional area of the  $i$ -th bar. The problem is subject to tensile and compressive stress constraints, bounds on displacements, and side constraints on the areas, as follows:

$$\begin{aligned} \sigma_{\min} &\leq \sigma_i \leq \sigma_{\max} & i &= 1, \dots, n \\ \delta_{\min} &\leq \delta_i \leq \delta_{\max} & i &= 1, \dots, m \\ A_i &\in \{\text{Available areas}\} & i &= 1, \dots, n_g \end{aligned}$$

where  $n$  is the number of members making up the structure,  $m$  is the number of nodes,  $n_g$  is the number of groups (number of design variables),  $\sigma_i$  and  $\delta_i$  are the stress and nodal deflection, respectively, and  $A_i$  is the cross-sectional area [9].

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### III. HILL CLIMBING

Hill climbing is a mathematical optimization technique which belongs to the family of local search. It is an iterative algorithm that starts with an arbitrary solution to a problem, then attempts to find a better solution by incrementally changing a single element of the solution. If the change produces a better solution, an incremental change is made to the new solution, repeating until no further improvements can be found.

Hill climbing is good for finding a local optimum (a solution that cannot be improved by considering a neighboring configuration) but it is not guaranteed to find the best possible solution (the global optimum) out of all possible solutions (the search space). The characteristic that only local optima are guaranteed can be cured by using restarts (repeated local search), or more complex schemes based on iterations, like iterated local search, on memory, like reactive search optimization and tabu search, on memory-less stochastic modifications, like simulated annealing.

In order to give a mathematical description of this method, Hill climbing attempts to maximize (or minimize) a target function  $f(x)$ , where  $x$  is a vector of continuous and/or discrete values. At each iteration, hill climbing will adjust a single element in  $x$  and determine whether the change improves the value of  $f(x)$ . With hill climbing, any change that improves  $f(x)$  is accepted, and the process continues until no change can be found to improve the value of  $f(x)$ . Then,  $x$  is said to be "locally optimal" [10].

### IV. SIMULATED ANNEALING

Simulated annealing (SA) is a generic probabilistic meta-heuristic for the global optimization problem of locating a good approximation to the global optimum of a given function in a large search space. It is often used when the search space is discrete. For certain problems, simulated annealing may be more efficient than exhaustive enumeration provided that the goal is merely to find an acceptably good solution in a fixed amount of time, rather than the best possible solution.

By analogy with this physical process, each step of the SA algorithm attempts to replace the current solution by a random solution (chosen according to a candidate distribution, often constructed to sample from solutions near the current solution). The new solution may then be accepted with a probability that depends both on the difference between the corresponding function values and also on a global parameter  $T$  (called the temperature), that is gradually decreased during the process. The dependency is such that the choice between the previous and current solution is almost random when  $T$  is large, but increasingly selects the better or "downhill" solution (for a minimization problem) as  $T$  goes to zero. The allowance for "uphill" moves potentially saves the method from becoming stuck at local optima which are the bane of greedier methods [11].

### V. PROPOSED ALGORITHMS

In order to solve the problem of size optimization for trusses, first of all a cost function has been implemented. In this function, the cost of a truss not having the desired conditions (stress and displacement conditions) is equal to infinity. For a truss having the desired conditions, the cost a solution is equal to the weight of the elements used.

Two algorithms have been proposed in this paper. The first algorithm uses in iterative hill climbing method. In the first iteration of this algorithm, an initial random solution is generated and hill climbing algorithm is used to get to a local minimum. Then, in each iteration, an initial random solution is generated until the cost of the generated solution is lower than the cost achieved in the last iteration. Then hill climbing is used again to get to another minimum. This loop iterates until no initial random solution can be generated having a lower cost than the minimum cost achieved in the last iteration.

The second proposed algorithm has just one difference in comparison with the first one. The difference is that simulated annealing is used instead of hill climbing to get to a minimum.

Using the proposed algorithms, we have a better chance to end up in a global minimum and not in a local minimum. The reason is that if in the first iteration we end up in a local minimum, in the second iteration we begin from a point having a lower cost than the last minimum achieved. Therefore, we will find another minimum in this iteration which surely has a lower cost than the last minimum. Repeating this process, in each iteration, a minimum with a lower cost is found and this will lead to achieving the global minimum.

### VI. EXPERIMENTAL RESULTS

In order to verify the proposed algorithms, we carried out the structural design optimization for two benchmarking design problems of 10-bar and 25-bar truss structures. Obtained results are compared to the solutions from other methods in the literature to demonstrate the efficiency of present approach. The algorithms were implemented and ran in an environment with the following characteristics:

Programming language: MATLAB 2010

CPU: Core 2 Duo 2.63GHz

Memory: 4GB

Operating System: Windows 7

#### A. Weight Optimization of a 10 Bars Plane Truss

The geometry of 10-bar plane truss structure is show in Fig. 1. This optimization problem has been studied by many researchers, and solutions by many different optimization approaches are available in the literature.

The objective function of the problem is to minimize the weight of the structure. The input data for this problem are Young's modulus,  $E = 6.895 \times 10^4$  MPa (104 Ksi), material density,  $\rho = 2767.990$  Kg/m<sup>3</sup> (0.1lb/in<sup>3</sup>) and vertical downward loads of 445.374 KN (100 Kips) at joints 2 and 4. The allowable displacement is limited to 5.08 cm (2 in) in both x and y directions at all nodes, and the allowable stress =  $\pm 172.375$  MPa (25 Ksi) for all members.

Design parameters:  $A_i$  (cm<sup>2</sup>)  $\in$  {10.45, 11.61, 12.84, 13.74, 15.35, 16.9, 18.58, 19.94, 20.19, 21.81, 22.39, 22.90, 23.42, 24.77, 24.97, 25.03, 26.97, 27.23, 28.97, 29.61, 30.97, 32.06, 33.03, 38.32, 46.58, 51.4, 74.19, 87.1, 89.68, 91.61, 99.99, 103.23, 109.03, 121.29, 128.39, 141.94, 147.74, 170.93, 183.87, 193.5, 216.13}

Obtained result from present work and other proposed methods are presented in Table I.

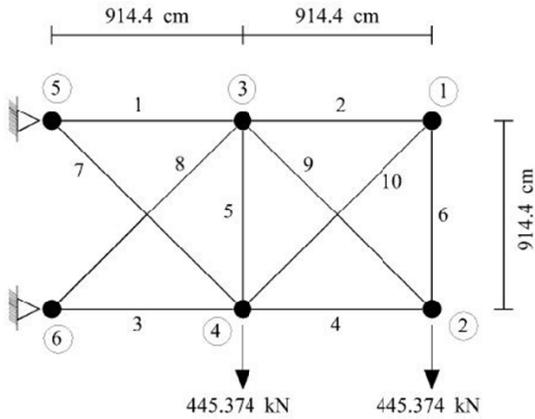


Fig. 1 Configuration of 10-bar truss

TABLE I  
OBTAINED RESULTS FROM PRESENT WORK AND OTHER PROPOSED METHODS FOR 10 BARS PLANE TRUSS

Method	proposed hill climbing	proposed simulated annealing	Rajeev, Krishnamoorthy[3]	Jang et al.[12]
Optimum weight(Kg)	2536.1	2500.5	2546.37	2626.3
A1	183.87	193.5	216.13	193.55
A2	12.84	10.45	10.45	16.9
A3	170.93	170.93	141.94	147.74
A4	103.23	103.23	99.99	147.74
A5	10.45	10.45	10.45	16.9
A6	10.45	10.45	10.45	16.9
A7	87.1	51.4	91.61	51.4
A8	121.29	141.94	128.39	147.74
A9	141.94	141.94	128.39	141.94
A10	10.45	10.45	16.90	10.45

**B. Weight Optimization of a 25 Bars Space Truss**

The second numerical example is a 25 bars space truss which is show in Fig. 2. Twenty Five members are categorized into eight groups. Member groupings are given in Table II. Loading conditions for this space truss are given in Table III. The assumed data are: Young's modulus,  $E = 6.895 \times 104$  MPa (104 Ksi), material density,  $\rho = 2767.990$  Kg/m<sup>3</sup> (0.1 lb/in<sup>3</sup>). The allowable displacement is limited to 8.89 mm (0.35 in) in x, y and z directions at all nodes, and the allowable stress =  $\pm 275.8$  MPa (40 Ksi) for all members.

Design parameters:  $A_i$  (cm<sup>2</sup>)  $\in$  {0.645, 1.29, 1.935, 2.58, 3.225, 3.87, 4.515, 5.16, 5.805, 6.45, 7.095, 7.74, 8.385, 9.03, 9.675, 10.32, 10.965, 11.61, 12.255, 12.9, 13.545, 14.19, 14.835, 15.48, 16.125, 16.77, 18.06, 19.35, 20.64, 21.93}.

Obtained result from present work and other proposed methods are presented in Table IV.

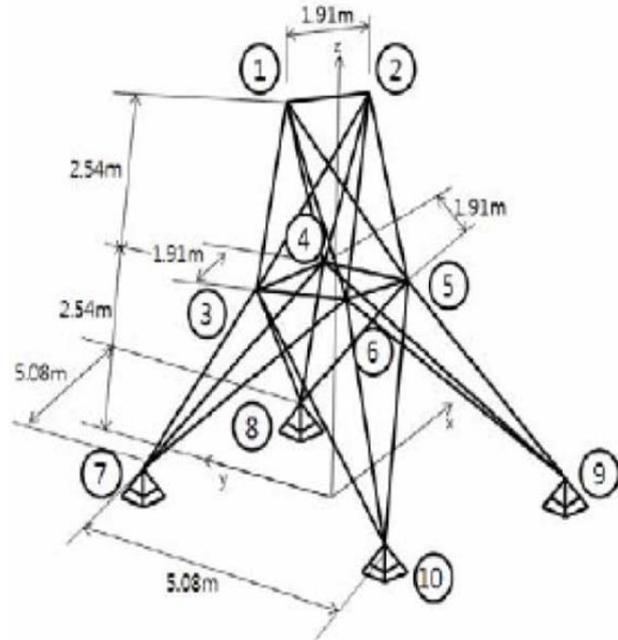


Fig. 2 Configuration of 25-bar truss

TABLE II  
MEMBER GROUPINGS FOR 25 BARS TRUSS

Design Variable Number	End Nodes of Members
1	(1, 2)
2	(1, 4), (1, 5), (2, 3), (2, 6)
3	(1, 3), (1, 6), (2, 4), (2, 5)
4	(3, 6), (4, 5)
5	(3, 4), (5, 6)
6	(3, 10), (4, 9), (5, 8), (6, 7)
7	(3, 8), (4, 7), (5, 10), (6, 9)
8	(3, 7), (4, 8), (5, 9), (6, 10)

TABLE III  
LOADING DATA

Nodal Number	Px(KN)	Py(KN)	Pz(KN)
1	4.454	-44.53	-44.53
2	0	-44.53	-44.53
3	2.227	0	0
4	2.672	0	0

TABLE IV  
OBTAINED RESULTS FROM PRESENT WORK AND OTHER PROPOSED METHODS  
FOR 25 BARS SPACE TRUSS

Method	proposed hill climbing	proposed simulated annealing	Rajeev and Krishnamoorthy[3]	Jang et al.[12]
Optimum weight(Kg)	236.95	223.97	247.67	247.91
X1	0.645	0.645	0.645	14.19
X2	10.965	1.29	11.61	1.29
X3	19.35	21.93	14.835	21.94
X4	0.645	0.645	1.29	9.68
X5	7.095	10.32	0.645	4.52
X6	7.095	5.805	5.16	4.52
X7	3.225	5.805	11.61	9.68
X8	20.64	21.93	19.53	19.53

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## VII. CONCLUSION

In this paper, an improved hill climbing and an improved simulated annealing algorithm were proposed to solve the size optimization problem for trusses. In order to verify the proposed algorithms, the structural design optimization was carried out for two benchmarking design problems of 10-bar and 25-bar truss structures. Obtained results represented that the proposed algorithms can be used more efficiently than some other well-known methods in the literature. The results also demonstrated that improved simulated annealing is more efficient than the improved hill climbing algorithm.

In future, we can use local beam search instead of hill climbing and simulated annealing to enhance the performance of the algorithm for this problem.

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