

An Analytical Electron Mobility Model based on Particle Swarm Computation for Silicon-based Devices

F. Djeffal, N. Lakhdar and T. Bendib

Abstract—The study of the transport coefficients in electronic devices is currently carried out by analytical and empirical models. This study requires several simplifying assumptions, generally necessary to lead to analytical expressions in order to study the different characteristics of the electronic silicon-based devices. Further progress in the development, design and optimization of Silicon-based devices necessarily requires new theory and modeling tools. In our study, we use the PSO (Particle Swarm Optimization) technique as a computational tool to develop analytical approaches in order to study the transport phenomenon of the electron in crystalline silicon as function of temperature and doping concentration. Good agreement between our results and measured data has been found. The optimized analytical models can also be incorporated into the circuits simulators to study Si-based devices without impact on the computational time and data storage.

Keywords—Particle Swarm, electron mobility, Si-based devices, Optimization.

I. INTRODUCTION

THE need to understand and predict the electron mobility has become increasingly important as electronic devices scale to and below 100-nm gate lengths [1]. Silicon-based materials need extremely large investments before it reaches a quality suitable for device production. Before such investments can be made, a thorough understanding of the electronic properties of the material is required. The most important parameter which affects directly the performance of an electronic device is the mobility of carriers. The latter can be considered as the limiting parameter in the electronic devices [1, 2]. There are several methods used in the calculation of the transport properties, namely: variational principle (VP), iterative method (IM), relaxation time approximation (RTA), Matthiessen rule (MR) formalism, and Monte Carlo (MC) method [3]. But from the circuit simulation point of view even 2-D or 3-D solution of numerical coupled

equations is overkill approach in term of both complexity and computational cost. For analytical modeling, in general, it is difficult or almost impossible to obtain closed form analytical models of mobility for Si-based devices. Thus, models are obtained by a simplification of the full physical model. The analytical models allow for fast system level simulation of the Si-based devices. However, the accuracy of such a model can be questionable because of the simplifications made during the model development phase. Model accuracy and simplicity are important for the design and optimization of Si-based devices. Particle Swarm Optimization (PSO) algorithm would be preferable and could provide practical solutions [4]. We call this type of solution approach as intelligent simulators [5,6].

In this paper, we present the applicability of the particle swarm optimization (PSO) technique for the analytical modeling of the electron mobility in Si-based devices. The database used for the optimization of our analytical electron mobility model is built on the basis of a numerical model of the velocity-applied electric field characteristics of the Si material developed using Monte Carlo simulations (MC). Optimized compact model obtained from this study can be used as the interface between device modeling and circuit simulators in order to analyze and study the Si-based integrated circuits.

II. PARTICLE SWARM COMPUTATION

The particle swarm optimization is an evolutionary computation technique inspired by social behavior of a flock of birds and insect swarms. Originally proposed by Kennedy and Eberhart [4], it has recently been introduced to study the complex and nonlinear systems and has found useful applications in engineering fields [7,8,9]. In contrast to genetic algorithms (GAs) and evolutionary strategies (ESs), which exploit the competitive characteristics of biological evolution (e.g., survival of the fittest), PSO exploits cooperative and social aspects, such as fish schooling, birds flocking, and insects swarming. In the original concept of the PSO, particles fly through the search space influenced by two factors: one is the individual's best position ever found (*pbest*); the other is the group's best position (*gbest*). Due to its simple mechanism and high performance for global optimization, PSO can be applied to study the electron

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transport phenomenon in semiconductor devices.

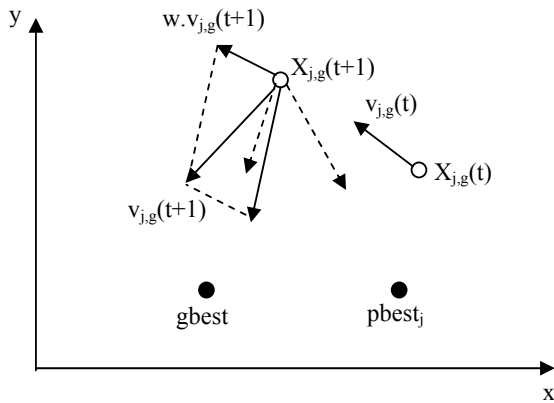


Fig. 1 Velocity and position updates in PSO computation for 2-D parameter space

The first step of our approach consists in an adequate approximation of the doping level dependence of the mobility at room temperature on the base of the well known Caughey-Thomas approximation [10]:

$$\mu_0 = \mu_{\min} + \frac{\mu_{\max} - \mu_{\min}}{1 + \left(\frac{N}{N_{ref}}\right)^\alpha} \quad (1)$$

where μ_{\min} , μ_{\max} , N_{ref} and α are fitting parameters, where μ_{\max} represents the mobility of undoped samples, μ_{\min} is the mobility in highly doped material, α is a measure of how quickly the mobility changes from μ_{\max} to μ_{\min} and N_{ref} represents the carrier concentration at which the mobility is half way between μ_{\max} and μ_{\min} [11]. Taking into account the effect of temperature on the electron transport, the temperature dependence of the low-field mobility can be given as

$$\mu_0(N, T) = \mu_{\max}(T_0) \times \frac{B(N) \times (T/T_0)^\beta}{1 + B(N) \times (T/T_0)^{\beta+\alpha}} \quad (2a)$$

$$\text{where } B(N) = \frac{\mu_{\min}(T_0) + \mu_{\max}(T_0) \times (N_{ref}/N)^\delta}{\mu_{\max}(T_0) - \mu_{\min}(T_0)} \quad (2b)$$

where $T_0 = 300\text{K}$ and μ_{\min} , μ_{\max} , N_{ref} , α , β and δ represent the fitting parameters of Equation (2) and $Par(T_0)$ is the value of the parameter Par at room temperature.

Plugging (2b) into (2a) yields to an analytical electron mobility model with six parameters which will be optimized using PSO approach.

In our study, the j^{th} particle is denoted by $x_j = (\mu_{\min}, \mu_{\max}, N_{ref}, \alpha, \beta \text{ and } \delta)$. The modified velocity and position of each particle can be calculated using the current velocity and the distance from the $pbest_{j,g}$ to $gbest_g$ (Fig.1) as shown in the following formulas [4,9]:

$$v_{j,g}^{(t+1)} = w.v_{j,g}^{(t)} + c_1.r_1 \times (pbest_{j,g} - x_{j,g}^{(t)}) + c_2.r_2 \times (gbest_{j,g} - x_{j,g}^{(t)}) \quad (3a)$$

$$x_{j,g}^{(t+1)} = x_{j,g}^{(t)} + v_{j,g}^{(t+1)}, \text{ with } j=1,2,\dots,n \text{ and } g=1,2,\dots,m \quad (3b)$$

where n represents the number of particles in the swarm; t is the number of generations; w is the inertia weight factor; c_1 and c_2 represent cognitive and social acceleration factors, respectively; m is the number of compounds for the vectors v_j and x_j (in our case $m=6$); r_1 , r_2 are random numbers uniformly distributed in the range (0,1); $x_{j,g}^{(t)}$ represents the g th component of the position of particle j at generation t ; $pbest_j$ represents the local best of particle j ; $gbest_j$ is the global best of the group $v_{j,g}^{(t)}$ represents the g th component of the velocity of particle j at generation t . In the present study, a mean squared error of the mobility for the j th particle is taken as the fitness function f , expressed in Table.1, where f is the fitness value; $\mu_{ps0,k}(N, T, x_j)$ is the predicted mobility based on x_j ; $\mu_{exp,k}(N, T)$ represents the target function (experimental and numerical (MC) mobility database); and S represents the number of samples (database size). It is aimed to minimize this fitness function in order to improve the accuracy of the electron mobility. The flowchart of PSO computation approach used in our study is detailed in Figure.2.

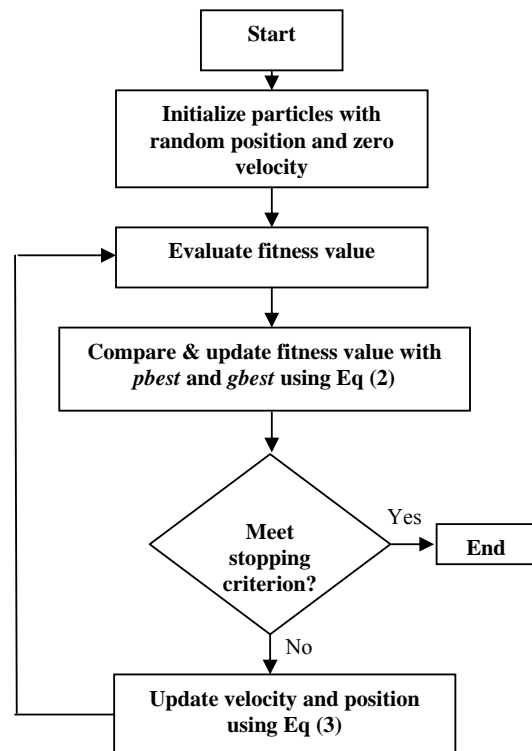


Fig. 2 Flowchart of PSO computation

III. RESULTS AND DISCUSSIONS

The optimization process was based on a population of 20 particles, and a maximum number of generation equal to 100, hence the stability of the fitness function has been obtained. The parameters of PSO were varied and the error was recorded. They allowed us to obtain an optimal configuration of the particles positions and their movement towards *gbest*. Table.1 shows the PSO parameters used in this study.

TABLE I
PARAMETER USED FOR OUR PSO APPROACH

PSO Parameters	Values
Population size (n)	20
Compounds number (m)	6
Maximum Generation (t)	100
Fitness function f	$f = \frac{1}{S} \sum (\mu_{exp,i}(N,T) - \mu_{ps0,i}(N,T,x_j))^2$
w	[0.6, 0.9]
c_1	1
c_2	2

For our PSO configuration, the obtained fitness function is equal to 0.08 and almost all cases have been correctly studied. Table.2 summarizes the obtained optimized parameters of our analytical electron mobility model for low applied electric field.

TABLE II
OBTAINED OPTIMIZED PARAMETERS OF THE ELECTRON MOBILITY MODEL

Parameters	Values
μ_{max}	1217 cm ² /V.s
μ_{min}	44.47 cm ² /V.s
N_{ref}	7.87.10 ¹⁶ cm ⁻³
α	2.12
β	0.33
δ	0.58

In order to validate the predictive property of the optimized PSO configuration, experimental set was compared to the PSO optimized mobility model. Fig.3 shows that a good agreement between experimental and predicted results was found. Hence, the optimized analytical model can be used to predict other combinations of input variables (doping and temperature for low electric field) in full range. This last observation shows the applicability of PSO technique to study of the transport phenomenon in Si-based devices.

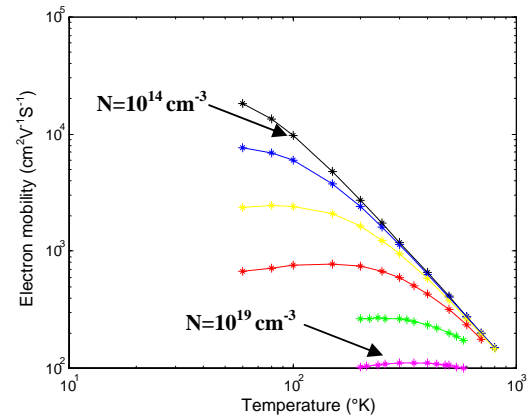


Fig. 3 Predicted low-field electron mobility in Si as function of temperature for different doping concentrations (from $N=10^{14}$ cm⁻³ to $N=10^{19}$ cm⁻³)

The modeling of high-field electron mobility has been the most critical part of the entire model development [8]. For high applied electric field, nonlinearities in the transport coefficients (mobility, diffusion constant,...) have been observed. The experimental database for electric field domain for GaN is quite limited. Therefore, the numerical results (Monte Carlo simulations) have been used to develop our optimized model for the high-field electron mobility. Under the influence of high electric field, the relation between velocity and field has to be described by mobility depending on the field. An expression suited to model such velocity-field characteristics is [10]:

$$v(E) = \frac{\mu_0 E}{\left(1 + (\mu_0 E / v_{sat})^\beta\right)^{\frac{1}{\beta}}} \quad (4)$$

where μ_0 represents the low-field mobility which depends on doping concentration and temperature (calculated from (2)), v_{sat} and β are fitting parameters. Basing on the work published in [11], these parameters can be given as temperature functions

$$Par(T) = Par_0 \times (a + bT + cT^2) \quad (5)$$

with Par represents the fitting parameter of Equation (5), which is v_{sat} or β , and a, b, c are constants that have to be determined by fitting. Plugging (5) into (4) yields to analytical high-field electron mobility model with eight parameters which will be optimised using PSO algorithm. In this case, the j^{th} particle is denoted by $x_j = (v_{sat0}, \beta_0, a_{v_{sat}}, b_{v_{sat}}, c_{v_{sat}}, a_\beta, b_\beta$

etc.). After the optimization process, the obtained fitness function (given in Table.1) is equal to 1.35 and almost all cases have been correctly studied. Table.3 summarizes the obtained optimized parameters of our analytical electron mobility model for high applied electric field. In order to

validate the predictive property of the optimized PSO configuration, numerical set was compared to the PSO optimized mobility model.

TABLE III
OBTAINED OPTIMIZED PARAMETERS OF THE HIGH-FIELD ELECTRON MOBILITY MODEL

Parameters	Values
v_{sat0}	$0.98 \cdot 10^7$ cm/s
β_0	2.05
$a_{v_{sat}}$	0.9157
$b_{v_{sat}}$	$3.8906 \cdot 10^{-4}$
$c_{v_{sat}}$	$-1.3007 \cdot 10^{-6}$
a_{β}	1.252
b	$-4.3645 \cdot 10^{-4}$
c	$2 \cdot 10^{-6}$

Figure.4 represents our optimized mobility model as function of electric field for different temperatures.

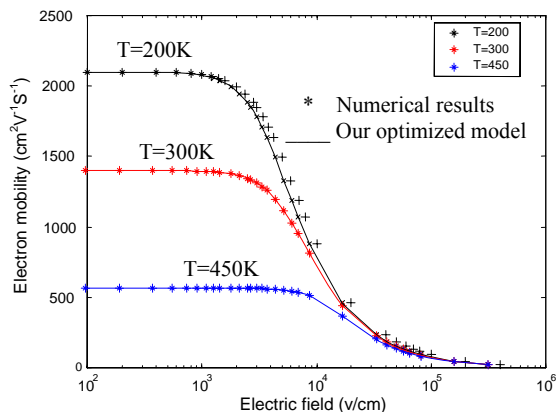


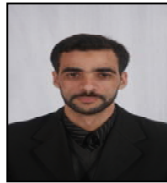
Fig. 4 Predicted high-field electron mobility in Si as function of applied electric field for different temperatures

IV. CONCLUSION

In this paper, we have presented a simple and accurate electron mobility model for Si-based devices based on particle swarm computation. In evolutionary computations, various Si-based materials parameters have simultaneously been considered, and therefore electron mobility parameters can be extracted according to the desired target. The optimised model describes the dependence of the electron mobility on wide doping concentration range (from 10^{14} to 10^{19} cm⁻³), wide temperature range (from 60 to 1000K), and low or high-electric field quite well. This work showed that PSO technique is certainly a promising candidate for silicon devices modeling.

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