

# An Effective Hybrid Genetic Algorithm for Job Shop Scheduling Problem

Bin Cai\*, Shilong Wang, and Haibo Hu

**Abstract**—The job shop scheduling problem (JSSP) is well known as one of the most difficult combinatorial optimization problems. This paper presents a hybrid genetic algorithm for the JSSP with the objective of minimizing makespan. The efficiency of the genetic algorithm is enhanced by integrating it with a local search method. The chromosome representation of the problem is based on operations. Schedules are constructed using a procedure that generates full active schedules. In each generation, a local search heuristic based on Nowicki and Smutnicki's neighborhood is applied to improve the solutions. The approach is tested on a set of standard instances taken from the literature and compared with other approaches. The computation results validate the effectiveness of the proposed algorithm.

**Keywords**—Genetic algorithm, Job shop scheduling problem, Local search, Meta-heuristic algorithm

## I. INTRODUCTION

THE job shop scheduling problem (JSSP) is one of the most difficult problems in combinatorial optimization that has garnered considerable attention due to both its practical importance and its solution complexity. Efficient methods for solving the JSSP have significant effects on profitability and product quality. During the last three decades, many solution methods have been proposed to solve the JSSP. Those approaches can be divided into two categories: exact methods and approximation algorithms. Exact methods, such as branch and bound, linear programming and decomposition methods, guarantee global convergence and have been successful in solving small instances. In manufacturing systems, most scheduling problems are very complex in nature and very complicated to be solved by exact methods to obtain a global optimal schedule. For the big instances there is a need for approximation algorithms, which include priority dispatch, shifting bottleneck approach, local search, and heuristic methods. Recently, using a high-level strategy to guide other heuristics, known as meta-heuristics, led to better and more appreciated results in a relatively short period. Therefore, a number of meta-heuristics were proposed in literature for the past two decades to deal with the JSSP such as genetic

algorithm (GA) [1]-[4], simulated annealing (SA) [5], taboo search (TS) [6], greedy randomized adaptive search procedure (GRASP) [7] etc. A comprehensive survey of job shop scheduling techniques has been done by Jain and Meeran [8].

Among the meta-heuristic algorithms, GA has been used with increasing frequency to address scheduling problems. The GA is based on the survival of the fittest and involves some selection, crossover and mutation operations. GA exhibits parallelism, contains certain redundancy and historical information of past solutions, and is suitable for implementation on massively parallel architecture. As GA became popular in the mid 1980s, many researchers started to apply this meta-heuristic method to the JSSP. Yamada and Nakano [1] designed a GA for solving the classical JSSP. Kobayashi, Ono and Yamamura [4] designed another GA for the classic problem, and reached solution with high quality. Cheng, Gen and Tsujimura [9]-[10] provided a tutorial survey of works on solving the classical JSSP using GA. Wang and Zheng [11] developed a hybrid optimization strategy for JSSP. Ombuki and Ventresca [12] proposed a local search genetic algorithm to solve JSSP. Goncalves, Mendes and Resende [13] developed another hybrid genetic algorithm for JSSP.

In this paper, an effective hybrid intelligent algorithm for JSSP based on genetic algorithm and local search is presented. The remainder of the paper is organized as follows. An introduction for the job shop scheduling problem is given in Section II. Detailed description of the proposed job shop scheduling algorithm is presented in Section III. Section IV discusses the experimental results. Finally, we summarize the paper and present our future work in Section V.

## II. JOB SHOP SCHEDULING PROBLEM

The problem studied in the paper is a deterministic and static  $n$ -job,  $m$ -machine JSSP. In this problem,  $n$  jobs are to be processed by  $m$  machines. Each job consists of a predetermined sequence of task operations, each of which needs to be processed without preemption for a given period of time on a given machine. Tasks of the same job cannot be processed concurrently and each job must visit each machine exactly once. Each operation cannot be commenced until the processing is completed, if the precedent operation is still being processed. A schedule is an assignment of operations to time slots on the machines. The makespan is the maximum completion time of the jobs. The objective of the JSSP is to find a schedule that minimizes the makespan.

Explaining the problem more specifically, let  $J=\{1, 2, \dots, n\}$  denote the set of jobs,  $M=\{1, 2, \dots, m\}$  represent the set of

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machines, and  $O=\{0, 1, 2, \dots, n \times m, n \times m + 1\}$  be the set of operations to be scheduled, where 0 and  $n \times m + 1$  represent the dummy initial and final operations, respectively. The operations are interrelated by the precedence constraints, which force each operation  $j$  to be scheduled after all predecessor operations  $P_j$  are completed. Moreover, operation  $j$  can only be scheduled if the required machine is idle. Furthermore, let  $T_j$  and  $F_j$  denote the fixed processing time and the finish time of operation  $j$ , respectively. Let  $A(t)$  be the set of operations being processed at time  $t$ , and let  $e_{jm}=1$  if operation  $j$  is required to process on machine  $m$  ( $e_{jm}=0$  otherwise).

The conceptual model of the JSSP can be stated as [13]

$$\min F_{n \times m + 1} \quad (1)$$

$$s.t. F_k \leq F_j - T_j, \quad j=1,2,\dots,n \times m + 1; k \in P_j \quad (2)$$

$$\sum_{j \in A(t)} e_{jm} \leq 1, \quad m \in M; \quad t \geq 0 \quad (3)$$

$$F_j \geq 0, \quad j=1,2,\dots,n \times m + 1. \quad (4)$$

The objective function (1) minimizes the finish time of the last operation, namely, the makespan. Constraint (2) imposes the precedence relations between operations. Constraint (3) represents that one machine can only process one operation at a time, and constraint (4) forces the finish times to be nonnegative.

### III. HYBRID GENETIC ALGORITHM FOR JSSP

The GA simulates the biological processes that allow the consecutive generations in a population to adapt to their environment. The adaptation process is mainly applied through genetic inheritance from parents to children and through survival of the fittest. The GA object determines which individuals should survive, which should reproduce, and which should die. To successfully apply a GA to solve a problem one needs to determine the following [14]:

- 1) The representation of possible solutions, or the chromosomal encoding;
- 2) The fitness function which accurately represents the value of the solution;
- 3) Genetic operators (selection, crossover and mutation) have to employ and the parameter values (population size, probability of applying operators, etc.) that are suitable.

#### A. Chromosome Representation

A proper chromosome representation has a great impact on the success of the used GA. Cheng, Gen and Tsujimura [9] gave a detailed tutorial survey on papers using different GA chromosome representations to solve classical JSSP. In this paper, an operation based representation is adopted, which uses an unpartitioned permutation with  $m$ -repetitions of job numbers for problems with  $n$  jobs and  $m$  machines. Within the representation, each job number occurs  $m$  times in the chromosome. By scanning the chromosome from left to right, the  $k$ -th occurrence of a job number refers to the  $k$ -th operation in the technological sequence of this job.

For example, suppose that a chromosome is given as [2 1 3 1 2 2 3 1 3] in a three jobs and three machines problem. Because each job consists of three operations, the job number occurs exactly three times in the chromosome. The fifth gene of the permutation implies the second operation of job 2 because number 2 has been repeated twice. Similarly, the sixth gene represents the third operation of job 2, and so on. The prominent advantage of operation based representation is that the permutation is always feasible. Moreover, it eliminates the deadlock schedules that are incompatible with the technological constraints and can never be finished. However, it will produce redundancy in the search space and will cause the search-space size to expand to  $(n \times m)! / (m!)^n$ .

#### B. Chromosome Decoding

The solution of the JSSP can be represented as the operation permutation of jobs on each machine. The total number of all possible schedules (both feasible and infeasible) is  $(n!)^m$  for problems with  $n$  jobs and  $m$  machines. Obviously, it is impossible to exhaust all the alternatives for finding the optimal solution even if the values of  $n$  and  $m$  are small. For example, for the Fisher-Thompson benchmark problem of ten jobs to ten machines, it has a search space with a size at about  $3.96 \times 10^{65}$ . Thus, it is necessary to restrict the search space and to guide the search process. The objective of the chromosome decoding procedure is to transform the chromosomes to schedules and obtain their makespans.

In general, schedules can be classified into three types: semiactive schedule, active schedule and non-delay schedule [15]. Semiactive schedules contain no excess idle time, but they can be improved by shifting some operations to the front without delaying others. Active schedules contain no idle time, and no operation can be finished earlier without delaying other operations. The set of non-delay schedules is a subset of active schedules. In a non-delay schedule, no machine is kept idle at a time when it could begin processing other operations. In order to further reduce the solution space, Zhang, Rao and Li [16] proposed a new type of schedule: full active schedule (FAS), which can be defined as a schedule with no more permissible left shifts and right shifts. Fig. 1 shows the relationships between the classes of schedules. The optimal schedule is guaranteed to be a full active schedule. Therefore, we only need to find the optimum solution in the set of full active schedules.

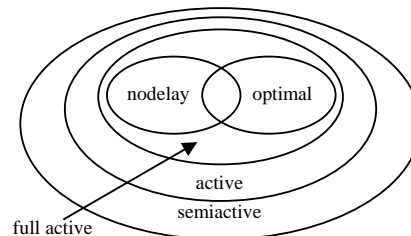


Fig. 1 Classes of schedules

#### C. Crossover Operation

Crossover operator plays an important role in genetic

algorithm approach. It intends to inherit the properties of two parent solutions to two offspring solutions. To apply crossover operation successfully to the JSSP, we must satisfy the following criteria: completeness, feasibility, non-redundancy and characteristics preservation [4]. In this paper, we use the set-partition crossover (SPX) [17] as crossover, which can preserve characteristics properly between parents and their children. Given chromosomes, *parent1* and *parent2*, crossover applied SPX generates the children, *child1* and *child2*, by the following procedure. Firstly, randomly divide the set of job numbers as  $\{1, 2, \dots, n\}$  into two nonempty exclusive subsets as  $J1$  and  $J2$ . Secondly, combine together those numbers of *parent1* in  $J1$  and those numbers of *parent2* in  $J2$ . The combination order is in an interweaving way, i.e. one by one from up-to-down and left-to-right. This part of procedure creates one new string. Exchange the two parents *parent1* and *parent2*, and do the combination once again to yield another new string. Fig. 2 shows an example of the three jobs and three machines problem; chromosome of *parent1* and *parent2* is  $\{1\ 2\ 3\ 3\ 2\ 1\ 3\ 2\ 1\}$  and  $\{1\ 2\ 2\ 2\ 3\ 1\ 3\ 3\ 1\}$  respectively. The crossover generates two children chromosomes, *child1*  $\{1\ 2\ 2\ 3\ 1\ 3\ 2\ 3\ 1\}$  and *child2*  $\{1\ 2\ 2\ 3\ 1\ 3\ 2\ 3\ 1\}$ .

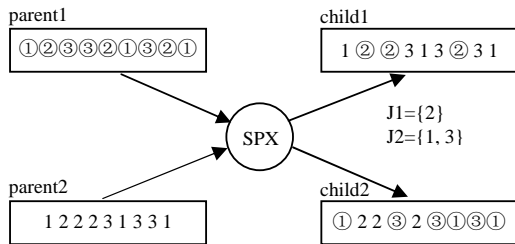


Fig. 2 Example of SPX crossover

**D. Mutation Operation**

Mutation is another important genetic operator that randomly changes a chromosome. This is done to maintain the diversity of the chromosomes and to introduce some extra variability into the population. In this paper, two types of mutation operators named forward insertion mutation (FIM) and backward insertion mutation (BIM) are used. Fig. 3 shows examples of the three jobs and three machines problem. In this work, the two mutation operators alternate randomly with equal probability. Two mutations are described as follows:

- 1) Forward insertion mutation selects two elements randomly and inserts the back one before the front one.
- 2) Backward insertion mutation selects two elements randomly and inserts the front one after the back one.

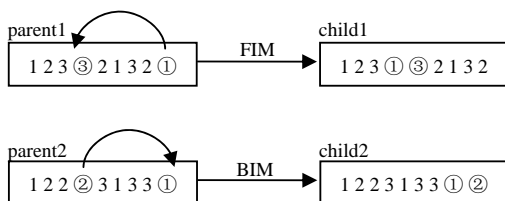


Fig. 3 Examples of the two mutation operators

**E. Local Search Procedure**

The use of local search techniques has been proven to be useful in solving combinatorial problems. Local search methods are applied to a neighborhood of a current solution. In the case of JSSP, a neighborhood is achieved by moving and inserting an operation in a machine sequence. In this paper, we focus particularly on the approach of Nowicki and Smutnicki [6], which is noted for proposing and implementing the most restrictive neighborhood in the literature. According to Nowicki and Smutnicki's work, a critical path in the solution is identified first. Then the operations on the critical path are called critical operations and the maximal sequence of adjacent critical operations that are processed on the same machine can be defined as blocks. The neighborhood is defined as interchanges of the last two or the first two critical operations of the blocks if the blocks are neither the first block nor the last block. In the first block only the last two operations and symmetrically in the last block of the critical path only the first two operations are swapped. If a block contains only one operation no swap is made. The Nowicki and Smutnicki's neighborhood is illustrated in Fig. 4.

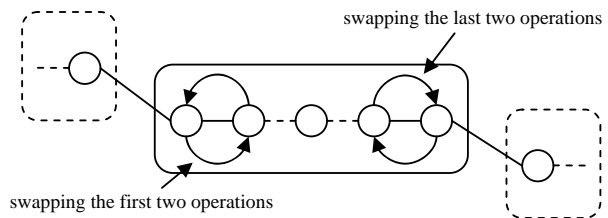


Fig. 4 The Nowicki and Smutnicki's neighborhood

The proposed local search starts with a feasible schedule  $S$  as an input. The input schedule is set to  $S_{best}$  which stands for the best found solution. Then, a single arbitrary critical path is generated and a neighborhood of schedule  $S_{best}$  is constructed. Randomly select a schedule  $S_{new}$  from the neighborhood. If  $S_{new}$  is better (i.e. has a lower makespan) than  $S_{best}$ , the  $S_{best}$  is replaced by  $S_{new}$ . The procedure is repeated until a maximum number of iterations ( $LOC\_ITER$ ) without improving the best found solution is reached. The pseudo-code of the local search heuristic is shown in Algorithm LS.

**Algorithm LS (local search)**

1. Calculate the makespan  $C_{max}(S_{best})$  of the current schedul  $S$ . Set iteration counter  $count$  to 1
2. **While**( $count < LOC\_ITER$ )**do**
3. Randomly selected a schedule  $S_{new}$  from the neighborhood of  $S_{best}$
4. **If**  $C_{max}(S_{new}) < C_{max}(S_{best})$  **Then**
5. Update  $S_{best}$  by setting  $S_{best} = S_{new}$
6. Set  $count$  to 1
7. **Else**
8.  $count++$
9. **End If**
10. **End While**

TABLE I  
COMPUTATIONAL RESULTS OF FT AND LA TEST INSTANCES

Instance	Size	BKS	our HGA	HGA param	LSGA	GRASP	GP+PR	TSAB	Beam Search	RCS	SBII
ft06	6×6	55	<b>55</b>	55	55	55	55	55	-	55	55
ft10	10×10	930	<b>930</b>	930	976	938	930	930	1016	930	930
ft20	20×5	1165	<b>1165</b>	1165	1209	1169	1165	1165	-	1165	1178
la01	10×5	666	<b>666</b>	666	-	666	666	666	666	666	666
la02	10×5	655	<b>655</b>	655	-	655	655	655	704	655	669
la03	10×5	597	<b>597</b>	597	-	604	597	597	650	597	605
la04	10×5	590	<b>590</b>	590	-	590	590	590	620	590	593
la05	10×5	593	<b>593</b>	593	-	593	593	593	593	593	593
la06	15×5	926	<b>926</b>	926	-	926	926	926	926	926	926
la07	15×5	890	<b>890</b>	890	-	890	890	890	890	890	890
la08	15×5	863	<b>863</b>	863	-	863	863	863	863	863	863
la09	15×5	951	<b>951</b>	951	-	951	951	951	951	951	951
la10	15×5	958	<b>958</b>	958	-	958	958	958	958	958	959
la11	20×5	1222	<b>1222</b>	1222	-	1222	1222	1222	1222	1222	1222
la12	20×5	1039	<b>1039</b>	1039	-	1039	1039	1039	1039	1039	1039
la13	20×5	1150	<b>1150</b>	1150	-	1150	1150	1150	1150	1150	1150
la14	20×5	1292	<b>1292</b>	1292	-	1292	1292	1292	1292	1292	1292
la15	20×5	1207	<b>1207</b>	1207	-	1207	1207	1207	1207	1207	1207
la16	10×10	945	<b>945</b>	945	959	946	945	945	988	945	978
la17	10×10	784	<b>784</b>	784	792	784	784	784	827	784	787
la18	10×10	848	<b>848</b>	848	857	848	848	848	881	848	859
la19	10×10	842	<b>842</b>	842	860	842	842	842	882	848	860
la20	10×10	902	<b>907</b>	907	907	907	902	902	948	907	914
la21	15×10	1046	<b>1047</b>	1046	1114	1091	1057	1047	1154	1069	1084
la22	15×10	927	<b>930</b>	935	989	960	927	927	985	927	944
la23	15×10	1032	<b>1032</b>	1032	1035	1032	1032	1032	1051	1032	1032
la24	15×10	935	<b>941</b>	953	1032	978	954	939	992	942	976
la25	15×10	977	<b>979</b>	986	1047	1028	984	977	1073	981	1017
la26	20×10	1218	<b>1218</b>	1218	1307	1271	1218	1218	1269	1218	1224
la27	20×10	1235	<b>1240</b>	1256	1350	1320	1269	1236	1316	1285	1291
la28	20×10	1216	<b>1216</b>	1232	1312	1293	1225	1216	1373	1216	1250
la29	20×10	1152	<b>1167</b>	1196	1311	1293	1203	1160	1252	1208	1239
la30	20×10	1355	<b>1355</b>	1355	1451	1368	1355	1355	1435	1355	1355
la31	30×10	1784	<b>1784</b>	1784	1784	1784	1784	1784	1784	1784	1784
la32	30×10	1850	<b>1850</b>	1850	1850	1850	1850	1850	1850	1850	1850
la33	30×10	1719	<b>1719</b>	1719	1745	1719	1719	1719	1719	1719	1719
la34	30×10	1721	<b>1721</b>	1721	1784	1753	1721	1721	1780	1721	1721
la35	30×10	1888	<b>1888</b>	1888	1958	1888	1888	1888	1888	1888	1888
la36	15×15	1268	<b>1278</b>	1279	1358	1334	1287	1268	1401	1292	1305
la37	15×15	1397	<b>1397</b>	1408	1517	1457	1410	1407	1503	1411	1423
la38	15×15	1196	<b>1202</b>	1219	1362	1267	1218	1196	1297	1278	1255
la39	15×15	1233	<b>1238</b>	1246	1391	1290	1248	1233	1369	1233	1273
la40	15×15	1222	<b>1228</b>	1241	1323	1259	1244	1229	1347	1247	1269
Average gap(%)			<b>0.14</b>	0.40	5.39	1.78	0.44	0.06	4.35	0.61	1.39
No. of instance			<b>43</b>	43	28	43	43	43	41	43	43
No. of BKS obtained			<b>32</b>	31	3	23	32	37	18	31	20

#### F. Designing a hybrid genetic algorithm for JSSP

In contrast to a simple genetic algorithm, a new generation alternation model is introduced for the proposed hybrid GA in this paper. Every pair of randomly selected distinct mates must pass either crossover or mutation, which are deployed in parallel. The crossover is performed with a probability  $P_c$ . When the mating process is carried out, crossover operator is applied to the two parents  $N$  times and  $2N$  offspring are generated; the best individual in those offspring is selected to the next generation. Otherwise, implements the mutation operator to the two parents  $N$  times respectively and  $2N$  offspring are generated too; the best individual is selected to the next generation. The crossover rate  $P_c$  is decreased linearly from 0.9 to 0.5 according to (5), where  $g$  represents the iterative number;  $MAX\_GEN$  is the maximum number of iterations.

Such a mechanism can improve the exploration ability of GA. For example, at the beginning of the evolution period, the crossover rate is big; whereas at the end of the convergence period, the crossover rate decreases and the mutation rate becomes big; this characteristic of the new crossover rate can avoid premature convergence better.

$$P_c = 0.9 - g / MAX\_GEN \times 0.4 \quad (5)$$

The brief outline of the proposed algorithm can be described as follows.

Step 1) Set values of  $pop\_size$ ,  $N$ ,  $MAX\_GEN$ ,  $LOC\_ITER$ .

Step 2) Generate a population  $P_0$  with  $pop\_size$  individuals randomly and evaluate the individuals with the decoding procedure; set generation counter  $g = 1$  and the current population  $P_{old} = P_0$ .

Step 3) Repeat Step 4) – 11) until  $g > MAX\_GEN$ .

Step 4) Copy the elite individual from  $P_{old}$  to the new population  $P_{new}$ . Set the new population size  $n = 1$ .

Step 5) Repeat Step 6) – 9) until  $n > pop\_size$ .

Step 6) Select a pair of individuals  $p1, p2$  from the  $P_{old}$

Step 7) Generate a random float  $rand\_num \in (0,1)$ , if  $rand\_num < P_c$  go to Step 8), else go to Step 9).

Step 8) Implement crossover on  $p1$  and  $p2$  for  $N$  times and generate  $2N$  offspring, select the best individual in the  $2N$  offspring to the next generation. Set  $n = n + 1$ .

Step 9) Implement mutation on  $p1$  and  $p2$   $N$  times respectively and generate  $2N$  offspring, select the best individual to the next generation. Set  $n = n + 1$ .

Step 10) Implement local search on every individual in  $P_{new}$ .

Step 11) Set  $P_{old} = P_{new}$

#### IV. COMPUTATIONAL RESULTS

To illustrate the effectiveness and performance, we use 43 instances that are taken from the ORLibrary [18] as test benchmarks to test our new proposed hybrid GA. In the 43 instances, FT06, FT10 and FT20 were designed by Fisher and Thompson in 1963 and instances LA01–LA40 that were designed by Lawrence in 1984. The algorithm was implemented in Visual C++ and the tests were run on a computer with Pentium IV2.4G and 1GB RAM. In our experiments, population size  $pop\_size = 100$ ,  $N = 5$ ,  $LOC\_ITER$  is the smallest integer number not less than  $n/2$ , and  $P_c$  is decreased linearly from 0.9 to 0.5. The algorithm was terminated when after  $MAX\_GEN = n \times m$  generations of the algorithm, and each instance is randomly run 20 times. Numerical results are compared with those reported in some existing literature works using some heuristic and meta-heuristic algorithms, including HGA-param [13], LSGA [12], GRASP [7], GP+PR [19], TSAB [6], Beam Search [20], RCS [21], and SBII [22].

Table I summarizes the results of the experiments. The contents of the table include the name of each test problem (Instance), the scale of the problem (Size), the value of the best known solution for each problem (BKS), the value of the best solution found by using the proposed algorithm (our HGA) and the best results reported in other research works.

TABLE II  
SUMMARY OF RESULTS FOR TYPICAL INSTANCES

Instance	Size	BKS	Best	BRD (%)	Mean	MRD (%)	t-avg (s)
ft06	6×6	55	55	0.00	55	0.00	0.62
ft10	10×10	930	930	0.00	936.85	0.74	8.21
ft20	20×5	1165	1165	0.00	1171.9	0.59	16.37
la01	10×5	666	666	0.00	666	0.00	1.90
la06	15×5	926	926	0.00	926	0.00	5.42
la11	20×5	1222	1222	0.00	1222	0.00	14.63
la16	10×10	945	945	0.00	947.15	0.23	7.65
la21	15×10	1046	1047	0.10	1057.15	1.07	24.49
la26	20×10	1218	1218	0.00	1218	0.00	62.48
la31	30×10	1784	1784	0.00	1784	0.00	202.81
la36	15×15	1268	1278	0.79	1286.55	1.46	56.20

It can be seen from Table I that the proposed algorithm is able to find the best known solution for 32 instances, i.e. in about 75% of the instances, and the deviation of the minimum found makespan from the best known solution is only on average 0.14%. The proposed algorithm yields a significant improvement in solution quality with respect to almost all other algorithms, expected for the approach proposed by Nowicki and Smutnicki that has a better performance in the  $15 \times 15$  problems mainly. The superior results indicate the successful incorporation of the improved GA and LS, which facilitates the escape from local minimum points and increases the possibility of finding a better solution. Therefore, it can be concluded that the proposed hybrid GA solves the JSSP fairly efficiently.

As mentioned above, the algorithm is performed 20 times for each instance. Table II lists the best solution (Best), the relative deviation of the best solution (BRD), the mean solutions (Mean), the relative deviation of the mean solution (MRD), and the average computing time (t-avg) of some typical instances with different size. The MRD is commonly zero for small-size problem and is not more than 1.5% for most other problems.

To illustrate the simulated results more intuitively, the problem la37 that is one of the hardest problems is specially described as an example. Fig. 5 plots the representative convergence curve finding best solution. Fig. 6 shows a Gantt chart of a best solution.

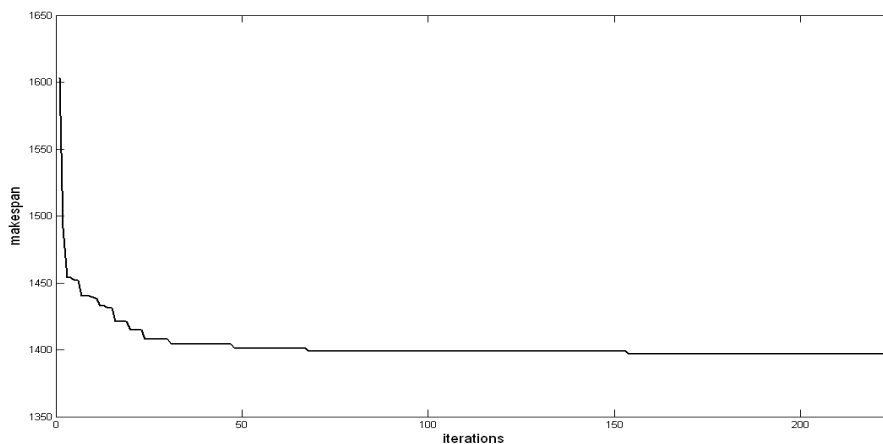


Fig. 5 Representative convergence curve for la37

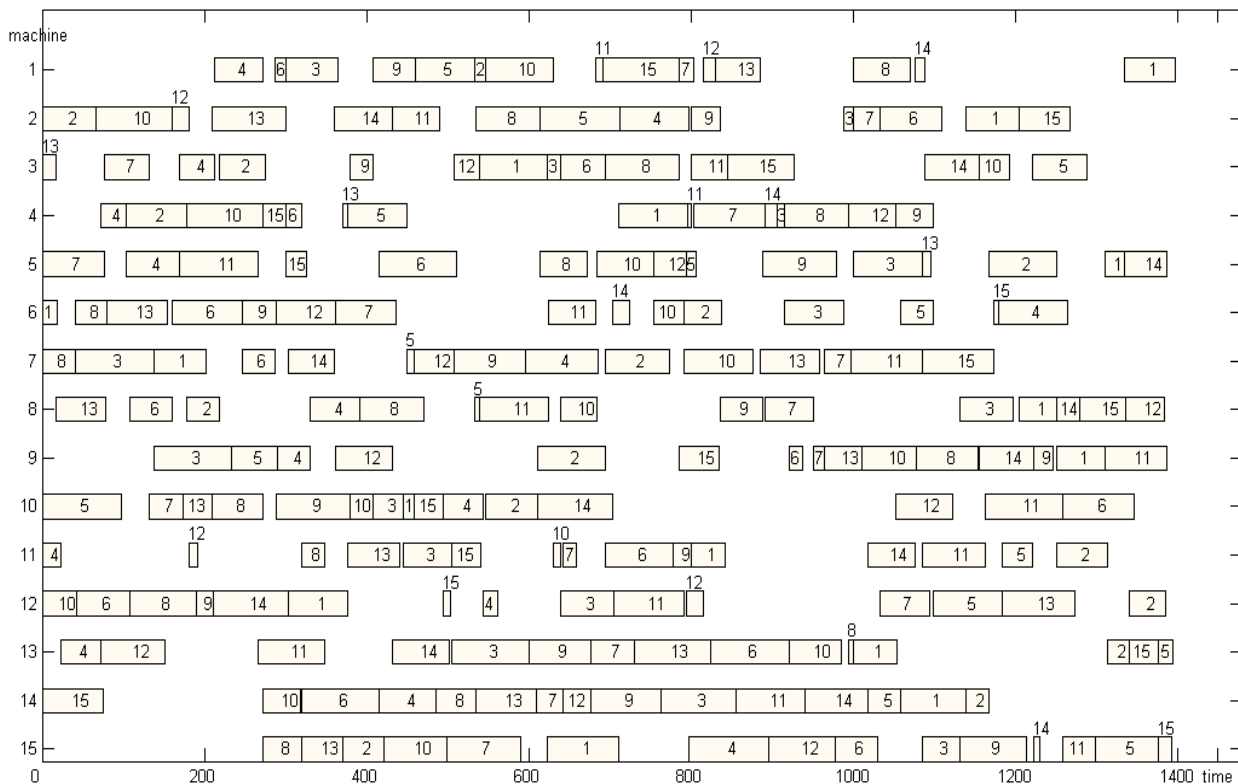


Fig. 6 Gantt chart of an optimal schedule for la37

### V. CONCLUSION AND PERSPECTIVES

This paper presents a hybrid algorithm combining genetic algorithm with local search for the JSSP. In the algorithm a new generation alternation model of genetic algorithm for JSSP is designed and a Nowicki and Smutnicki's neighborhood based local search algorithm is incorporated. This allows the GA to explore more solution space whereas LS does the exploitation part. The approach is tested on a set of 43 standard instances taken from the literature and compared with other approaches. The computational results show that the algorithm produced optimal or near-optimal solutions on all instances tested. Overall, the algorithm produced solutions with an average relative deviation of 0.14% to the best known solution. In our future work we aim to extend the proposed algorithm in order that it can be applied to more practical and integrated manufacturing problems such as dynamic arrivals, machine breakdown, or other factors that affect job status over time.

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