International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:5, No:8, 2011

Mean square exponential synchronization of stochastic neutral type chaotic neural networks with mixed delay

Zixin Liu, Huawei Yang, Fangwei Chen

Abstract—This paper studies the mean square exponential synchronization problem of a class of stochastic neutral type chaotic neural networks with mixed delay. On the Basis of Lyapunov stability theory, some sufficient conditions ensuring the mean square exponential synchronization of two identical chaotic neural networks are obtained by using stochastic analysis and inequality technique. These conditions are expressed in the form of linear matrix inequalities (LMIs), whose feasibility can be easily checked by using Matlab LMI Toolbox. The feedback controller used in this paper is more general than those used in previous literatures. One simulation example is presented to demonstrate the effectiveness of the derived results.

Keywords—Exponential synchronization, stochastic analysis, chaotic neural networks, neutral type system.

I. Introduction

SINCE the seminal works of Pecora and Carroll [1], [2], chaos synchronization has been intensively researched because of its potential applications in various fields such as secure communication, biological systems, information science, etc (see [3]-[13]). On the other hand, delayed neural networks as special complex dynamical systems, have also been found to exhibit unpredictable behaviors such as periodic oscillations, bifurcation and attractors. The study on chaos synchronization of delayed neural networks have also been proposed (see [3]-[11]). In [5], Lou and Cui studied the synchronization of neural networks based on parameter identification and via output or state coupling. In [6] Li and Yang discussed the adaptive exponential synchronization of delayed neural networks with reaction-diffusion terms. In [10] Yang and Cao researched the exponential lag synchronization of a class of chaotic delayed neural networks with impulsive effects. In [11] exponential synchronization of chaotic neural networks with mixed delays was investigated.

It's worth pointing out that neutral type dynamical model as one of the most important dynamical system is ubiquitous in both nature and man-made systems. However, to the best of our knowledge, few results for mean square exponential synchronization of a class of stochastic neutral type chaotic neural networks with mixed delay are reported.

Motivated by the above analysis, in this paper, we'll focus on the mean square exponential synchronization of a class of stochastic neutral type chaotic neural networks with mixed delay. And the same, a more general feedback controller which relates not only to the discrete delay but also to the distributed delay is considered.

Compared with the existing results on the analysis of exponential synchronization, the work of our paper has three features. First, we consider the drive system with distributed and discrete delays. Secondly, the feedback controller used in this paper is more general than that used in previous literatures. Thirdly, both of the drive system and the response system are neutral type neural networks, and the response system includes stochastic factors.

Z. Liu, H. Yang, and F. Chen are with the School of Mathematics and Statistics, Guizhou College of Finance and Economics, Guiyang, 550004, China. E-mail: (xinxin905@163.com).

II. Preliminaries

Notations. The notations are used in our paper except where otherwise specified. Let $\mathbb{N}$ denotes the nature number set, $(\Omega, \mathscr{F}, \{\mathscr{F}\}_{t\geq 0}, P)$ denotes a complete probability space with filtration $\{\mathscr{F}\}_{t\geq 0}$, $\|\cdot\|$ a vector or a matrix norm; $\mathscr{R}, \mathscr{R}^n$ are real and n-dimension real number sets respectively, $\mathscr{R}^{n\times n}$ denotes $n\times n$ matrix set. $\lambda_{min}(\cdot)(\lambda_{min}(\cdot))$ the smallest(largest) eigenvalue of given matrix, $\mathbb{E}(\cdot)$ denotes the mathematical expectation with respect to the given probability measure P. $\mathscr{L}$ denotes the well-known $\mathscr{L}$-operator given by the Itô's formula, I denotes the identity matrix,

In this paper, we consider the following neutral type chaotic neural networks

$$\begin{cases} d[x(t)-Ex(t-h)] = [-Cx(t)+Af(x(t)) \\ \quad + Bf(x(t-\tau)) + D\int_{-\infty}^{t} K(t-s)f(x(s))ds \\ \quad + I']dt, t>0 \\ x(\theta)=\phi(\theta), \qquad\qquad \theta\in(-\infty,0], \end{cases} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is a real n-vector which denotes the state variable, $C = diag(c_1, c_2, \ldots, c_n)$, $c_i > 0, i = 1, 2, \ldots, n$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and the external inputs; $A = (a_{ij})_{n\times n}$ represents the connection weight matrix, $B = (b_{ij})_{n\times n}, D = (d_{ij})_{n\times n}, E = (e_{ij})_{n\times n}$ represent the delayed connection weight matrices, I' represents the external input, f is activation function, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$, $f(x(t-\tau)) = (f_1(x_1(t-\tau)), f_2(x_2(t-\tau)), \ldots, f_n(x_n(t-\tau)))^T$, $\tau > 0, h > 0$ are the transmission delays, $K(t-s) = diag(k_1(t-s), k_2(t-s), \ldots, k_n(t-s))$ is delayed kernel function, $\phi(\cdot)$ denotes the continuous initial function.

In order to synchronize system (1) via the feedback control, we introduce the respond system from the unidirectional linear coupling approach as follows:

$$\begin{cases} d[y(t)-Ey(t-h)] = [-Cy(t)+Af(y(t)) \\ +Bf(y(t-\tau))+D\int_{-\infty}^{t}K(t-s)f(y(s))ds \\ +I'+u(t)]dt+\sigma(t,e(t),e(t-\tau))d\omega(t), t>0. \\ y(\theta)=\varphi(\theta), \qquad \theta\in(-\infty,0], \end{cases} \tag{2}$$

where $\sigma:\mathscr{R}\times\mathscr{R}^n\times\mathscr{R}^n\to\mathscr{R}^{n\times m}$, $\omega(t)=(\omega_1(t),\omega_2(t),\cdots,\omega_m(t))^T$ denotes a m-dimensional standard Brownian motion defined on a complete probability space $(\Omega,\mathscr{F},\{\mathscr{F}\}_{t\geq 0},P)$, $u(t)=F_1e(t)+F_2\int_{t-\tau}^{t}e(s)ds$ is the state feedback controller given to achieve the synchronization between drive-response system, and F_1,F_2 are the feedback gain parameters to be scheduled, $e(t)=y(t)-x(t)$ represents the error, $\varphi(\cdot)$ denotes the continuous initial function.

Throughout this paper, we always make the following assumptions:

(A_1) $\quad \|f_i(u)-f_i(v)\|\leq l_i\|u-v\|, \forall u,v\in\mathscr{R}^n.$

(A_2) $\quad trace[\sigma^T(t,e(t),e(t-\tau))\sigma(t,e(t),e(t-\tau))]$
$\leq \|M_1e(t)\|^2+\|M_2e(t-\tau)\|^2.$

(A_3) $\quad \int_0^\infty k_j(s)ds=1, \int_0^\infty e^{\varepsilon s}k_j(s)ds=k<\infty.$

where M_1,M_2 are constant matrices with appropriate dimensions, $\varepsilon>0$ is a constant scalar. From system (1) and (2) we can obtain the error system as follows:

$$\begin{cases} d[e(t)-Ee(t-h)] = [(F_1-C)e(t)+Ag(e(t)) \\ +Bg(e(t-\tau))+D\int_{-\infty}^{t}K(t-s)g(e(s))ds \\ +F_2\int_{t-\tau}^{t}e(s)ds]dt+\sigma(t,e(t),e(t-\tau))d\omega(t), t>0, \\ e(\theta)=\psi(\theta), \qquad \theta\in(-\infty,0], \end{cases} \tag{3}$$

where $g(e(t))=f(y(t))-f(x(t))$, $g(e(t-\tau))=f(y(t-\tau))-f(x(t-\tau))$, $\psi(t)=\varphi(t)-\phi(t)$.

For further discussion, we introduce the following definition and lemmas.

Definition 2.1: The drive system (1) and the response system (2)are said to be mean square exponentially synchronized if for a suitably designed feedback controller, there exist positive scalars $\alpha>0,\beta>1$, such that for any $t\geq 0$ and $\psi\in C([-\tau,0],\mathscr{R}^n)$

$$\mathbb{E}(\|e(t\|)^2\leq\alpha\mathbb{E}(\|\psi\|_\Delta)^2e^{-\beta t},$$

where $\|\psi\|_\Delta\triangleq\sup_{-\infty<\theta\leq 0}\|\psi(\theta)\|$, and the constant β is defined as the exponential synchronization rate.

Lemma 2.1: [7] For any constant symmetric positive defined matrix W, scalars $a<b$, vector function $f:[a,b]\to R^n$ such that the integrations concerned are well defined, then

$$(\int_a^b f(t)dt)^TW(\int_a^b f(t)dt)\leq(b-a)\int_a^b f(t)^TWf(t)dt.$$

III. Main results

In this section, we'll consider the synchronization problem between the drive system (1) with the response system (2).

Theorem 3.1: Suppose that $\|E\|<1/2$, then, if there exist positive definite matrices P,N_1,N_2,N_3, positive definite diagonal matrices M,N_4 and positive scalar ε such that

$$\Xi=\begin{pmatrix} \Xi_{11} & \Xi_{12} & 0 & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} \\ * & \Xi_{22} & 0 & \Xi_{24} & \Xi_{34} & \Xi_{26} & \Xi_{27} \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & 0 & 0 \\ * & * & * & * & * & -M & 0 \\ * & * & * & * & * & * & \Xi_{77} \end{pmatrix}<0,$$

where

$$\Xi_{11}=\varepsilon P+P(-C+F_1)+(-C+F_1)^TP$$
$$+N_1+N_3+kLML+\tau^2F_2^TF_2+\lambda_{max}(P)M_1^TM_1,$$

$$\Xi_{12}=-\varepsilon PE-(F_1-C)^TPE,$$

$$\Xi_{14}=PA+\frac{1}{2}LN_4, \Xi_{15}=PB, \Xi_{16}=PD, \Xi_{17}=PF_2,$$

$$\Xi_{22}=\varepsilon E^TPE-N_1e^{-\varepsilon h},$$

$$\Xi_{24}=-E^TPA, \Xi_{26}=-E^TPD, \Xi_{27}=-E^TPF_2,$$

$$\Xi_{33}=-N_3e^{-\varepsilon\tau}+\lambda_{max}(P)M_2^TM_2,$$

$$\Xi_{34}=-E^TPB, \Xi_{44}=N_2-N_4, \Xi_{55}=-N_2e^{-\varepsilon\tau}$$

$$\Xi_{77}=-e^{\varepsilon\tau}F_2^TF_2, M=diag(m_1,m_2,\ldots,m_n),$$

then the drive system (1) and the response system (2) are mean square exponentially synchronized.

Proof. Set $\mathscr{D}(t)=e(t)-Ee(t-h)$, we construct the Lyapunov functional for system (3) by $V(t,e(t))=V_1+V_2+V_3+V_4+V_5+V_6$, where

$$V_1=e^{\varepsilon t}\mathscr{D}^T(t)P\mathscr{D}(t),$$

$$V_2=\int_{t-h}^{t}e^{\varepsilon s}e^T(s)N_1e(s)ds,$$

$$V_3=\int_{t-\tau}^{t}e^{\varepsilon s}g^T(e(s))N_2g(e(s))ds,$$

$$V_4=\int_{t-\tau}^{t}e^{\varepsilon s}e^T(s)N_3e(s)ds,$$

$$V_5=\tau\int_{t-\tau}^{t}(s-t+\tau)e^{\varepsilon s}e^T(s)F_2^TF_2e(s)ds,$$

$$V_6=\sum_{j=1}^{n}m_j\int_0^\infty k_j(\xi)\int_{t-\xi}^{t}e^{\varepsilon(\xi+s)}g_j^2(e_j(s))dsd\xi,$$

then $\mathscr{L}V = \mathscr{L}V_1 + \mathscr{L}V_2 + \mathscr{L}V_3 + \mathscr{L}V_4 + \mathscr{L}V_5 + \mathscr{L}V_6$, where

$$\begin{aligned}
\mathscr{L}V_1 &= \varepsilon e^{\varepsilon t}\mathscr{D}^T(t)P\mathscr{D}(t) + 2e^{\varepsilon t}\mathscr{D}^T(t)P[(F_1 - C)e(t) \\
&+Ag(e(t)) + Bg(e(t-\tau)) \\
&+D\int_{-\infty}^{t} K(t-s)g(e(s))ds + F_2\int_{t-\tau}^{t} e(s)ds] \\
&+e^{\varepsilon t}trace[\sigma^T(t,e(t),e(t-\tau))P\sigma(t,e(t),e(t-\tau))] \\
&= \varepsilon e^{\varepsilon t}(e^T(t) - e^T(t-h)E^T)P(e(t) - Ee(t-h)) \\
&+2e^{\varepsilon t}\mathscr{D}^T(t)P[(F_1 - C)e(t) \\
&+Ag(e(t)) + Bg(e(t-\tau)) \\
&+D\int_{-\infty}^{t} K(t-s)g(e(s))ds + F_2\int_{t-\tau}^{t} e(s)ds] \\
&+e^{\varepsilon t}trace[\sigma^T(t,e(t),e(t-\tau))P\sigma(t,e(t),e(t-\tau))] \\
&\leq \varepsilon e^{\varepsilon t}(e^T(t) - e^T(t-h)E^T)P(e(t) - Ee(t-h)) \\
&+2e^{\varepsilon t}\mathscr{D}^T(t)P[(F_1 - C)e(t) \\
&+Ag(e(t)) + Bg(e(t-\tau)) \\
&+D\int_{-\infty}^{t} K(t-s)g(e(s))ds + F_2\int_{t-\tau}^{t} e(s)ds] \\
&+\lambda_{max}(P)e^{\varepsilon t}trace[\sigma^T(t,e(t), \\
&e(t-\tau))\sigma(t,e(t),e(t-\tau))] \\
&\leq \varepsilon e^{\varepsilon t}(e^T(t) - e^T(t-h)E^T)P(e(t) - Ee(t-h)) \\
&+2e^{\varepsilon t}\mathscr{D}^T(t)P[(F_1 - C)e(t) \\
&+Ag(e(t)) + Bg(e(t-\tau)) \\
&+D\int_{-\infty}^{t} K(t-s)g(e(s))ds + F_2\int_{t-\tau}^{t} e(s)ds] \\
&+\lambda_{max}(P)e^{\varepsilon t}[e^T(t)M_1^T M_1 e(t) \\
&+e^T(t-\tau))M_2^T M_2 e(t-\tau)] \\
&= e^{\varepsilon t}\{e^T(t)[\varepsilon P + P(F_1 - C) + (F_1 - C)^T P \\
&+\lambda_{max}(P)M_1^T M_1]e(t) - 2e^T(t)[\varepsilon PE \\
&+(F_1 - C)^T PE]e(t-h) + 2e^T(t)PAg(e(t)) \\
&+2e^T(t)PBg(e(t-\tau)) \\
&+2e^T(t)PD\int_{-\infty}^{t} K(t-s)g(e(s))ds \\
&+2e^T(t)(PF_2)\int_{t-\tau}^{t} e(s)ds \\
&+e^T(t-h)\varepsilon E^T PEe(t-h) \\
&-2e^T(t-h)E^T PAg(e(t)) \\
&-2e^T(t-h)E^T PBg(e(t-\tau)) \\
&-2e^T(t-h)E^T PD\int_{-\infty}^{t} K(t-s)g(e(s))ds \\
&-2e^T(t-h)(E^T PF_2)\int_{t-\tau}^{t} e(s)ds \\
&+\lambda_{max}(P)e^T(t-\tau)M_2^T M_2 e(t-\tau)\}. \qquad (4)
\end{aligned}$$

$$\mathscr{L}V_2 = e^{\varepsilon t}[e^T(t)N_1 e(t) - e^{-\varepsilon h}e^T(t-h)N_1 e(t-h)]. \quad (5)$$

$$\mathscr{L}V_3 = e^{\varepsilon t}[g^T(e(t))N_2 g(e(t)) - e^{-\varepsilon\tau}g^T(e(t-\tau))N_2 g(e(t-\tau))]. \quad (6)$$

From Assumption A_1, we have $g^T(e(t))N_4 Le(t) = \sum_{i=1}^{n} g_i(e_i(t))n_{4_i}l_i e_i(t) \geq \sum_{i=1}^{n} n_{4_i}g_i^2(e_i(t)) = g^T(e(t))N_4 g(e(t))$, where $N_4 = diag(n_{4_1}, n_{4_2}, \ldots, n_{4_n})$ and $L = diag(l_1, l_2, \ldots, l_n)$ are positive definite diagonal matrices, thus

$$\begin{aligned}
\mathscr{L}V_3 \leq\ & e^{\varepsilon t}[g^T(e(t))(N_2 - N_4)g(e(t)) + g^T(e(t))N_4 Le(t) \\
& -e^{-\varepsilon\tau}g^T(e(t-\tau))N_2 g(e(t-\tau))]. \qquad (7)
\end{aligned}$$

$$\mathscr{L}V_4 = e^{\varepsilon t}[e^T(t)N_3 e(t) - e^{-\varepsilon\tau}e^T(t-\tau)N_3 e(t-\tau)]. \quad (8)$$

$$\begin{aligned}
\mathscr{L}V_5 &= e^{\varepsilon t}\tau^2 e^T(t)F_2^T F_2 e(t) \\
&\quad -\tau\int_{t-\tau}^{t} e^{\varepsilon s}e^T(s)F_2^T F_2 e(s)ds \\
&\leq e^{\varepsilon t}\tau^2 e^T(t)F_2^T F_2 e(t) \\
&\quad -e^{\varepsilon(t-\tau)}\tau\int_{t-\tau}^{t} e^T(s)F_2^T F_2 e(s)ds \\
&= e^{\varepsilon t}[\tau^2 e^T(t)F_2^T F_2 e(t) \\
&\quad -e^{-\varepsilon\tau}\tau\int_{t-\tau}^{t} e^T(s)F_2^T F_2 e(s)ds] \\
&\leq e^{\varepsilon t}[\tau^2 e^T(t)F_2^T F_2 e(t) \\
&\quad -e^{-\varepsilon\tau}(\int_{t-\tau}^{t} e(s)ds)^T F_2^T F_2(\int_{t-\tau}^{t} e(s)ds)]. \qquad (9)
\end{aligned}$$

$$\begin{aligned}
&\mathscr{L}V_6 = \sum_{j=1}^{n} m_j\int_0^\infty k_j(\xi)[e^{(t+\xi)\varepsilon}g_j^2(e_j(t)) \\
&-e^{\varepsilon t}g_j^2(e_j(t-\xi))]d\xi \\
&= \sum_{j=1}^{n} m_j\int_0^\infty k_j(\xi)e^{(t+\xi)\varepsilon}g_j^2(e_j(t))d\xi \\
&-\sum_{j=1}^{n} m_j\int_0^\infty k_j(\xi)e^{\varepsilon t}g_j^2(e_j(t-\xi))d\xi \\
&= e^{\varepsilon t}[\sum_{j=1}^{n} m_j g_j^2(e_j(t))\int_0^\infty k_j(\xi)e^{\xi\varepsilon}d\xi \\
&-\sum_{j=1}^{n} m_j\int_0^\infty k_j(\xi)g_j^2(e_j(t-\xi))d\xi] \\
&= e^{\varepsilon t}[\sum_{j=1}^{n} m_j g_j^2(e_j(t))k \\
&-\sum_{j=1}^{n} m_j(\int_0^\infty k_j(\xi)d\xi)(\int_0^\infty k_j(\xi)g_j^2(e_j(t-\xi))d\xi)] \\
&\leq e^{\varepsilon t}[\sum_{j=1}^{n} m_j g_j^2(e_j(t))k \\
&-\sum_{j=1}^{n} m_j(\int_0^\infty k_j(\xi)g_j(e_j(t-\xi))d\xi)^2] \\
&\leq +e^{\varepsilon t}[ke^T(t)LMLe(t) - (\int_{-\infty}^{t} k(t-s)g(e(s))ds)^T \times \\
&M(\int_{-\infty}^{t} k(t-s)g(e(s))ds)] \qquad (10)
\end{aligned}$$

From (4) to (10) we can get

$$\begin{aligned}
\mathscr{L}V \le\ & e^{\varepsilon t}\{e^T(t)[\varepsilon P + P(F_1 - C) + (F_1 - C)^T P \\
& + N_1 + N_3 + kLML + \tau^2 F_2^T F_2 \\
& + \lambda_{max}(P) M_1^T M_1]e(t) + e^T(t)[-2\varepsilon PE \\
& - 2(F_1 - C)^T PE]e(t-h) \\
& + e^T(t)[2PA + LN_4]g(e(t)) \\
& + e^T(t)2PBg(e(t-\tau)) + e^T(t)(2PF_2)\int_{t-\tau}^{t} e(s)ds \\
& + e^T(t)(2PD)\int_{-\infty}^{t} K(t-s)g(e(s))ds \\
& + e^T(t-h)[\varepsilon E^T PE - N_1 e^{-\varepsilon h}]e(t-h) \\
& - 2e^T(t-h)E^T PAg(e(t)) \\
& - e^T(t-h)2E^T PBg(e(t-\tau)) \\
& - e^T(t-h)2E^T PD\int_{-\infty}^{t} K(t-s)g(e(s))ds \\
& - e^T(t-h)2E^T PF_2\int_{t-\tau}^{t} e(s)ds \\
& + e^T(t-\tau)[-N_3 e^{-\varepsilon\tau} \\
& + \lambda_{max}(P) M_2^T M_2]e(t-\tau) \\
& + g^T(e(t))[N_2 - N_4]g(e(t)) \\
& - g^T(e(t-\tau))[N_2 e^{-\varepsilon\tau}]g(e(t-\tau)) \\
& - (\int_{-\infty}^{t} k(t-s)g(e(s))ds)^T \times \\
& M(\int_{-\infty}^{t} k(t-s)g(e(s))ds) \\
& - e^{-\varepsilon\tau}(\int_{t-\tau}^{t} e(s)ds)^T F_2^T F_2(\int_{t-\tau}^{t} e(s)ds)\} \\
=\ & e^{\varepsilon t}\eta^T(t)\Xi\eta(t) < 0 \qquad (11)
\end{aligned}$$

where $\eta^T(t) = [e(t), e(t-h), e(t-\tau), g(e(t)), g(e(t-\tau)), \int_{-\infty}^{t} K(t-s)g(e(s))ds, \int_{t-\tau}^{t} e(s)ds]$. On the other hand, by Itô's formula we have

$$dV(t,e(t)) = \mathscr{L}V(t,e(t))dt + \frac{\partial V(t,e(t))}{\partial e(t)}\sigma(t,e(t),e(t-\tau))d\omega(t). \qquad (12)$$

Integrating both side of equation (12) from 0 to t, we can obtain

$$\begin{aligned}
V(t,e(t)) =& V(0,e(0)) + \int_0^t \mathscr{L}V(s,e(s))ds \\
& + \int_0^t \frac{\partial V(s,e(s))}{\partial e(s)}\sigma(s,e(s),e(s-\tau))d\omega(s). \qquad (13)
\end{aligned}$$

Taking mathematical expectation of the both side of equation (13), we have

$$\mathbb{E}V(t,e(t)) = \mathbb{E}V(0,e(0)) + \int_0^t \mathbb{E}\mathscr{L}V(s,e(s))ds. \qquad (14)$$

In views of inequality(11), we can obtain

$$\begin{aligned}
& \mathbb{E}V(t,e(t)) \le \mathbb{E}V(0,e(0)) \\
& = \mathbb{E}\{[e(0) - Ee(-h)]^T P[e(0) - Ee(-h)] \\
& + \int_{-h}^{0} e^{\varepsilon s} e^T(s) N_1 e(s) ds \\
& + \int_{-\tau}^{0} e^{\varepsilon s} g^T(e(s)) N_2 g(e(s)) ds \\
& + \int_{-\tau}^{0} e^{\varepsilon s} e^T(s) N_3 e(s) ds \\
& + \tau\int_{-\tau}^{0} (s+\tau) e^{\varepsilon s} e^T(s) F_2^T F_2 e(s) ds \\
& + \sum_{j=1}^{n} m_j \int_0^\infty k_j(\xi)\int_{-\xi}^{0} e^{\varepsilon(\xi+s)} g_j^2(e_j(s)) ds d\xi\} \\
& \le \mathbb{E}\{\lambda_{max}(P)(1+\|E\|)^2\|\psi\|_\Delta^2 \\
& + \lambda_{max}(N_1)\|\psi\|_\Delta^2 \int_{-h}^{0} e^{\varepsilon s} ds \\
& + \lambda_{max}(N_2)\|L^2\|\|\psi\|_\Delta^2 \int_{-\tau}^{0} e^{\varepsilon s} ds \\
& + \lambda_{max}(N_3)\|\psi\|_\Delta^2 \int_{-\tau}^{0} e^{\varepsilon s} ds \\
& + \tau^2\|F_2\|^2\|\psi\|_\Delta^2 \int_{-\tau}^{0} e^{\varepsilon s} ds \\
& + \sum_{j=1}^{n} m_j \int_0^\infty k_j(\xi)\int_{-\xi}^{0} e^{\varepsilon(\xi+s)} l_j^2 e_j^2(s) ds d\xi\} \\
& \le \mathbb{E}\{\lambda_{max}(P)(1+\|E\|)^2\|\psi\|_\Delta^2 \\
& + \frac{1}{\varepsilon}[\lambda_{max}(N_1)\|\psi\|_\Delta^2 + \lambda_{max}(N_2)\|L^2\|\|\psi\|_\Delta^2 \\
& + \lambda_{max}(N_3)\|\psi\|_\Delta^2 + \tau^2\|F_2\|^2\|\psi\|_\Delta^2] \\
& + \sum_{j=1}^{n} m_j \int_0^\infty k_j(\xi) e^{\varepsilon\xi} l_j^2 |e_j^2(s)|_\Delta \int_{-\xi}^{0} e^{\varepsilon s} ds d\xi\} \\
& \le \mathbb{E}\{\lambda_{max}(P)(1+\|E\|)^2\|\psi\|_\Delta^2 \\
& + \frac{1}{\varepsilon}[\lambda_{max}(N_1)\|\psi\|_\Delta^2 + \lambda_{max}(N_2)\|L^2\|\|\psi\|_\Delta^2 \\
& + \lambda_{max}(N_3)\|\psi\|_\Delta^2 + \tau^2\|F_2\|^2\|\psi\|_\Delta^2 \\
& + \sum_{j=1}^{n} m_j l_j^2 |e_j^2(s)|_\Delta \int_0^\infty k_j(\xi) e^{\varepsilon\xi} d\xi]\} \\
& \le \{\lambda_{max}(P)(1+\|E\|)^2 + \frac{1}{\varepsilon}[\lambda_{max}(N_1) \\
& + \lambda_{max}(N_2)\|L^2\| + \lambda_{max}(N_3) + \tau^2\|F_2\|^2 \\
& + k\|M\|\|L\|^2]\}\mathbb{E}\|\psi\|_\Delta^2. \qquad (15)
\end{aligned}$$

This means that

$$\lambda_{max}(P)e^{\varepsilon t}\mathbb{E}\|\mathscr{D}(t)\|^2 \le \mathbb{E}V(t,e(t)) \le \alpha'\mathbb{E}\|\psi\|_\Delta^2, \quad (16)$$

where $\alpha' \triangleq \lambda_{max}(P)\alpha = \lambda_{max}(P)(1+\|E\|)^2 + \frac{1}{\varepsilon}[\lambda_{max}(N_1) + \lambda_{max}(N_2)\|L^2\| + \lambda_{max}(N_3) + \tau^2\|F_2\|^2 + k\|M\|\|L\|^2]$. Namely

$$\mathbb{E}\|\mathscr{D}(t)\|^2 \le \alpha\mathbb{E}\|\psi\|_\Delta^2 e^{-\varepsilon t}. \qquad (17)$$

On the other hand, since $\|E\| < 1/2$, we can obtain

$$\begin{aligned}\mathbb{E}\|e(t)\|^2 &\leq 2\mathbb{E}(\|\mathscr{D}(t)\|^2) + 2\|E\|^2\mathbb{E}(\|e(t-h)\|^2) \\ &\leq 2\mathbb{E}(\|\mathscr{D}(t)\|^2) + \|E\|\mathbb{E}(\|e(t-h)\|^2). \quad (18)\end{aligned}$$

In what follows, we'll prove that

$$\mathbb{E}(\|e(t)\|^2) \leq [\frac{2\alpha\mathbb{E}(\|\psi\|^2_\Delta)}{1-\|E\|e^{\beta h}} + \mathbb{E}(\|\psi\|^2_\Delta)]e^{-\beta t}, \quad (19)$$

where β is a positive scalar and satisfies $\beta < \varepsilon, \|E\|e^{\beta h} < 1$.

When $t = 0$, from (18) we can obtain

$$\begin{aligned}\mathbb{E}(\|e(0)\|^2) &\leq 2\mathbb{E}(\|\mathscr{D}(0)\|^2) + \|E\|\mathbb{E}(\|e(-h)\|^2) \\ &\leq 2\|\mathbb{E}(\mathscr{D}(0)\|^2) + \|E\|\mathbb{E}(\|\psi\|^2_\Delta) \\ &\leq \frac{2\alpha\mathbb{E}(\|\psi\|^2_\Delta)}{1-\|E\|e^{\beta h}} + \mathbb{E}(\|\psi\|^2_\Delta), \quad (20)\end{aligned}$$

this means that inequality (19) hold when $t = 0$.

If inequality (19) not hold when $t > 0$, then there exists t^* such that

$$\mathbb{E}(\|e(t^*)\|^2) = [\frac{2\alpha\mathbb{E}(\|\psi\|^2_\Delta)}{1-\|E\|e^{\beta h}} + \mathbb{E}(\|\psi\|^2_\Delta)]e^{-\beta t^*}, \quad (21)$$

and

$$\mathbb{E}(\|e(t)\|^2) < [\frac{2\alpha\mathbb{E}(\|\psi\|^2_\Delta)}{1-\|E\|e^{\beta h}} + \mathbb{E}(\|\psi\|^2_\Delta)]e^{-\beta t}, \forall t \in (0, t^*).$$

Then we can obtain

$$\begin{aligned}\mathbb{E}(\|e(t^*)\|^2) &\leq 2\mathbb{E}(\|\mathscr{D}(t^*)\|^2) + \|E\|\mathbb{E}(\|e(t^*-h)\|^2) \\ &\leq 2\alpha\mathbb{E}(\|\psi\|^2_\Delta)e^{-\varepsilon t^*} \\ &\quad +\|E\|[\frac{2\alpha\mathbb{E}(\|\psi\|^2_\Delta)}{1-\|E\|e^{\beta h}} + \mathbb{E}(\|\psi\|^2_\Delta)]e^{-\beta(t^*-h)} \\ &< 2\alpha\mathbb{E}(\|\psi\|^2_\Delta)e^{-\beta t^*} \\ &\quad +\frac{2\alpha\|E\|\mathbb{E}(\|\psi\|^2_\Delta)}{1-\|E\|e^{\beta h}}e^{-\beta(t^*-h)} \\ &\quad +\|E\|\mathbb{E}(\|\psi\|^2_\Delta)e^{-\beta(t^*-h)} \\ &< [\frac{2\alpha\mathbb{E}(\|\psi\|^2_\Delta)}{1-\|E\|e^{\beta h}} + \mathbb{E}(\|\psi\|^2_\Delta)]e^{-\beta t^*}, \quad (22)\end{aligned}$$

this contradicts with equality (21), which means that for all $t > 0$, we have

$$\mathbb{E}\|e(t)\|^2 \leq [\frac{2\alpha}{1-\|E\|e^{\beta h}} + 1]\mathbb{E}\|\psi\|^2_\Delta e^{-\beta t},$$

which complete the proof.

Corollary 3.1: Suppose that $\|E\| < 1/2$, then, if there exist positive definite matrices N_1, N_2, N_3, positive definite diagonal matrices M, N_4 and positive scalar ε, μ such that

$$\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} & 0 & \Xi_{14} & \mu B & \mu D & \mu F_2 \\ * & \Xi_{22} & 0 & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} & 0 & 0 \\ * & * & * & * & * & -M & 0 \\ * & * & * & * & * & * & \Xi_{77} \end{pmatrix} < 0,$$

where

$$\begin{aligned}&\Xi_{11} = \varepsilon\mu I + \mu(-C + F_1) + \mu(-C + F_1)^T + N_1 \\ &+N_3 + kLML + \tau^2 F_2^T F_2 + \mu M_1^T M_1, \\ &\Xi_{12} = -\mu\varepsilon E - \mu(F_1 - C)^T E, \\ &\Xi_{14} = \mu A + \frac{1}{2}LN_4, \\ &\Xi_{22} = \mu\varepsilon E^T E - N_1 e^{-\varepsilon h}, \\ &\Xi_{24} = -\mu E^T A, \Xi_{25} = -\mu E^T B, \\ &\Xi_{26} = -\mu E^T D, \Xi_{27} = -\mu E^T F_2, \\ &\Xi_{33} = -N_3 e^{-\varepsilon\tau} + \mu M_2^T M_2, \\ &\Xi_{44} = N_2 - N_4, \Xi_{55} = -N_2 e^{-\varepsilon\tau}, \\ &\Xi_{77} = -e^{\varepsilon\tau} F_2^T F_2, M = diag(m_1, m_2, \ldots, m_n).\end{aligned}$$

then the drive system (1) and the response system (2) are mean square exponentially synchronized.

Proof. Let $P = \mu I$. We can obtain Corollary 3.1 directly by Theorem 3.1.

If $E = 0$ in system (1) and (2), then the considered models degenerate to the general stochastic neural networks with mixed delay, in this case, we can obtain the following Corollaries.

Corollary 3.2: If there exist positive definite matrices P, N_1, N_2, N_3, positive definite diagonal matrices M, N_4 and positive scalar ε such that

$$\Xi = \begin{pmatrix} \Xi_{11} & 0 & 0 & \Xi_{14} & PB & PD & PF_2 \\ * & \Xi_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 \\ * & * & * & * & -N_2 e^{-\varepsilon\tau} & 0 & 0 \\ * & * & * & * & * & -M & 0 \\ * & * & * & * & * & * & \Xi_{77} \end{pmatrix} < 0,$$

where

$$\begin{aligned}&\Xi_{11} = \varepsilon P + P(-C + F_1) + (-C + F_1)^T P + N_1 + N_3 \\ &+kLML + \tau^2 F_2^T F_2 + \lambda_{max}(P) M_1^T M_1, \\ &\Xi_{14} = PA + \frac{1}{2}LN_4, \Xi_{22} = -N_1 e^{-\varepsilon h}, \\ &\Xi_{33} = -N_3 e^{-\varepsilon\tau} + \lambda_{max}(P) M_2^T M_2, \Xi_{44} = N_2 - N_4, \\ &\Xi_{77} = -e^{\varepsilon\tau} F_2^T F_2, M = diag(m_1, m_2, \ldots, m_n).\end{aligned}$$

then the drive system (1) and the response system (2) are mean square exponentially synchronized.

Corollary 3.3: If there exist positive definite matrices N_1, N_2, N_3, positive definite diagonal matrices M, N_4 and positive scalar ε, μ such that

$$\Xi = \begin{pmatrix} \Xi_{11} & 0 & 0 & \Xi_{14} & \mu B & \mu D & \mu F_2 \\ * & \Xi_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55} = & 0 & 0 \\ * & * & * & * & * & -M & 0 \\ * & * & * & * & * & * & \Xi_{77} \end{pmatrix} < 0,$$

where

$$\begin{aligned}&\Xi_{11} = \varepsilon\mu I + \mu(-C + F_1) + \mu(-C + F_1)^T \\ &+N_1 + N_3 + kLML + \tau^2 F_2^T F_2 + \mu M_1^T M_1,\end{aligned}$$

$$\Xi_{14} = \mu A + \frac{1}{2} L N_4, \Xi_{22} = -N_1 e^{-\varepsilon h},$$

$$\Xi_{33} = -N_3 e^{-\varepsilon \tau} + \mu M_2^T M_2,$$

$$\Xi_{44} = N_2 - N_4, \Xi_{55} = -N_2 e^{-\varepsilon \tau},$$

$$\Xi_{77} = -e^{\varepsilon \tau} F_2^T F_2, M = diag(m_1, m_2, \ldots, m_n).$$

then the drive system (1) and the response system (2) are mean square exponentially synchronized.

IV. Numerical Example

In this section, we shall present one numerical example to illustrate the validity of our results.

Example. consider the following neutral type delayed neural networks

$$\begin{cases} d[x(t) - Ex(t-h)] = [-Cx(t) + Bf(x(t-\tau)) \\ + Af(x(t)) + D\int_{-\infty}^{t} K(t-s)f(x(s))ds + I']dt, \\ x(\theta) = \phi(\theta), \qquad \theta \in (-\infty, 0], \end{cases} \quad (23)$$

where

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, A = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{pmatrix},$$

$$B = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{pmatrix}, D = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix},$$

$$E = \begin{pmatrix} 0.1 & 0.01 \\ 0.01 & 0.1 \end{pmatrix},$$

$$f(x(t)) = tanh(x(t)), \tau = h = 1, K(t) = diag(te^{-t}, te^{-t}).$$

In order to synchronize system (23) via the feedback controller, we introduce the respond system from the unidirectional linear coupling approach as follows:

$$\begin{cases} d[y(t) - Ey(t-h)] = [-Cy(t) + Bf(y(t-\tau)) \\ + Af(y(t)) + D\int_{-\infty}^{t} K(t-s)f(y(s))ds + I' \\ + u(t)]dt + \sigma(t, e(t), e(t-\tau))dt\omega(t), t > 0, \\ y(\theta) = \varphi(\theta), \qquad \theta \in (-\infty, 0], \end{cases} \quad (24)$$

where

$$\sigma(t, e(t), e(t-\tau)) = \begin{pmatrix} \sqrt{0.2}e_1(t) & \sqrt{0.3}e_2(t-\tau) \\ \sqrt{0.3}e_1(t-\tau) & \sqrt{0.2}e_2(t) \end{pmatrix}.$$

Then we have $L = I, M_1^T M_1 = 0.2I, M_2^T M_2 = 0.3I, k \simeq 1.4286$.

By system (23) and (24) we can obtain the error system as follows

$$\begin{cases} d[e(t) - Ee(t-h)] = [(F_1 - C)e(t) + Ag(e(t) \\ + Bg(e(t-\tau)) + D\int_{-\infty}^{t} K(t-s)g(e(s))ds \\ + F_2\int_{t-\tau}^{t} e(s)ds]dt + \sigma(t, e(t), e(t-\tau))dt\omega(t)), \\ e(\theta) = \psi(\theta), \qquad \theta \in (-\infty, 0], \end{cases} \quad (25)$$

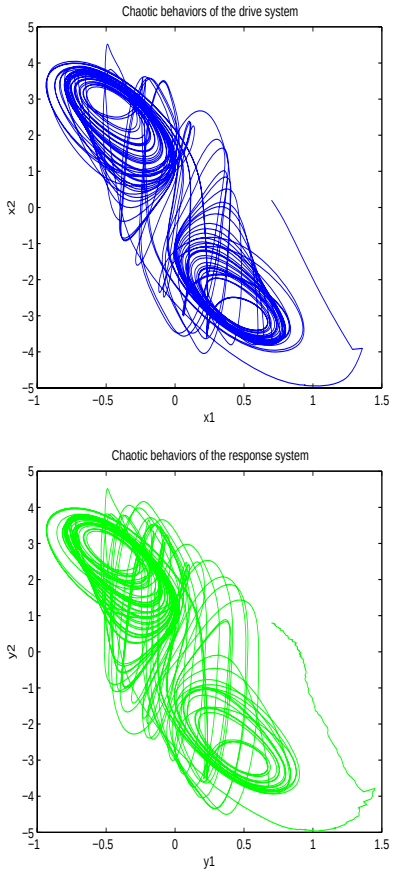

Fig. 1. Phase trajectories and state trajectories of drive and response system

where F_1, F_2 are the gain matrix which need to be estimated in the feedback controller $u(t) = F_1 e(t) + F_2 \int_{t-\tau}^{t} e(s)ds$. Set $\varepsilon = 0.3, \mu = 0.3$, from Corollary 3.1 and using LMI toolbox in Matlab, we can obtain the following feasible solution:

$$F_1 = \begin{pmatrix} -83.4826 & 6.4319 \\ 1.1299 & -89.2625 \end{pmatrix}, M = \begin{pmatrix} 2.0000 & 0 \\ 0 & 2.0000 \end{pmatrix}.,$$

$$N_1 = \begin{pmatrix} 16.1562 & 0.1353 \\ 0.1353 & 15.9428 \end{pmatrix}, N_2 = \begin{pmatrix} 0.4498 & 0.0350 \\ 0.0350 & 0.4870 \end{pmatrix},$$

$$N_3 = \begin{pmatrix} 16.0922 & -0.0675 \\ -0.0675 & 16.1525 \end{pmatrix}, N_4 = \begin{pmatrix} 1.0000 & 0 \\ 0 & 1.0000 \end{pmatrix},$$

$$F_2 = \begin{pmatrix} 6.5944 & -0.0622 \\ 0.0126 & 6.6165 \end{pmatrix}, M = \begin{pmatrix} 2.0000 & 0 \\ 0 & 2.0000 \end{pmatrix}.$$

V. Conclusions

In this paper, by constructing appropriate Lyapunov functional, we have derived some sufficient conditions to guaranteeing the mean square exponential synchronization of two identical chaotic neural networks. And the feedback controller are designed by LMI toolbox in MATLAB. One simulation numerical example shows that our results are valid.

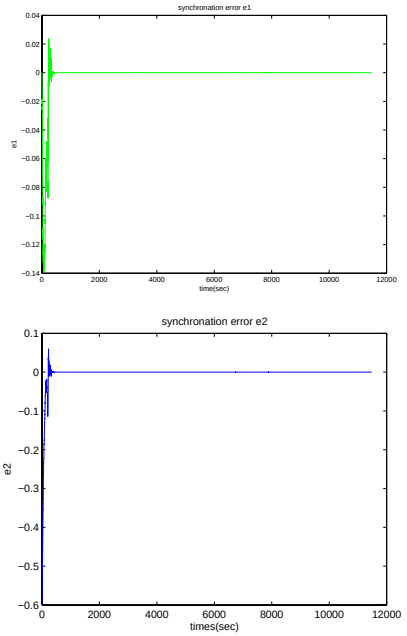

Fig. 2. State trajectories of error system

ACKNOWLEDGMENT

This work was supported by Science and technology Foundation of Guizhou Province of China (Grant No. [2010]2139).

REFERENCES

[1] T.L. Carroll, L.M. Pecora, Synchronization in chaotic systems, Phys. Rev. Lett 64 (1990) 821-824.
[2] T.L. Carroll, L.M. Pecora, Synchronizing chaotic circuits, IEEE Trans. Circ. Syst 38 (1991) 453-456.
[3] T. Li, S.M. Fei, K.J. Zhang, Synchronization control of recurrent neural networks with distributed delays, Physica A 387 (2008) 982-996.
[4] T. Liu, G. M. Dimirovskib, J. Zhao, Exponential synchronization of complex delayed dynamical networks with general topology, Physics Letters A 387 (2008) 643-652.
[5] X. Lou, B. Cui, Synchronization of neural networks based on parameter identification and via output or state coupling, Journal of Computational and Applied Mathematics (2007), doi:10.1016/j.cam.2007.11.015.
[6] Sheng L et al., Adaptive exponential synchronization of delayed ..., Chaos, Solitons, Fractals (2007), doi:10.1016/j.chaos.2007.08.047.
[7] Tao Li, Shu-Min Fei, Qing Zhu and Shen Cong, Exponential synchronization of chaotic neural networks with mixed delays, Neurocomputing (2008), doi:10.1016/j.neucom.2007.12.029.
[8] J. Yan, J. Lin, M. Hung, T. Liao, On the synchronization of neural networks containing time-varying delays and sector nonlinearity, Phys. Lett. A 361 (2007) 70-77.
[9] J.J. Yan, J.S. Lin,etc, On the synchronization of neural networks containing time-varying delays and sector nonlinearity, Physics Letters A 361 (2007) 70-77.
[10] Y.Q. Yang, J.D. Cao, Exponential lag synchronization of a class of chaotic delayed neural networks with impulsive effects, Physica A 386 (2007) 492-502.
[11] W. Yu, J. Cao, Adaptive synchronization and lag synchronization of uncertain dynamical system with time delay based on parameter identification, Phys. A 375 (2007) 467-482.
[12] Q. J, Zhang, J.A. Lu, Chaos synchronization of a new chaotic system via nonlinear control, Chaos, Solitons and Fractals 37 (2008) 175-179.
[13] H.G. Zhang, Y.H. Xie, Z.L. Wang, Adaptive synchronization between two different chaotic neural networks with time delay, IEEE Transaction on Neural Networks 18(2007) 1841-1845.