International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:7, No:2, 2013

Conjugate Heat Transfer in an Enclosure Containing a Polygon Object

Habibis Saleh and Ishak Hashim

Abstract—Conjugate natural convection in a differentially heated square enclosure containing a polygon shaped object is studied numerically in this article. The effect of various polygon types on the fluid flow and thermal performance of the enclosure is addressed for different thermal conductivities. The governing equations are modeled and solved numerically using the built-in finite element method of COMSOL software. It is found that the heat transfer rate remains stable by varying the polygon types.

Keywords—Natural convection, Polygon object, COMSOL

I. Introduction

NATURAL convection heat transfer in a differentially heated enclosures from side or below has received considerable attention over the past few decades, largely due to a wide variety of applications, which include double pane windows, electronic boxes, solar collector technology, energy storage, nuclear reactor technology, etc. Another practical application of natural convection is encountered when an obstacle such as an inserted object placed inside the enclosure. Many authors reported that this inserted object changes the flow field and the heat transfer characteristics of the enclosure.

Ref. [1] reported the heat transfer may be enhanced or reduced by a square object with a thermal conductivity ratio less or greater than unity. Ref. [2] found placing the solid objects near to the walls reduce the rate of heat transfer due to the blockage effects, but placing low conductor objects far from the boundary layer region may enhance the rate of heat transfer compared with enclosures without obstacles. Ref. [3] reported a critical size of the adiabatic object below which the increasing the size increases the heat transfer and above which the increasing the size decreases the heat transfer. Ref. [4] studied when the enclosure were given an inclination angle. Ref. [5] moved to an inserted cylinder object in the center of the enclosure. They concluded that the thermophysical properties of the cylinder object were important on the overall heat transfer process across the enclosure.

Present work aims to investigate the fluid flow and heat transfer for various polygon types placed inside the center of the square enclosure. Complete two dimensional numerical simulation and systematical generalization of the conjugate heat transfer behavior occurring in the enclosure by varying the obstacles shaped is carried out.

II. Mathematical Formulation

A schematic diagram of a square enclosure having a conductive regular polygon placed at the center is shown in in

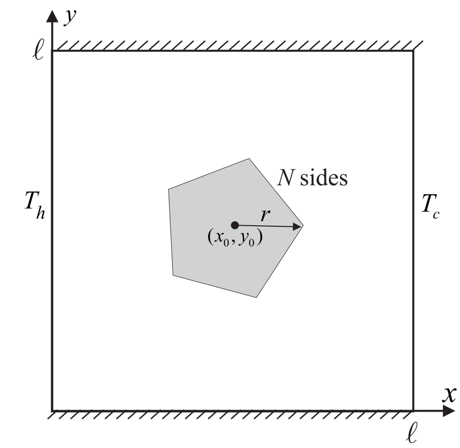


Fig. 1. Schematic representation of the model

Fig. 1. The left surface is heated to a constant temperature T_h, and the right surface of the enclosure is cooled to a constant temperature T_c, while the horizontal surfaces are kept adiabatic. Thermophysical properties of the fluid in the flow field are assumed to be constant except the density variations causing a body force term in the momentum equation. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changes, and to couple in this way the temperature field to the flow field. Under the above assumptions, the governing equations for steady natural convection flow using conservation of mass, momentum and energy can be written in its dimensionless form as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y} = 0 \qquad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) +Ra\,Pr\,\Theta_f \qquad (3)$$

$$U\frac{\partial \Theta_f}{\partial X}+V\frac{\partial \Theta_f}{\partial Y} = \left(\frac{\partial^2 \Theta_f}{\partial X^2}+\frac{\partial^2 \Theta_f}{\partial Y^2}\right) \qquad (4)$$

and the energy equation for the conducting polygon is:

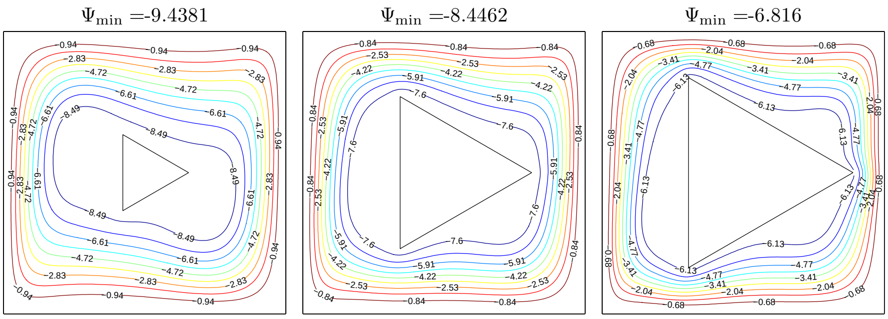
$$\frac{\partial^2 \Theta_s}{\partial X^2}+\frac{\partial^2 \Theta_s}{\partial Y^2} = 0 \qquad (5)$$

The values of the non–dimensional velocity are zero in the solid region and on the solid-fluid interfaces. The boundary

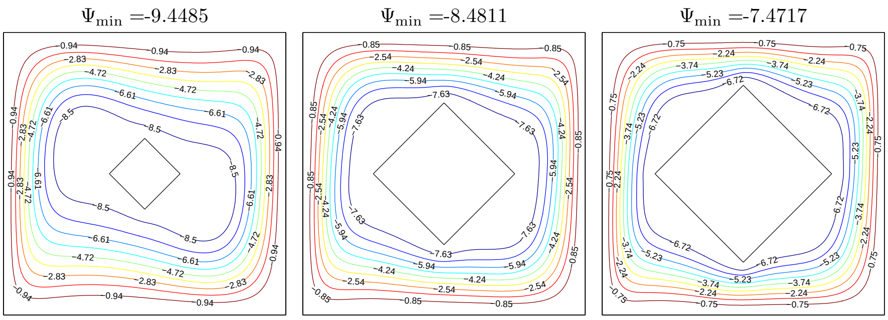
H. Saleh and I. Hashim are with School of Mathematical Sciences, Universiti Kebangsaan Malaysia, 43600 Bangi Selangor, Malaysia, e-mail: Dr.habibissaleh@gmail.com and ishak_h@ukm.my.

Manuscript received January , 2013; revised January , 2013.

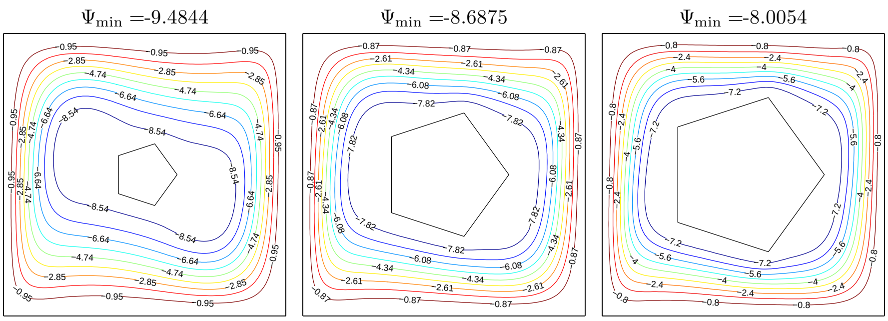
International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:7, No:2, 2013



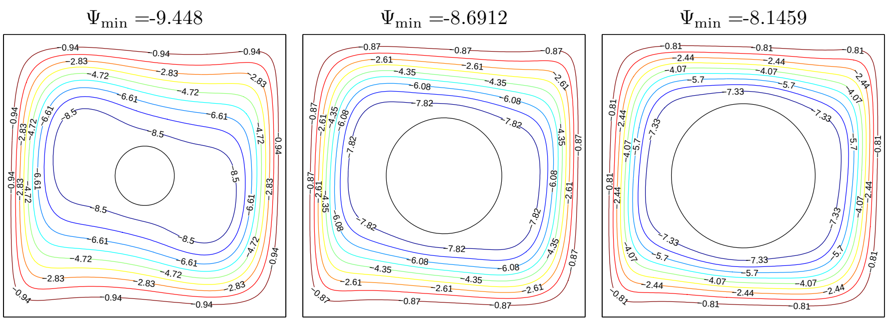
(a) $N = 3$



(b) $N = 4$



(c) $N = 5$



(d) $N = \infty$

Fig. 2. Streamlines for different polygon size, A and number of polygon sides, N at $K_r = 1$ and $Ra = 10^5$.

International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:7, No:2, 2013

conditions for the non-dimensional temperatures are:

$$\Theta = 1 \text{ at } X = 0, \quad \Theta = 0 \text{ at } X = 1 \tag{6}$$

$$\frac{\partial \Theta}{\partial Y} = 0 \text{ at } Y = 0 \text{ and } Y = 1 \tag{7}$$

$$\Theta = \Theta_s \text{ at the outer polygon surface} \tag{8}$$

$$\frac{\partial \Theta}{\partial \eta} = K_r \frac{\partial \Theta_s}{\partial \eta} \text{ at the inner polygon surface} \tag{9}$$

where $K_r = k_s/k_f$ is the thermal conductivity ratio. All sides of the solid polygon are equal in length. Number of sides is denoted by N, where for $N = 3$ the shape is triangle, for $N = 4$ the shape is quadrilateral, for $N = 5$ the shape is pentagon, for $N = 6$ the shape is hexagon, for $N = 7$ the shape is heptagon, for $N = 8$ the shape is hexagon, for $N = 9$ the shape is nonagon, for $N = 10$ the shape is decagon, for $N = 11$ the shape is hendecagon and for $N = \infty$ the shape becomes cylinder. The polygon to enclosure area ratio is defined by:

$$A = R^2 N \frac{\sin(\frac{360}{N})}{2} \tag{10}$$

The physical quantities of interest in this problem are the average Nusselt number along the hot wall which is defined by:

$$\overline{Nu} = \int_0^1 \frac{-\partial \Theta_f}{\partial Y} \mathrm{d}Y \tag{11}$$

III. Computational Methodology

The governing equations along with the boundary conditions are solved numerically by the CFD software package COMSOL Multiphysics. COMSOL Multiphysics (formerly FEMLAB) is a finite element analysis, solver and simulation software package for various physics and engineering applications. We consider the following application modes in COMSOL Multiphysics. The Incompressible Navier-stokes Equations mode (ns) for Eqs. (1)–(3), the Convection–Conduction Equations mode (cc) for Eq. (4) and the Diffusion Equations mode (di) for Eq. (5). In this study, mesh generation on square enclosure containing polygon object is made by using triangles. Several grid sensitivity tests were conducted to determine the sufficiency of the mesh scheme and to ensure that the results are grid independent. We use a finer mesh sizes for all the computations done in this paper.

IV. Results and Discussion

Fig. 2 illustrates the streamlines for various types of solid polygon where the solid area attains the value $\pi/100$, $\pi/25$ and $\pi/16$, respectively. The thermal conductivity ratio is fixed at $K_r = 1$ and the Rayleigh number at $Ra = 10^5$. The fluid temperature adjoining the hot surface rises and move from the left to the right, falling along the cold surface, then rising again at the hot surface. This movement creates a clockwise circulation cell in free space between the polygon and walls enclosure. The cell shape near the polygon was distorted by the presence polygon, $N \leq 5$. The distortion takes higher as the polygon size is made bigger. The strength of the flow circulation decreases by increasing the solid area for the same polygon type. It is obvious that increasing the A leads to smaller space for the flow to circulate. At fixed A, increasing N increases the flow strength, as indicated from $|\Psi|_{\min}$ values. This increasing occurs except at small cylinder, $A = \pi/100$, where the flow strength much weaker than the small solid pentagon inserted in enclosure.

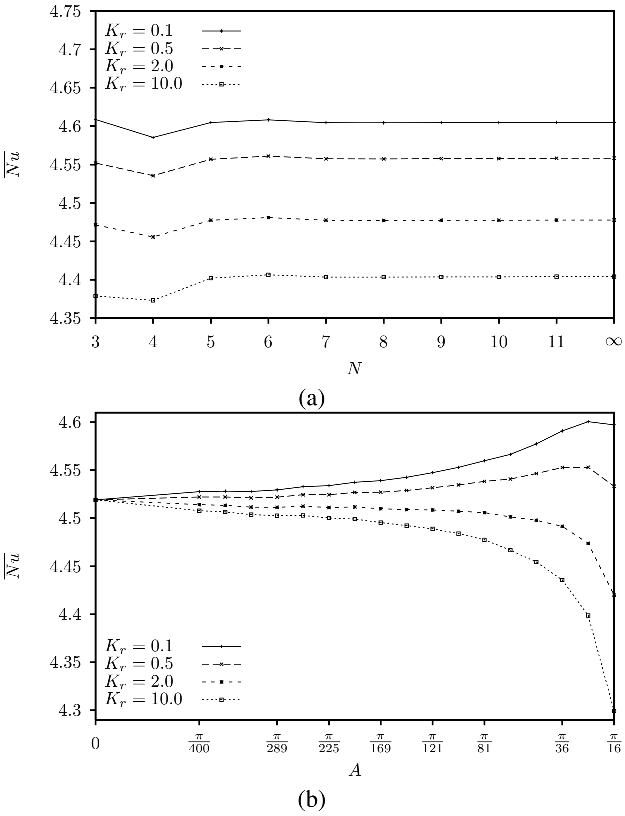


Fig. 3. Variation of $\overline{Nu}$ with N (a), with A (b) for different values of K_r.

Variations of the average Nusselt number with the number of polygon sides are shown in Fig. 3(a) for different thermal conductivity ratio at $Ra = 10^5$ and $A = \pi/25$. The average Nusselt number was shown stable for solid pentagon and above. Increasing thermal conductivity ratio increases the $\overline{Nu}$ at fixed N. Variations of the average Nusselt number with the polygon areas are shown in Fig. 3(b) for different thermal conductivity ratio at $Ra = 10^5$ and $N = 5$. Fig. 3(b) exhibits a critical size of the solid pentagon is exist at low conductivities, $K_r = 0.1$, 0.5; below which, the increasing the size increases the $\overline{Nu}$ and above which the increasing the size decreases the $\overline{Nu}$.

V. Conclusion

The present numerical simulations study the effects various solid polygon properties on natural convection inside a square enclosure. The dimensionless forms of the governing equations were solved using the COMSOL Multiphysics software. Detailed computational results for fluid flow and heat transfer characteristics in the enclosure have been presented in graphical forms. We conclude that the heat transfer rate remains stable by varying the number of polygon sides. The theoretical prediction in this paper is hoped to be a useful guide for the experimentalists to study the various combinations of the polygon shaped and its thermal conductivity properties to control the fluid flow and thermal performance of enclosure at different size. The factors of polygon location, orientation and rotation with different angular speed will be the focus of our research undertaking.

References

[1] J. House, C. Beckermann, and T. Smith, "Effect of a centered conducting body on natural convection heat transfer in an enclosure," *Numer. Heat Transf. Part A*, vol. 18, pp. 213–225, 1990.

[2] A. A. Merrikh and A. A. Mohamad, "Blockage effects in natural convection in differentially heated enclosures," *J. of Enhanced Heat Transfer*, vol. 8, pp. 55–72, 2001.

[3] P. Bhave, A. Narasimhan, and D. A. S. Rees, "Natural convection heat transfer enhancement using adiabatic block: Optimal block size and Prandtl number effect," *Int. J. Heat Mass Transf.*, vol. 49, pp. 3807–3818, 2006.

[4] M. Das and K. Reddy, "Conjugate natural convection heat transfer in an inclined square cavity containing a conducting block," *Int. J. Heat Mass Transf.*, vol. 49, pp. 4987–5000, 2006.

[5] V. Costa and A. Raimundo, "Steady mixed convection in a differentially heated square enclosure with an active rotating circular cylinder," *Int. J. Heat Mass Transf.*, vol. 53, pp. 1208–1219, 2010.