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Thermal Carpet Cloaking Achieved by Layered Metamaterial

Bang-Shiuh Chen and Lien-Wen Chen

Abstract**—**We have devised a thermal carpet cloak theoretically and implemented in silicon using layered metamaterial. The layered metamaterial is composed of single crystalline silicon and its phononic crystal. The design is based on a coordinate transformation. We demonstrate the result with numerical simulation. Great cloaking performance is achieved as a thermal insulator is well hidden under the thermal carpet cloak. We also show that the thermal carpet cloak can even the temperature on irregular surface. Using thermal carpet cloak to manipulate the heat conduction is effective because of its low complexity.

Keywords**—**Metamaterial, heat conduction, cloaking, phononic crystal.

I. Introduction

TRANSFORMATION optics proposed by Pendry [1] and Leonhordt et al. [2] in 2006, is a powerful design tool to manipulate electromagnetic waves within homogeneous and anisotropic medium. It had been applied to many state-of-the-art devices such as the concentrator [3], the rotator [4], and the invisible cloak [1], etc. The invisible cloak is a remarkable invention. The electromagnetic wave is smoothly detoured around by the cloak, and travels in the same direction as passing through the empty volume of space. Pendry [1] proposed a circular electromagnetic cloak with the further heterogeneous and anisotropic materials because of the nature of the coordinate transformation in a cylindrical coordinate system.

Ground cloaking strategy was introduced by Li et al. [5] in 2008. They investigated an EM wave cloak to mimic a flat ground plane. The object is hiding under a carpet cloak, and then it is indistinguishable from the surface. Using the quasiconformal mapping, they generated an EM cloak with isotropic material parameters without extreme values. However, the cloak occupies a large space and the material is heterogeneous. Another type of ground cloaks have been developed based on the linear transformation for the acoustic cloaking [6], [7]. Unlike the quasiconformal mapping, it produced a cloak with anisotropic material parameters. Nevertheless, the material is homogeneous and can be implemented in a compact way. These two methods can simplify the transformed material parameters and let the cloaking realizable. The ground cloaking strategy is found to be a feasible way to realize cloaking.

Recently, the idea of Transformation optics has been extended to acoustics [8], quantum mechanics [9], elastodynamics [10], and fluid dynamics [11]. In thermal conduction, the heat diffusion equation has been verified the invariance under curvilinear transformations [12]. Guenneau et al. [12] used mutilayers of homogenous materials to realize the circular thermal cloak. The temperature within the cloak is smoothly detoured around the invisibility area, and the field outside the cloak stays undisturbed. However, the conductivity of the thermal cloak varies over a large range of values, and the heat resistance of the layers has to be considered. One should select the material with the appropriate conductivity.

Phonon is the main heat carrier in the nonmetal material. The thermal conductivity can be manipulated by altering the phonon propagation. Hopkins et al. [13] employed Callaway-Holland model [14], [15] to calculate the effective conductivity of microporous solids, and the Bolzmann Transport equation was used to estimate the diffusion of phonons. The conductivity is well closed to the experimental results [16].

However, the Callaway-Holland model cannot have good estimation of thermal conductivity as the lattice constant of periodic structures is in nanometer-scale. Recent studies have revealed the reason [17], [18]. The periodic structure (phononic crystal) alters the phononic spectrum [19], which plays an important role in determining the thermal conductivity. Using the phononic crystal provides a way to manipulate the thermal conductivity.

In this study, we devise a ground cloak for thermodynamics by using the linear transform method. We select layered material composed of single crystalline silicon and its phononic crystal structure for implementing the thermal cloak. The cloaking effect is simulated by Finite Element Method. This work provides a pragmatic way to achieve the thermodynamics cloak.

II. Design Method

A. The Transformation of Heat Conduction Equation

The heat conduction equation without source term can be written as

$$\rho(x)C_p(x)\frac{\partial T}{\partial t} - \nabla(\kappa(x)\nabla T) = 0 \qquad (1)$$

B. S. Chen is with the Department of Mechanical Engineering, National Cheng Kung University, Tainan 701, Taiwan (e-mail: lovejben@gmail.com).

L. W. Chen is with the Department of Mechanical Engineering, National Cheng Kung University, Tainan 701, Taiwan (e-mail: chenlw@mail.ncku.edu.tw).

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where ρ is the density, C_p is the heat capacity, T is the temperature, and κ is the thermal conductivity. Upon the coordinate transformation $\mathbf{x} = (x, y) \rightarrow \mathbf{x}' = (x', y')$, the new equation in the system we are interested takes the form [12]:

$$\rho(\mathbf{x}')C_p(\mathbf{x}')/\det(\Lambda)\frac{\partial T}{\partial t} - \nabla(\Lambda\kappa(\mathbf{x}')\Lambda^T/\det(\Lambda)\nabla T) = 0 \quad (2)$$
$$\Lambda_i^{i'} = \frac{\partial x_{i'}}{\partial x^i}$$
$$\rho'(x')C_p'(x') = \rho(x')C_p(x')/\det(\Lambda)$$
$$\kappa'(x') = \Lambda\kappa(x')\Lambda^T/\det(\Lambda)$$

We note that the heat conduction equation is invariant to transformation; the physical phenomenon remains the same [12]. We mention that it is not necessary to transform the density term for steady state ($\frac{\partial T}{\partial t} = 0$).

B. Thermal Carpet Cloak

Here, we consider 2D heat conduction in solid. We design a triangular carpet cloak, which has already been used in acoustics cloaking [6], [7]. Fig. 1 illustrates the installation and the coordinate transformation. The coordinate transforms from Cartesian to the triangular-carpet-cloak coordinate. Mathematically, the equation of transformation is defined by [6], [7].

$$x' = x,\ y' = \frac{c-a}{c}y + \frac{b - x\,\mathrm{sgn}(x)}{b}a \quad (3)$$

$$\kappa' = \kappa_0\begin{pmatrix} \frac{c}{c-a} & -\frac{ac}{(c-a)b}\mathrm{sgn}(x) \\ -\frac{ac}{(c-a)b}\mathrm{sgn}(x) & \frac{c-a}{c} + \frac{c}{c-a}\left(\frac{a}{b}\right)^2 \end{pmatrix} \quad (4)$$

$$\rho' C_p' = \frac{c}{c-a}(\rho C_p)_0 \quad (5)$$

where a, b and c are the geometrical parameter. The matrix of heat conduction in (4) can be diagonalized by a proper rotation. By calculating the eigenvalues of the matrix, the two non-zero components are obtained.

$$\kappa'_{pr} = \kappa_0\begin{pmatrix} F + \sqrt{F^2-1} & 0 \\ 0 & F - \sqrt{F^2-1} \end{pmatrix} \quad (6)$$

where

$$F = \frac{b^2c^2 + (c-a)^2b^2 + a^2c^2}{2(c-a)b^2c} \quad (7)$$

and the angle of rotation is

$$\alpha = \mathrm{sgn}(x)\left\{\arctan\left[\frac{bc - b(c-a)\left(F+\sqrt{F^2-1}\right)}{ac}\right] + \frac{\pi}{2}\right\} \quad (8)$$

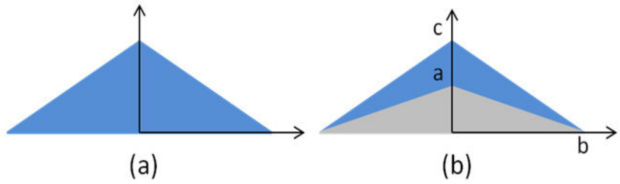

Fig. 1 System of Coordinate transformation (a) the original space (b) the transformed space

Enlightening by the Maxwell-Garnett effective medium theory, we treat a large number of layers with piecewise constant thermal conductivity as an anisotropic material; it has already been verified and simulated [12]. Consider a periodic layered system formed by two homogenous materials with heat conductivity κ_a and κ_b, respectively.

$$\kappa_x = \frac{\kappa_a + \kappa_b}{1+\eta}, \frac{1}{\kappa_z} = \frac{1}{1+\eta}\left(\frac{1}{\kappa_a} + \frac{\eta}{\kappa_b}\right) \quad (9)$$

where η is the thickness ratio of the layers($\eta = r_b/r_a$).

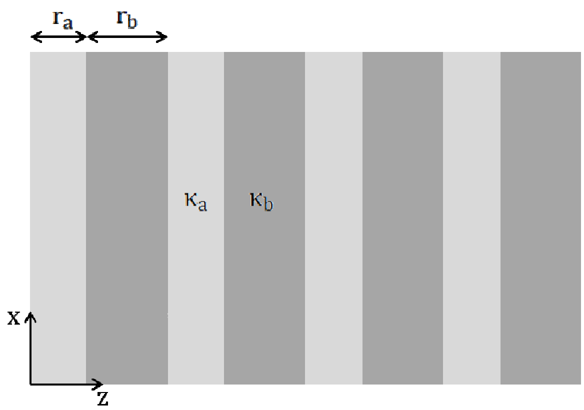

Fig. 2 The mutilayered metamaterial

Equations (6)-(9) are design equations for homogeneous thermal carpet cloak. Notice that the layered system composed of two homogeneous materials is aligned with the angle of rotation.

III. Example of Clock Implementation

Next we select a phononic crystal structure composed of silicon (host material) and air (phonon scatterer) for the thermal cloak implementation. According to the experiment data [17], the thermal conductivity of single crystalline silicon wafer with a thickness of 500nm is $39.2 \pm 4.8\ \mathrm{W\,m^{-1}K^{-1}}$. And the phononic crystal we select is arranged in a simple cubic lattice with air holes of diameter 300nmand lattice constant 600nm on 500nm thick wafer. The thermal conductivity of the phononic crystal has been measured $4.81 \pm 1.0\mathrm{W\,m^{-1}K^{-1}}$[17].

Fig. 3 illustrates the thermal cloaking implementation. By calculating the parameters through (6)-(9) with $\eta = 1$, we get one possible solution $a:b:c = 1:3.3:3.7$, and $\alpha = 60.9^\circ$.

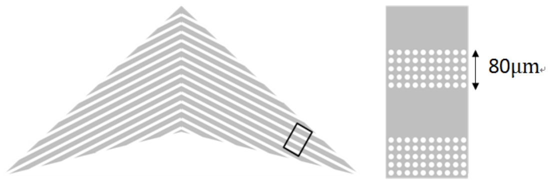

Fig. 3 The implementation of the cloak

IV. Result and Discussion

Fig. 3 illustrates the temperature distribution of the thermal conductivity for the cloaking implementation. We consider a triangular thermal insulator in the bottom of the simulated area. The result was obtained in numerical simulations performed with the heat transfer in solid application mode of Comsol Multiphysics.

In Figs. 3 (a)-(c), the left boundary of the material is subjected to a constant temperature 320℃, and a convective cooling boundary condition on the right boundary with $h = 100\ \mathrm{W/m^2 \cdot {}^\circ C}$, external temperature 300℃. The upper and lower boundaries are thermal insulation. One could see that the stream line of heat conduction flux is bent in the original case in Fig. 3 (a). Therefore, the temperature on the right boundary is no uniform.

Fig. 3 (b) shows that the stream lines of heat conduction are all horizontal outside the cloak after implemented the theoretical cloak, and the temperature of the right boundary is uniform. The thermal cloak alters the path of the heat flux, and the stream line is no longer disturbed by the triangular insulator.

Fig. 3 (c) illustrates the temperature distribution with implemented the layered metamaterial of the ideal cloak. The metamaterial has 24 layers (12 layers of original silicon wafer and 12 layers of silicon wafer of phononic crystal pattern). The result is well closed to the ideal case.

We quantify the cloak performance through its far-field temperature distribution. Fig. 4 shows the temperature gradient in y direction on the right boundary in Figs. 3 (a)-(c). The temperature gradient of non-cloaked case is much higher than the cloaked cases, and the temperature gradient of the layered cloak is slightly above zero. The layered metamaterial fulfills the thermal cloaking.

In Fig. 3 (d), we consider a circular source. The inner circular boundary is subjected to a constant temperature 320℃, and the outer circular boundary is subjected to the convective cooling as same condition of Figs. 3(a)-(c). The temperature field is also distorted by the triangular insulator. Hence, the temperature field does not remain uniform on the outer boundary.

Another application of the thermal ground cloak is demonstrated in Fig. 5. We subject a constant temperature 320℃ on upper boundary, and apply the convective cooling condition on lower boundary with $h = 100\ \mathrm{W/m^2 \cdot {}^\circ C}$, external temperature 300℃. In thissituation, the temperature distribution on the lower boundary is uniform despite the triangular concave. The even temperature distribution on surface could benefit some manufacturing process like glazing, because of the lack of thermal stress.

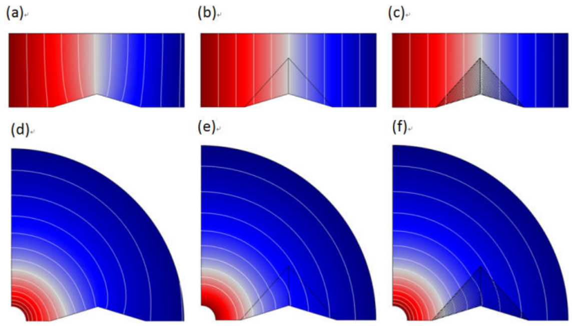

Fig. 3 Simulated temperature field (the difference of temperature between two adjacent isothermal line is 1℃): (a) diffusion of heat from the left boundary with obstruction of a triangular heat insulator; (b) Temperature distribution around theoretical cloak; (c) Temperature distribution around the metamaterial implementation of the ideal cloak; (d) Diffusion of heat from the center with obstruction of a triangular heat insulator; (e) Same as (b); and (f) same as (c)

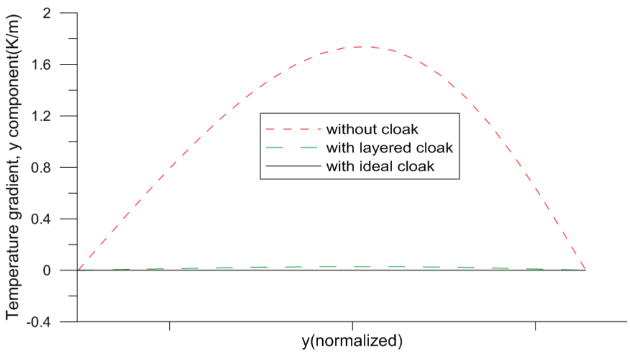

Fig. 4 Temperature gradient on the right boundary

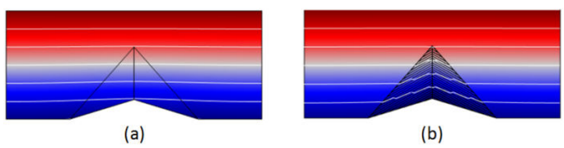

Fig. 5 Special case in ground cloaking (the difference of temperature between two adjacent isothermal line is 1℃) (a) temperature distribution of non-cloaking case (b) temperature distribution of the layered metamaterial implementation

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Bang-Shiuh Chen received his BS degree in mechanical engineering from National Cheng Kung University, Taiwan, in 2011. He is currently an NSC student and a research assistant ofPhotonic Technology Lab in National Cheng Kung University.

Lien-Wen Chen received his BS degree in mechanical engineering from National Cheng Kung University, Taiwan, in 1972, his MS degree in engineering mechanics from the University of Kentucky in 1977, and his PhD degree in mechanical engineering from Rensselaer Polytechnic Institute, in 1980. Since 1984, he has been a professor with the Department of Mechanical Engineering, National Cheng Kung University, Taiwan. His research activities include mechanical vibrations, mechanics of composite materials, smart material structures, photonic and phononic technology. He has authored and coauthored over 200 scientific publications.