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Reliability Analysis of Underground Pipelines Using Subset Simulation

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***Abstract*—**An advanced Monte Carlo simulation method, called Subset Simulation (SS) for the time-dependent reliability prediction for underground pipelines has been presented in this paper. The SS can provide better resolution for low failure probability level with efficient investigating of rare failure events which are commonly encountered in pipeline engineering applications. In SS method, random samples leading to progressive failure are generated efficiently and used for computing probabilistic performance by statistical variables. SS gains its efficiency as small probability event as a product of a sequence of intermediate events with larger conditional probabilities. The efficiency of SS has been demonstrated by numerical studies and attention in this work is devoted to scrutinise the robustness of the SS application in pipe reliability assessment. It is hoped that the development work can promote the use of SS tools for uncertainty propagation in the decision-making process of underground pipelines network reliability prediction.

***Keywords*—**Underground pipelines, Probability of failure, Reliability and Subset Simulation.

I. Introduction

PIPE structural reliability algorithms have been received greater attention recently, though predictions of small failure probabilities techniques are very few till now. In recent years, attention has been focused on reliability problems with complex system characteristics in high dimensions (i.e., with a large number of uncertain or random variables) [1]. Prediction of small failure probabilities is one of the most important and challenging computational problems in reliability engineering [2]. The probabilistic assessment of the engineering system performance may involve a significant number of uncertainties in system behaviour. To implement probabilistic assessment for an engineering system, main difficulties arise from: (1) the relationship between the random variables, (2) too many random variables involved, (3) information about rare scenarios and (4) many interactive response variables in the description of performance criteria. Like other engineering systems, analysing of buried pipeline systems are characterised by a large number of degrees of freedom, time-varying and response dependent nonlinear behaviour. In the presence of uncertainty, the performance of an underground pipeline can be quantified in terms of ‘performance margin’ with respect to specified design objectives. In reliability engineering ‘performance margin’ is denoted as reliability index, probability of failure and safety margin etc. The failure events in the pipe failure analysis can be formulated as the exceedance of a critical response variable over some specified threshold level with an acceptable factor of safety. By predicting the pipeline reliability, the safe service life can be estimated with a view to prevent unexpected failure of underground pipelines by prioritising maintenance based on failure severity and system reliability [3], [4].

There is no general algorithm available to estimate the reliability for a buried pipeline system. The pipeline reliability is usually given by an integral over a high dimensional uncertain parameter space. Methods of reliability analysis such as first order reliability method (FORM), second-order reliability method (SORM), point estimate method (PEM) and Monte Carlo simulation (MCS), etc. are available in literature [5]–[7]. In this context, a robust uncertainty propagation method whose applicability is insensitive to complexity nature of the problem is most desirable. Many of the methods are inefficient when there is much number of random variables and failure probabilities are small. Furthermore, some need a large number of samples which is a time consuming procedure. Advanced Monte Carlo methods, often called ‘variance reduction techniques’ have been developed over the years. In this respect, a promising and robust approach is SS which is originally developed to solve the multidimensional problems of structural reliability analysis [8]. The structural systems fail when the applied load or stress level exceeds the capacity or the resistance. SS is well suited for the quantitative analysis of the functional failures systems, where the failures are specified in terms of one or more safety variables, e.g., temperatures, pressures, flow rates, etc. In the SS approach, the functional failure probability is expressed as a product of conditional probabilities of adaptive chosen intermediate events. For example, if a structure is assumed to be failed when the load is exceeded 400kN, then intermediate events could be represented by the load exceeding 250, 300 and 350kN, respectively. The problem of evaluating the small probabilities of functional failures is thus tackled by performing a sequence of simulations of more frequent events in their conditional probability spaces; then the necessary conditional samples are generated through successive Markov Chain Monte Carlo (MCMC) simulations in a way to gradually populate the intermediate conditional regions until the final functional failure region is reached [9].

This paper focuses on application of SS for computing time dependent reliability of flexible buried metal pipelines. Failure probabilities for corrosion induced failure of deflection, buckling, wall thrust and bending have been predicted in this paper. First, the SS is applied for estimating the failure

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probabilities for every failure case and then an upper and lower bounds of failure probabilities are predicted as a series system. Then probability of failure and corresponding coefficient of variations (COV) and a sensitivity analysis for pipe failure due to corrosion induced deflection, as an example of failure event has also been assessed to illustrate the robustness and effectiveness of SS method. All applications of SS method also have been assessed by MCS method. Finally, the applications of SS method are verified with respect to standard MCS.

II. Formulations for Prediction of Pipe Failure

For a buried pipe structure, the number of potential failure modes is very high for all systems failure definitions. This is true in spite of the simplifications imposed by assumptions such as having a finite number of failure elements at given points of the structure and only considering the proportional loadings. It is, therefore, important to have a method by which the most critical failure modes can be identified. The critical failure modes are those contributing significantly to the reliability of the system at the chosen level. The failure criterions are adopted here due to loss of structural strength of pipelines by external loadings and these failure criterions are influenced by corrosion through reduction of the pipe wall thickness. The chosen dominating failure modes are as below:

A. Metal Pipes Corrosion

Metal pipe corrosion pit is a continuous and variable process. Under certain environmental conditions, metal pipes can become corroded based on the properties of the pipe, soil, liquid properties and stray electric currents. The corrosion pit depth can be modelled with respect to time [10], [11] as (1). Due to reduction of wall thickness given by (1), the moment of inertia of pipe wall per unit length, I and the cross-sectional area of pipe wall per unit length, as defined by [12] can be modified as shown below.

The corrosion pit depth,

$$D_T = kT^n \tag{1}$$

where D_T is pit depth, T is exposure time and k and n are empirical constants.

Moment of inertia, $I = (t - D_T)^3 / 12$

Cross-sectional area, $A_S = t - D_T$

In this paper, failure criteria of flexible pipes are characterised by deflection, buckling, wall thrust and bending stress [13].

B. Deflection

The performance of flexible pipes in its ability to support load is typically assessed by measuring the deflection from its initial shape. Deflection is quantified in terms of the ratio of the horizontal (or vertical) increased diameter to the original pipe diameter. The actual deflection can be calculated as (2) [14]:

$$\Delta_y = \frac{K_b (D_L W_c + P_s) D}{\left(\frac{8EI}{D^3} + 0.061E' \right)} \tag{2}$$

where K_b is deflection coefficient, DL is deflection lag factor, D is mean diameter = D_i + 2c, Di is inside diameter and c is distance from inside diameter to neutral axis, E is modulus of elasticity of pipe material, I is moment of inertia per unit length and E' is modulus of soil reaction.

Soil load, $W_c = \gamma_s H$, and live load, $P_s = \frac{W_s I_f}{L_1 L_2}$.

where γs is unit weight of the soil, H= height of soil on the top of pipe, Ws is traffic load, I_f is impact factor = 1.1 for 0.6m < H < 0.9m, or 1 for H ≥ 0.9m, L_1 is load width parallel to direction of travel = 0.253 + 1.75H and L_2 is load width perpendicular to direction of travel = 0.51 + 1.75H for 0.6m < H < 0.76m, or (13.31 + 1.75H)/8 for H ≥ 0.76m [15].

C. Buckling

Buckling is a premature failure in which the structure becomes unstable at a stress level that is well below the yield strength of the structural material [5]. The actual buckling pressure should be less than the critical buckling pressure for the safety of structure. The actual buckling pressure, p_a and the critical buckling pressure, P_{cr} can be calculated as below [16]:

$$p = R_W \gamma_S + \gamma_W H_W + P_s \tag{3}$$

$$p_{cr} = \frac{1}{FS} \sqrt{\left(32 R_W B' E' \frac{EI}{D^3} \right)} \tag{4}$$

where R_w is water buoyancy factor = 1 – 0.33 (H_w/H), γ_w is unit weight of water, H_w is height of groundwater above the pipe. B' is empirical coefficient of elastic support = $1/(1+4e^{-0.213H})$, *FS* is design safety factor for buckling.

D. Wall Thrust

Wall thrust or stress on the pipe wall is determined by the total load on the pipe including soil loads, traffic loads and hydrostatic loads. Two wall thrust analyses are required: (1) accounts both the dead load and live load and employs the short term material properties throughout the procedure, (2) accounts only the dead load and employs the long-term material properties throughout the process. Then, the most limiting value is used for wall thrust analysis [17].

The critical wall thrust, $T_{cr} = F_y A_s \phi_p = F_y (t - D_T) \phi_p$ (5)

where F_y is the minimum tensile strength of pipe, A_s is cross-sectional area of pipe wall per unit length and ϕ_p is capacity modification factor for pipe.

The actual wall thrust,

$$T_a = 1.3(1.5W_A + 1.67P_S C_L + P_w)(D_0 / 2) \quad (6)$$

where soil arch load,

$$W_A = P_{sp} VAF$$

The geostatic load,

$$P_{sp} = \gamma_s (H + 0.11 \times 10^{-7} (D_0))$$

The vertical arching factor,

$$VAF = 0.76 - 0.71((S_h - 1.17)/(S_h + 2.92))$$

E. Bending Stress and Strain

Under the effect of earth and surface loads, the buried pipe may bend through pipe wall. The allowable strain for flexible pipes is 0.15% to 2% [18]. The bending stress and strain are important to ensure that these are within material capability. Bending stress and strain can be calculated as (7) and (8) [19]:

Bending stress,

$$\sigma_b = 2D_f E\Delta_y y_0 SF_b / D^2 \quad (7)$$

Bending strain,

$$\varepsilon_b = 2D_f \Delta_y y_0 SF_b / D^2 \quad (8)$$

where D_f is shape factor, y_0 is distance from centroid of pipe wall to the furthest surface of the pipe = the greater of (Do – D)/2 or (D – Di)/2; SF_b = Safety factor for bending.

III. Reliability Prediction

A. Basic Equations of SS

SS is an adaptive stochastic simulation procedure for efficiently computing a small failure probability. The basic idea of subset simulation is shown in Fig. 1 for a two-dimensional case. For simplification, F is denoted as the failure event as well as its corresponding failure region in the uncertain parameters space. Given a failure event F, let $F_1 \supset F_2 \supset F_3 .... \supset F_m = F$. If the failure of a system is defined as an exceedance of on uncertain demand P over a given capacity Q, that is $F = (P > Q)$ then a sequence of decreasing failure events can simply be defined as $F_i = \{P > Q_i\}$ where $Q_1 < Q_2 < Q_3 ..... < Q_m = Q$ where i= 1, 2, 3,..., m; m = number of conditional events. This enables the computation of the failure probability as a product of conditional probabilities $P(F_{i+1} \mid F_i)$ and $P(F_i)$. A conceptual illustration of the SS method is presented in Fig. 1.

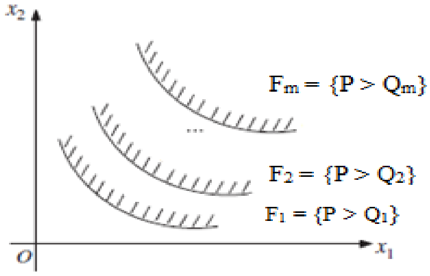

Fig. 1 Illustration of failure events in SS method

The probability of failure (P_f) can be calculated based on the above sequence of failure domains (or subsets) as follows [8]:

$$\begin{aligned} P_f &= P(F_m) = P(F_m|F_{m-1})P(F_{m-1}|F_{m-2}).......P(F_2|F_1) \\ &= P(F_1)\prod_{i=1}^{m-1} P(F_{i+1}|F_i) \end{aligned} \quad (9)$$

It is natural to compute the conditional failure probabilities based on an estimator similar to (10), which requires the simulation of samples according to the conditional distribution θ given that lies in F_i [8]. Direct MCS can be used to estimate $P(F_1)$. Computing the conditional probabilities in (12), MCMC simulation provides a powerful method for generating samples conditional on the failure region F_i , i = 1, 2, ..., (m – 1) [8], [20]. In the first step, the probability $P_1 = P(F_1)$ can be determined by application of the direct MCS simulation as (10).

$$P(F_1) \approx \tilde{P} = \frac{1}{N}\sum_{k=1}^{N} I_{F_1}(\theta_k) \quad (10)$$

With the application of the MCMC simulation by modified the Metropolis-Hastings algorithm samples may be generated in a numerically efficient way as (11).

$$P(F_{i+1}|F_i) \approx \tilde{P}i{+}1 = \frac{1}{N}\prod_{k=1}^{N} I_{F_{i+1}}(\theta_k^{(i)}) \quad (11)$$

Based on (10) and (11), (9) can be rewritten as below:

$$P_f = \frac{1}{N}\sum_{k=1}^{N} I_{F_1}(\theta_k)\frac{1}{N}\prod_{k=1}^{N} I_{F_{i+1}}(\theta_k^{(i)}) \quad (12)$$

On the basis of reliability analysis of SS, the failure probability P_f can be transformed in to a set of conditional failure probabilities P_i (i =$1, 2, 3 ... m$). Based on (12), the partial derivative of the failure probability with respect to the distributional parameter α (the mean μ_{xi} or the standard deviation σ_{xi}) of normal random variables x_i can be obtained, which is called reliability sensitivity by (13) [21]:

$$\frac{\partial P_f}{\partial \alpha} = \sum_{i=1}^{m} \frac{P_f}{P_i} \frac{\partial P_i}{\partial \alpha} \tag{13}$$

Reliability sensitivity analysis can reflect the significance of the distributional parameter (μ and σ) with respect to the failure probability.

B. Procedures

The SS procedures can be expressed as a product of larger conditional failure probabilities for a sequence of intermediate failure events, thereby converting a rare event simulation problem into a sequence of more frequent ones [22]. During the simulation process, the conditional samples are generated from specially designed Markov chains, so that they gradually populate each intermediate failure region until they reach the final target failure region [8].

The SS procedure for adaptively generating samples of corresponding to specified target probabilities is summarized as follows. In this work, the intermediate threshold values are chosen adaptively in such a way that the estimated conditional probabilities are equal to a fixed value $p_0 = 0.1$ has been used, according to [8], [9], [20].

Procedure of SS algorithm can be summarised as below:

1: Generate N samples $\theta_{1,\,k}$: $k = 1, 2, ..., N$ of the probability density function (PDF) $f(\cdot)$ by direct MCS;
2: Compute the corresponding response values $P(\theta_{1,\,k})$;
3: Choose the first threshold value Q_1 as the p_0N^{th} position in the descending list of the response values (note that it is implicitly assumed that p_0N is an integer value);
4: Define the first intermediate failure level as $F_1 = \{\theta: P(\theta) > Q_1\}$. The failure probability $P_1 = P(F_1)$ can be estimated by $P_1 = p_0$;
5: From these initial p_0N samples that lie in F_1, use MCMC algorithm to generate $N(1-p_0)$ additional conditional samples so that there are a total of N samples in the i^{th} level;
6: Compute the corresponding response values $P(\theta_{i,k})$;
7: Choose the i^{th} threshold value Q_i as the p_0N^{th} position in the descending list of the response values;
8: Define the i^{th} intermediate failure level as $F_i = \{\theta: P(\theta) > Q_i\}$. The conditional failure probability $P_i = P(F_i \mid F_{i-1}$ can be estimated by $\widetilde{P}_i = p_0$.
9: Repeat for higher conditional levels until $Q_i > Q$. The conditional failure probability $P_m = P(F_m \mid F_{m-1})$ can be estimated by where N_f is the number of samples that lie in the target failure event F_m;
10: The target failure probability can be estimated by

$$\tilde{P} = p_0^{\,m-1}\left(\frac{N_f}{N}\right)$$

11: Return P_f

Note that the total number of samples employed is

$N_T = N + (m - 1)(1 - p_0)N.$

IV. Numerical Illustration

The structural time dependent reliability for an underground flexible metal pipe has been predicted in this example, where pipe failure probability, sensitivity and COV analysis are conducted by applying SS. Calculations are presented for a steel buried pipe under a heavy roadway subject to corrosion and external loadings conditions. A typical pipe section is shown in Fig. 2. Numerical values are based on practice and have been obtained from the literature [11], [23].

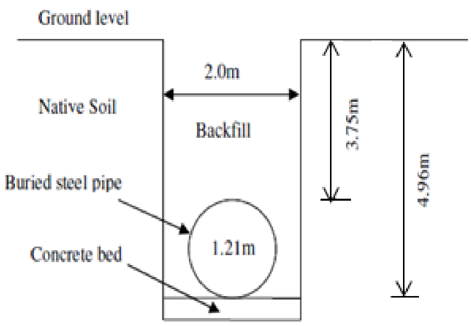

Fig. 2 Geometrical details of the buried flexible metal pipe section (not to scale)

The materials properties and parameters are listed in Table I. There are 9 random variables where the means and COVs are listed in Table II. The pipe corrosion rate is modelled using (1). Assuming the changing of pipe wall surface due to corrosion is uniform over the entire surface area.

TABLE I
Materials Properties and Parameters

Symbol description	Numerical value
Buoyancy factor, R_w	1.00
Trench width, B_d	2.00 m
Outside pipe diameter, D_o	1.231 m
Inside pipe diameter, D_i	1.189 m
Soil constrained modulus, M_s	2.02 MPa
x-sectional area of pipe wall/ unit length, Ap	0.021 sq. m/m
Shape factor, Df	4.0
Capacity modification factor for pipe, ϕ_p	1.00
Capacity modification factor for pipe, ϕ_s	0.90
Factor of safety for Bending	1.5
Factor of safety for Buckling	2.5
Poisson ratio	0.3
Allowable strain	0.2%
Tensile strength of pipe	450 MPa
Buoyancy factor, R_w	1.00

TABLE II
STATISTICAL PROPERTIES OF RANDOM VARIABLE

Material properties	Mean	COV (%)	Standard Deviation (σ)
Elastic modulus of pipe, E	213.74×10^6 kPa	1.0	2.1374×10^6kPa
Backfill soil modulus, E_s	1000 kPa	5.0	50 kPa
Unit of weight of soil, γ	18.0kPa	2.5	0.45 kPa
Wheel load (Live load), P_s	80.0 kPa	3.0	2.4 kPa
Deflection coefficient, K_b	0.11	1.0	0.0011
Multiplying constant, k	2.0	10.0	0.1
Exponential constant, n	0.3	5.0	0.015
Thickness of pipe, t	0.021 m	1.0	0.00021 m
Height of the backfill, H	3.75 m	1.0	0.00375 m

V. RESULTS AND DISCUSSION

The failure probabilities for every failure case, COVs with respect to failure probability and sensitivity over time (corrosion induced deflection failure as an example) have been predicted using the MATLAB software. In SS method, the failure probability (P_f) for every failure mode is predicted as a sum of the sub failure events within each failure event. For example, in case of failure due to corrosion induced buckling, at time, $T = 1$ year, the failure event is subdivided as 400 events and then the sum of the 400 sub failure events produced the final P_f at that year.

In case of underground pipes, the assessment of probability of failure on year basis is useful which enables calculation of reliability over time. The probabilities of failure for corrosion induced excessive deflection, bucklin g, wall thrust and bending stress with respect to time have been estimated using the material properties and random variables which are presented in Tables I and II. The study reveals that excessive bending stress is the most critical failure event whereas buckling has the lowest probability of failure during the whole service life of the pipe. Considering the failure probability of 0.1 (10%) as a threshold level for the safe service life [4], the study illustrates that the safe service life in the worst case scenario is less than 50 years. According to the example data, the correlations between failures events are found within range 0 to + 1. So, applying the theory of systems reliability, the probability of failure for a series system, $P_{f,s}$ is estimated by (14) [24].

$$Max[P_{f,j}] \le P_{f,s} \le 1 - \prod_{j=1}^{r}[1 - P_{f,j}] \tag{14}$$

where P_{fj} is the probability of failure due to j^{th} failure mode of pipe and r is the number of failure modes considered in the system.

In Figs. 4 – 8 total number of samples, N for MCS = 10^6 is considered in all the failure cases (corrosion induced deflection, buckling, wall stress and bending), where SS is applied with a conditional failure probability at each level equal to $p_0 = 0.1$. It is found that the sample numbers less than 500 do not provide precise results as shown in Fig. 3 (pipe failure due to corrosion induced bending, as an example), where number of samples, N more than 200 and less than 500, the graph is not worthy enough to predict the failure probability accurately. The study also illustrates that for SS method N = 500 or more than 500 gives the more precise results as shown in Figs. 4 – 8, which are very small compare to MCS method, indicates the supremacy and accurateness of the current SS method. The expected value of P_f for series system is determined in between upper and lower values of failure probability curve as shown in Fig. 8. The number of conditional levels is chosen to cover the required response level whose failure probability is estimated.

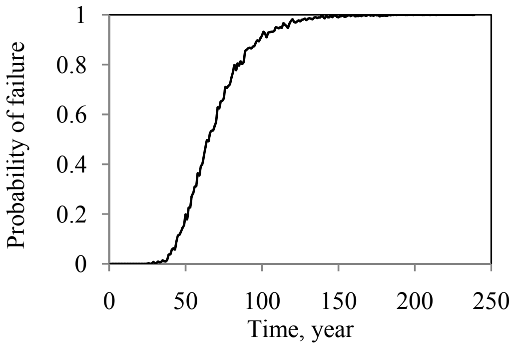

Fig. 3 P_f in bending stress/strain vs. time (N < 500)

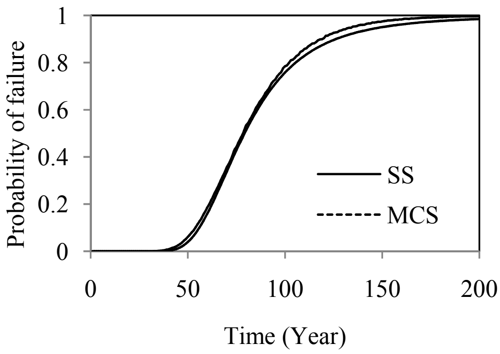

Fig. 4 P_f due to deflection vs. time

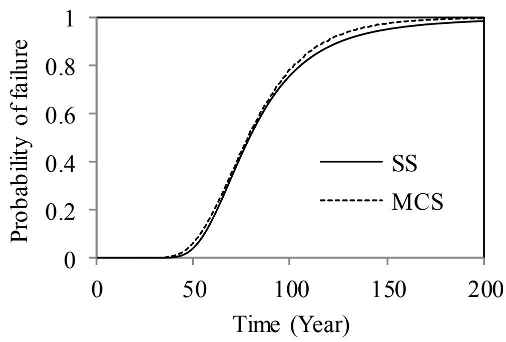

Fig. 5 P_f due to buckling vs. time

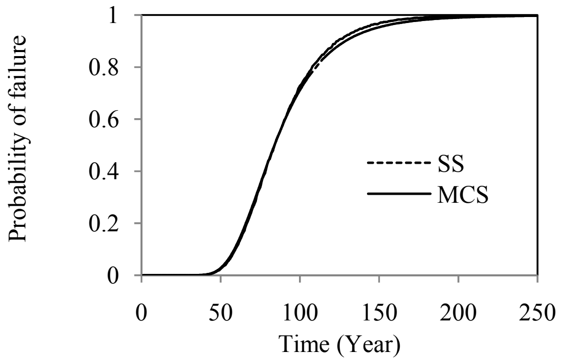

Fig. 6 P_f for wall thrust vs. time

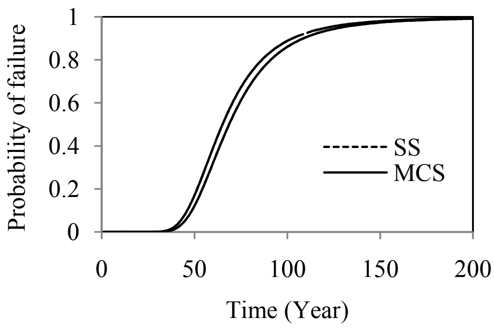

Fig. 7 P_f in bending stress/strain vs. time

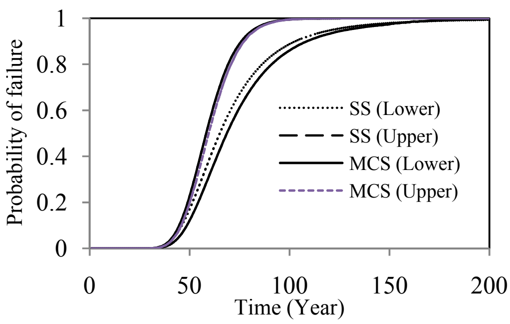

Fig. 8 P_f in series system vs. time

Next, the COV at particular failure event of corrosion induced deflection (as an example) using the same total number of samples is plotted in Fig. 9. The results indicate that pipe service life from installation to 40 years, the probability of failure is insignificant (both lower and upper bounds) (Fig. 8). Therefore, the calculations are performed for the failure probability and corresponding COV for corrosion induced deflection failure at 50 years of service life (as a representative of the significant probability of failure service life time) using 30 independent runs with 500 samples. Thus the sample average value and COV of failure probability have been obtained, where some points are deleted for clarification (Fig. 9). The results show that COV achieved by SS and MCS are approximately same in the large probability region. The result also shows that the values of COV for SS and MCS coincide at $P_f = 0.1$, since according to the SS procedure with $p_0 = 0.1$, this probability is computed based on an initial MCS. While the COV for SS grows approximately in a linear fashion with decreasing failure probabilities, the COV for MCS grows exponentially. Thus, SS becomes more and more efficient compared with MCS as the target probability of failure gets smaller.

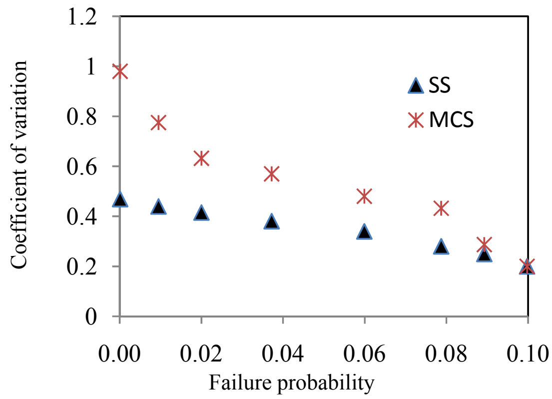

Fig. 9 Failure probability versus COV for pipe failure due to corrosion induced deflection at 50 years of service life

Finally, two examples are presented below to demonstrate the application of SS method in pipe reliability sensitivity estimation. The results are shown in Figs. 10 and 11. For empirical corrosion constants (multiplying constant, k and exponential constant, n) as can be seen in (1) are considered as the dominating influencing parameters in pipe reliability due to aforementioned failure event to demonstrate the computational analysis of the presented reliability sensitivity based on SS technique. It is found that the reliability sensitivity results calculated by the presented methods are in good agreement with the MCS results, while the efficiency, which is indicated by the sampling size, of the SS method is higher than that of MCS.

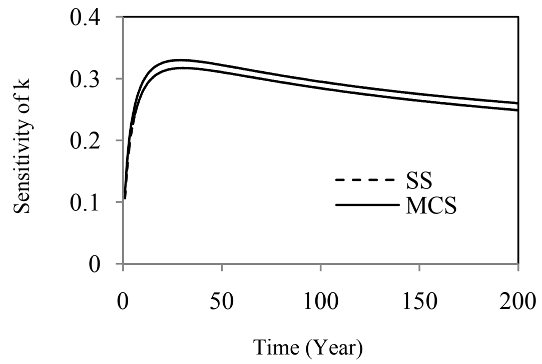

Fig. 10 Sensitivity of multiplying constant (k) for corrosion induced deflection during pipe service life

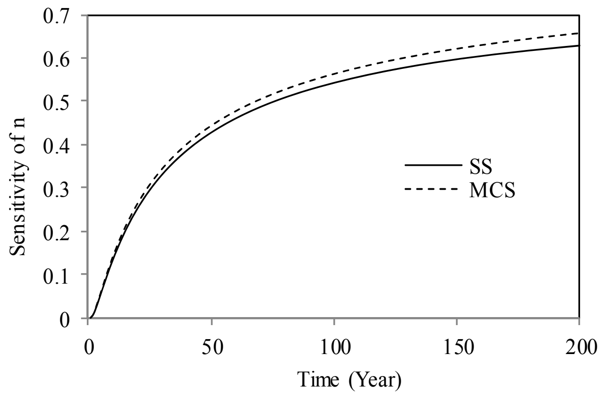

Fig. 11 Sensitivity of exponential constant (n) for corrosion induced deflection during pipe service life

The study shows that at the early stage of the pipe service life, the impact of multiplying constant (k) has about 10% contribution and sharply increases to 30% about 25 years of the pipe life and then this contribution slightly decreases over time. The sensitivity of the exponential constant (n) has zero impact and increases significantly with the pipe age for all the four failure criteria as shown in Figs. 10 and 11. This is attributed to the fact that corrosion does not cause any problem to new pipes but mainly the root cause of failure and collapse for aging and external loadings on pipes. .

VI. Conclusions

A subset simulation approach is presented for time-dependent reliability analysis for buried pipeline system subject to corrosion induced different failures modes. The analysis shows that this method is robust to the choice of the intermediate failure events. One of the major complications to estimating small failure probabilities is to simulate rare events. SS resolves this by breaking the problem into the estimation of a sequence of larger conditional probabilities. It is quite clear that the above procedure can be extended to any random dimension in a trivial way. In general, the accuracy of this SS method depends on amendment factors such as the choice of m, N, p parameters and the proposal distribution of the system. The analysis shows that behaviour of buried pipes is considerably influenced by uncertainties due to external loads, corrosion parameters, pipe materials and surrounding soil properties etc. where excessive bending stress is the most critical failure event whereas buckling is the least susceptible during the whole service life of the pipe. The estimation of failure probability may then be utilised to predict the maintenance and repair options during expected service life time and hence, a renewal strategy can be applied to avoid unexpected collapse or failure of the pipe network.

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